Arithmetic

OR LOWER
ECONDARY SCHOOLS

G. K. GOKHALE, B. A

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ARITHMETIC FOR LOWER SECONDARY SCHOOLS.

ARITHMETIC

FOR

LOWER SECONDARY SCHOOL

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PREFACE.

THIS book has been specially written for the lower classes of Indian High Schools, for whom my "Arithmetic for High Schools" is found to be too difficult.

G. K. GOKHALE.

FERGUSSON COLLEGE,
POONA, *April*, 1902.

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ARITHMETIC.

CHAPTER I.

NOTATION AND NUMERATION.

1. A *unit* denotes a single object or thing; as, a boy, a rupee, a foot.

A unit is also called *unity*.

2. A *number* shows how many times a unit is taken.

3. *Arithmetic* is the science of numbers.

4. Numbers are divided into *abstract* and *concrete*.

5. A number is said to be *abstract* when it is taken without any relation to particular objects; as, two, five, seven, eight.

6. A number is said to be *concrete* when it is considered as belonging to a particular kind; as, five mangoes, six boys, ten feet.

7. *Numeration* is the art of expressing numbers by sounds or words.

8. *Notation* is the art of denoting numbers by symbols or figures.

9. The unit taken by itself is called *one*. Then follow

Two (One and one)
Three (Two and one)
Four (Three and one)
Five (Four and one)
Six (Five and one)
Seven (Six and one)
Eight (Seven and one)
Nine (Eight and one)

The next number, *i.e.* nine and one, is called *ten*. It is considered to be the unit of a new kind, and is termed the *unit of the second order*, as distinguished from the *simple* unit, which is called *unit of the first order*. By counting by tens as we before counted by one, we have one ten, two tens, three tens, . . . nine tens, or as they are briefly called twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

There are nine numbers between ten and twenty, and their names are eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen. The nine numbers between twenty and thirty, thirty and forty, . . . eighty and ninety, as also the nine numbers that follow ninety are expressed by placing in order the names of the first nine numbers after twenty, thirty, . . . ninety; thus we get twenty-one, twenty-two, . . . ninety-nine.

The number which follows ninety-nine, *i.e.* ninety-nine and one, or ten tens, is called *hundred*, and is considered to be the *unit of the third order*. By counting by hundreds as we counted by simple units we get:

One hundred, two hundreds (or better, two hundred), . . . nine hundred.

The ninety-nine numbers between one hundred and two hundred, two hundred and three hundred, . . . eight hundred and nine hundred, as also the ninety-nine numbers that follow nine hundred, are expressed by placing in order the names of the first ninety-nine numbers after one hundred, . . . nine hundred. Thus we get one hundred and one, . . . nine hundred and ninety-nine.

The number which follows nine hundred and ninety-nine, *i.e.* nine hundred and ninety-nine and one, or ten hundred, is called *thousand*, and is considered to be *unit of the fourth order*. Proceeding as before we get *ten thousand* as the *unit of the fifth order*, a *hundred thousand* as the *unit of the sixth order* (there are no independent names for these units), and a *thousand thousands* as the *unit of the seventh order*. This last unit has an independent name and is called a million; and proceeding in this way far enough we get a *million millions*, the *unit of the thirteenth order*. This unit is called a *billion*. Proceeding again far enough we get a *million* *billions*, which is called a *trillion*, a *million trillions*, which is called *quadrillion*, and so on. In this way the nomenclature of all imaginable numbers can be formed. We give below the numeration table by way of recapitulation :

etc.	Tens of Billions.	Billions.	Hundreds of T. of Millions.	Tens of T. of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.

10. After determining a set of words to express numbers the next thing we want is a set of symbols or figures to denote them in writing. The first nine numbers are represented by independent symbols, which are 1, 2, 3, 4, 5, 6, 7, 8, 9. These nine symbols are called *significant* symbols because they have values of their own, which are called intrinsic values as distinguished from their *local* or *place* values, which depend upon the position they occupy in expressing a number. There is one more independent symbol also used. It is called zero, cipher, or nought, and is written as 0. This symbol does not represent any number and therefore has no value of its own. It is used only to determine the positions and therefore the local values of the significant symbols to which it may be affixed.

These ten symbols, which are also called *digits*, enable us to denote any number in writing. For instance, let it be required to represent the number twenty-seven. Since this number consists of two tens and seven units, it would be represented as ‘ 27.’ Again, let it be required to represent the number two hundred and forty-seven. Since this number consists of two hundreds, four tens and seven, it would be represented as ‘ 247.’ Thus, to express in figures a number which is given in words, we should find the number of units of each order in that number and then write down the corresponding figures in a line beginning with the highest order, after which should come the next highest and so on, writing from left to right and placing a cipher where there are no units of any order.

Ex. 1. Express in figures six thousand five hundred and eighty-five.

This number consists of 6 thousands, 5 hundreds, 8 tens, and 5 simple units. Therefore *it should be written as 6585.*

It will be seen that the value of the first 5 in the number expressed above is not the same as that of the second 5. Though the intrinsic values of both are the same, viz. 5 units, the units which they represent are of different orders; the *first* 5 representing hundreds and being a unit of the third order, while the second 5 representing simple units and being a unit of the first order. The *local* or *place* value of the first 5 is therefore five hundred, whereas that of the second 5 is only five units.

Ex. 2. Express in figures four thousand and seventy-one.

This number consists of 4 thousands, no hundreds, 7 tens, and one simple unit. Therefore it is represented by 4071.

Ex. 3. Express in figures thirty-seven thousand three hundred and seventy-two.

This number consists of 3 tens of thousands, 7 simple thousands, *i.e.* of 37 thousands, 3 hundreds, 7 tens, and 2 simple units. Therefore it should be expressed as 37372.

Ex. 4. Express in figures two hundred and forty-five thousand eight hundred and three.

This number consists of 2 hundreds of thousands, 4 tens of thousands, and 5 thousands, *i.e.* of 245 thousands, 3 hundreds, no tens, and 3 simple units. Therefore it should be expressed as 245803.

Ex. 5. Express in figures two million one hundred thousand and four hundred.

This number consists of 2 millions, 100 thousands, 4 hundreds, no tens, and no simple units. Therefore it should be written as 2100400.

Ex. 6. Express in figures two hundred and five million thirty-four thousand and five.

This number consists of two hundreds of millions, no tens of millions, and 5 simple millions, *i.e.* of 205 millions; no hundreds of thousands, 3 tens of thousands, and 4 simple thousands, *i.e.* of 34 thousands; no hundreds; no tens, and 5 simple units.

Therefore it is expressed as 205034005.

11. Similarly any number which is represented in figures may easily be expressed in words. For instance, let it be required to read the number 374. Since the figures represent units of different orders in succession, beginning with the highest, 3 represents 3 hundreds, 7, seven tens, and 4, four simple units; and therefore the number may readily be read as 'three hundred and seventy-four.' The number 4583 may similarly be read as four thousand five hundred and eighty-three; the number 281475 as two hundred and eighty-one thousand four hundred and seventy-five; the number 4579823 as four millions 579 (*i.e.* five hundred and seventy-nine) thousand eight hundred and twenty-three; and the number 14,605283,279478 as fourteen billions 605283 (*i.e.* six hundred and five thousand two hundred and eighty-three) millions, 279 (*i.e.* two hundred and seventy-nine) thousand, four hundred and seventy-eight.

12. When a number consists of more than 6 digits it is convenient to mark off by a comma the first three digits to the right, then the next three, and then, if the number of digits be more than twelve, the next six; and if more than eighteen, the next six, and so on; the number should then be read from left to right beginning with the highest order of units.

The system of numeration and notation explained above is called the *decimal* or *denary* system, because it takes *ten* units of any particular order to form *one* unit of the next higher order.

EXERCISE I.

Express in words:

1. 5. **2.** 25. **3.** 125. **4.** 1250. **5.** 10205.
6. 50. **7.** 100. **8.** 2000. **9.** 30000. **10.** 400000.

11. What is the place-value of the digit 5 in each of the following numbers:

75, 57, 105, 510, 5010, 105000.

Express in figures:

12. Two hundred and thirty-four.

13. Three hundred and twenty-four.

Express in figures :

14. Four thousand and thirty-two.

15. Three thousand four hundred and twenty.

Write down in figures :

16. Ten thousand nine hundred and eighty-six.

17. Two hundred thousand four hundred and seven.

18. Four hundred and sixty-eight thousand nine hundred and thirty-five.

19. One million two thousand five hundred and seventy-eight.

20. Twelve million thirteen thousand and fourteen.

Write down in words :

21. 435. **22.** 9676. **23.** 12349. **24.** 60705.
25. 43216. **26.** 248532. **27.** 4372180. **28.** 6020075.
29. 1001. **30.** 30402. **31.** 7600060. **32.** 3020106809.

33. Express in figures the greatest number of three digits and the least number of four digits.

Express in figures :

34. Thirty-three tens.

35. Four hundred tens.

36. Eight thousand hundreds.

37. Ten thousand thousands.

38. Eight hundred and nine hundreds.

39. Express in figures the greatest and the least numbers of eight digits.

Write down in words :

40. 357914719. **41.** 123456789. **42.** 2003000400005.
43. 6402100706070. **44.** 1020300450060008007.

Write down in figures :

45. One million two thousand and three hundred.

46. Four thousand and five million eight hundred and fifty thousand and seventy-five.

47. Three hundred and fifty-one thousand six hundred and nine million forty thousand and eight.

48. Five trillion four billion three million one thousand and one.

49. Express in words the number which consists of three significant figures 4, 6, and 8 (from the left) each being followed by a zero.

50. A certain number of 7 digits has the figure 1 in the place of hundreds, 3 in that of millions, 2 in that of units and 8 in that of ten thousands. Express the number in figures and in words.

CHAPTER II.

THE FUNDAMENTAL OPERATIONS.

13. Addition, Subtraction, Multiplication, and Division are called the first four fundamental operations or Simple Rules in Arithmetic.

ADDITION.

14. *Addition* is the operation of finding a single number equal to two or more given numbers taken together. This single number is called the *sum* of the original numbers, which are called the *summands.*

15. To find the sum of two or more numbers :

Rule. Write down the numbers to be added under one another so that the units of the same order may be in the same vertical column, *i.e.* units under units, tens under tens, hundreds under hundreds, and so on, and draw a line underneath. Add together the simple units of all the numbers ; if the sum be nine or less, set it down under the same column. But if it be greater than nine, write the units' figure of the sum under the units' column and carry the tens to the next column. Add together the figure thus carried and the digits in the tens' column, and set down and carry as before ; and proceed in this way till the figures in all the columns are exhausted, always bearing in mind that when the sum of the units of any order exceeds 9 the tens of that order have to be carried to the next higher order as units and to be added to the latter.

Ex. 1. Add together 254, 321 and 412.

$$\begin{array}{r}254\\321\\412\\\hline 987\end{array}$$

Write down the numbers under one another as directed in the rule. The process of adding is as follows.

Add up the figures in the units' column, thus : 2 and 1 are 3 (see Addition Table at the end of the chapter), 3 and 4 are 7 ; set down 7 below the units' column. Now add up the figures in the tens' column, thus : 1 and 2 are 3, 3 and 5 are 8 ; set down 8 below the tens' column. Lastly, add up the figures in the hundreds' column, thus : 4 and 3 are 7, 7 and 2 are 9 ; set down 9 below the hundreds' column. Therefore the required **sum is 987.**

Ex. 2. Add together 45018, 7203, 32472, and 765.

45018
7203
32472
765
85458

Write down the numbers as above. The process of adding is as follows:

5 and 2 are 7, 7 and 3 are 10, 10 and 8 are 18. Set down 8 and carry 1 to the tens' column. 1 and 6, 7, and 7, 14, and 1, 15. Set down 5 and carry 1. 1 and 7, 8, and 4, 12, and 2, 14. Set down 4 and carry 1. 1 and 2, 3, and 7, 10, and 5, 15. Set down 5 and carry 1. 1 and 3, 4, and 4, 8. Therefore the required sum is 85458.

16. The truth of a result in Addition may be tested thus: If in adding the given numbers you have begun at the bottom and added upwards, for testing the correctness of your result begin at the top and add downwards, observing in other respects the same rule as before. If the result in the second case is the same as in the first, the work is correct. If in the original process you have added downwards, in the process for testing proceed upwards.

EXERCISE II.

Add together:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
76	536	628	480	46576	21324	37451
32	342	451	602	52213	65273	90147

8.	**9.**	**10.**	**11.**	**12.**	**13.**
43672	68543	13579	752108	253762	48126
65437	12345	76852	687950	846895	27658
21253	98765	89364	908064	784319	75293
			321569	469785	19657

14.	**15.**	**16.**	**17.**
573945	384762	51326898	954267387
46731	153479	27961842	60458892
540918	986456	89437536	1084910638
23074	261834	15657818	45903
674596	394095	41761827	36156690
		407219	173468585
		6208	286548112
		73458361	3819751458

18. Add together 3612, 84, 105, 5829, and 321.

19. Find the sum of 2369, 184, 207, and 46.

20. Find the sum of 1008, 36, 5400, 90, and 108.

21. Add together 1218, 301, 75, 4050, and 406.

22. Add together seventy million eighty thousand seven hundred and nine, five million and eight, nine hundred and eighty-five thousand six hundred and thirty, ninety-three million seven hundred and four thousand six hundred and twenty-seven, sixty thousand eight hundred and thirty-one and two hundred and six thousand and one million seven hundred thousand four hundred and fifty.

23. Find the sum of five hundred thousand and sixty-one, ten million eight thousand three hundred and fifty-four, three thousand two hundred million and thirty-one, forty-seven million nine thousand and thirty-eight, one million eighty thousand two hundred and ninety-seven, and eight hundred and five thousand three hundred and sixty.

24. Add together nine thousand and forty-one, ninety thousand and seven hundred, nine hundred thousand and two hundred, and nine million and fifty-nine, and write the answer in words.

25. Add together four thousand and three, eighty thousand and twenty, seven hundred and six, five thousand and seventy, and nine hundred and ten thousand and two hundred, and express the answer in words.

ADDITION TABLE.

4	2 and 2			
5	2 and 3			
6	2 and 4	3 and 3		
7	2 and 5	3 and 4		
8	2 and 6	3 and 5	4 and 4	
9	2 and 7	3 and 6	4 and 5	
10	2 and 8	3 and 7	4 and 6	5 and 5
11	2 and 9	3 and 8	4 and 7	5 and 6
12	3 and 9	4 and 8	5 and 7	6 and 6
13	4 and 9	5 and 8	6 and 7	
14	5 and 9	6 and 8	7 and 7	
15	6 and 9	7 and 8		
16	7 and 9	8 and 8		
17	8 and 9			
18	9 and 9			

SUBTRACTION.

17. *Subtraction* is the operation of finding what number remains when a smaller number is taken from a greater. The greater number is called the *minuend*, the smaller one, the *subtrahend*, and the number left, the *remainder*.

18. To subtract one number from another :

Rule. Write the smaller number under the greater so that the units of the same order may be under one another and draw a line below. Beginning at the units' column subtract each figure of the smaller number from the one above it in the greater and place the remainder immediately below.

If any figure in the smaller number be greater than the figure above it, add ten to the latter and subtract from the sum the former ; then add one to next figure of the smaller number.

Ex. 1. Subtract 7432 from 8796.

$$\begin{array}{r} 8796 \\ 7432 \\ \hline 1364 \end{array}$$

Write the smaller number under the greater. The process then is as follows :

2 from 6 leaves 4 ; set down 4 in the units' place ; 3 from 9 leaves 6 ; set down 6 in the tens' place ; 4 from 7 leaves 3 ; set down 3 in the hundreds' place ; 7 from 8 leaves 1 ; set down 1 in the thousands' place.

The result is therefore 1364.

Ex. 2. Subtract 23872 from 34725.

$$\begin{array}{r} 34725 \\ 23872 \\ \hline 10853 \end{array}$$

Writing down the smaller number under the greater, we have to proceed as follows :

2 from 5 leaves 3 ; set down 3 in the units' place. Since 7 is greater than 2, add 10 to 2, which thus becomes 12. Take 7 from 12. This leaves 5 ; set down 5 in the tens' place and carry 1, *i.e.* add 1 to 8.

1 and 8 are 9. Since 9 is greater than 7, add 10 to 7 ; then 9 from 17 leaves 8 ; set down 8 in the hundreds' place, and carry 1. 1 and 3 are 4, 4 from 4 leaves 0 ; set down 0 in the thousands' place. 2 from 3 leaves 1 ; set down 1 in the place of tens of thousands. The result therefore is 10853.

19. The accuracy of the result in Subtraction may be tested thus :

Add the smaller number and the remainder together ; if the sum be equal to the greater number, the work is correct.

EXERCISE III.

Perform the following subtractions:

1. 87	2. 654	3. 584	4. 7895	5. 34185
65	321	302	6470	28092
6. 81645	7. 10263	8. 435127	9. 588501	10. 302010
60728	9187	381493	397074	270605

11. 45278	12. 324356	13. 290428	14. 394567
19484	235679	180934	289701

15. Subtract 1405192 from 4038001.

16. From 76892431 take 68945973.

17. Find the difference between 1020304 and 250819.

18. Subtract 2057312 from 5287201 and 2057312 from the remainder.

19. If five hundred and forty thousand and sixty-eight be subtracted from seventeen million one hundred thousand and seventy-one, what remains ?

20. How much does one million one thousand and one exceed one hundred thousand one hundred and ten ?

21. Find the difference between four thousand and fifty and five thousand and forty.

22. A boy wrote 6005 when he was asked to write six hundred and five, how much more or less than required did he write ?

23. By how much does the first 4 exceed the second four in the number 647045 ?

24. Find the difference between the values of 7 and 3 in the number 837.

25. Which is greater, two hundred thousand and three hundred or one million and two hundred, and by how much ?

PROBLEMS IN ADDITION AND SUBTRACTION.

Ex. 1. A boy had 12 mangoes, 18 plantains, and 24 oranges ; his brother had 20 plantains, 16 mangoes, and 6 oranges ; and his sister had 9 oranges, 14 plantains, and 15 mangoes ; find how many fruits there were in all, and how many of each kind ?

	mangoes.	plantains.	oranges.
	12	18	24
	16	20	6
	15	14	9
Total	43	52	39

And by adding 43, 52, and 39 together we get the total number of fruits. Thus

$$\begin{array}{r}43\\52\\39\\\hline 134\end{array}$$

Hence the total number of fruits is 134.
„ „ mangoes is 43.
„ „ plantains is 52.
And „ „ oranges is 39.

Ex. 2. A man has 4809 rupees; how many rupees must he borrow from his friend in order to be able to buy a house worth 6870 rupees?

The question comes to this: The sum of two numbers is 6870, and one of them is 4809; find the other.

Subtract 4809 from 6870.

$$\begin{array}{r}6870\\4809\\\hline 2061\end{array}$$

Therefore 2061 rupees is the answer.

Ex. 3. In a school of 401 pupils there were 89 girls and 97 infants; how many boys were there?

The *number of pupils is equal to the sum of the numbers of boys, girls, and infants.* Therefore we must subtract from 401 the sum of 89 and 97. Thus

$$\begin{array}{rr}\text{Addition.} & \text{Subtraction.}\\ 89 & 401\\ 97 & 186\\ \cline{1-2} 186 & 215\end{array}$$

Hence 215 boys is the answer.

EXERCISE IV.

1. In a school there are five classes consisting of 29, 23, 31, 34, 19 boys respectively; if 4 more boys be admitted into each class, find the total number of boys in the school.

2. *A* had 95 sheep, 7 cows, and 10 horses; *B* had 19 sheep, 32 horses, and 21 cows; and *C* had 41 horses, 16 cows, and 147 sheep: find how many animals there were in all, and how many of each kind.

3. A man owed 6489 rupees to *A*, 708 rupees to *B*, and five thousand and six rupees to *C*; he sold his property for 30207 rupees out of which he paid off his debts; find the amount of the balance left.

4. Sir Isaac Newton was born in 1642 and died in 1726; how long did he live ?

5. A man sold his horse for 7985 rupees and his farm for 6709 rupees ; how many rupees more did the house fetch than the farm ?

6. A man bought a horse and a carriage for 469 rupees ; the carriage cost him 378 rupees : find the cost of the horse.

7. In an orchard there were 79 fig trees, 123 plantain trees, 71 tamarind trees, 97 mango trees and 317 guava trees ; how many trees were there altogether ?

8. A boy had 51 mangoes, his sister had 15 more and his brother 29 less ; find the total number of mangoes which the three had among them.

X 9. Three boys collected together five hundred and sixty-seven mangoes ; of these the first and the second together collected 395, and the first and the third together 401 ; find how many mangoes did each collect ?

10. A gentleman left 4985 rupees to be distributed among his 2 sons and 1 daughter. If the eldest son got 2401 rupees and the daughter 598, find the share of the younger son.

X 11. A gentleman was born in 1789 ; how old was he in 1813 ?

12. A gentleman had three children, two sons and a daughter. He was 27 years old when his eldest son was born ; his daughter was 3 years older than his younger son and 2 years younger than his eldest son. How old was the father when his younger son was 37 years old ?

X 13. A farmer had 97 more sheep than oxen and cows together ; if the number of cows, which was 45, exceeded the number of oxen by 15, find how many animals he had.

14. A man was born in 1879 ; when will he be 75 years of age ?

15. The united ages of a man and his wife amount to 91 years ; if the man is 7 years older than his wife, what will be his age 5 years h[illegible]ce ?

16. A railway train had nine carriages containing 36, 27, 19, 43, 35, 28, 39, 51 and 47 passengers respectively ; if 7 passengers were to get down from each at a certain station, find the number of passengers the train will carry from that station.

17. A farmer bought a horse, a cow and a buffalo, for 150 rupees ; the horse and the cow together cost 112 rupees, and the buffalo 7 rupees less than the cow ; find the cost of the cow.

18. Rama and Govind play at marbles. Rama had 253 to begin with and Govind had 149 ; Govind wins 54 from Rama ; how many has each now ?

X 19. A man whose age is now 57 years was 23 years old when his daughter was born ; how old is she now ?

20. A man was 27 years old when his eldest son was born; if the present age of the son is 39 years, find that of the father.

21. The population of Poona, according to the last census, consists of 56605 males and 54780 females; find how many more males there are than females in the city and also the total number of inhabitants.

22. A man had a son and a daughter. The man bought for himself 19 mangoes, 24 oranges and 36 plantains; for his son he bought 53 oranges, 12 plantains and 39 mangoes; and for his daughter, 18 plantains, 15 oranges and 21 mangoes; how many fruits of each kind did he buy?

23. In a city consisting of fifty-four thousand and eighty inhabitants, there were two thousand and 95 births in a week, while the number of deaths amounted to 3209; find the exact population in the city on the first day of the next week.

24. *A*, *B* and *C* had together 9859 rupees; of these *A* and *B* had each 3067; find how many rupees *C* had.

25. In three schools there are 143 boys in each, in a fourth 431, and 413 in the fifth; find the total number of boys.

26. A man earned 492 rupees, of which he spent 379; how many did he save?

27. In a certain week a gentleman gave 234 pice in charity on Monday, 123 on Tuesday, 243 on Wednesday, 432 on Thursday, 143 on Friday, 431 on Saturday, and on Sunday he gave as many as on all the previous days together. Find the amount of his charity in the week.

28. In a regiment there are 576 men, in another 423, and in a third as many as in the first and the second together; find the total number of men in the three regiments.

29. Three regiments had together 13209 men; the first and the second together had 9955, and the second and the third together had 8677; how many more men were there in the second than in the third?

30. The sum of three numbers is ten thousand and ten; one of these is greater than three thousand by one hundred and another less than four thousand by ninety-nine; find the third.

MULTIPLICATION.

20. *Multiplication* is the operation of finding the sum of a given number repeated as many times as there are units in another given number. The sum thus formed is called the *product* of the two numbers; the number to be repeated is called the *multiplicand* and the other the *multiplier*. The multiplicand and the multiplier are called the *factors* of the product.

The following Multiplication Table should be learnt by heart:

1	2	3	4	5	6	7	8	9	0
2	4	6	8	10	12	14	16	18	0
3	6	9	12	15	18	21	24	27	0
4	8	12	16	20	24	28	32	36	0
5	10	15	20	25	30	35	40	45	0
6	12	18	24	30	36	42	48	54	0
7	14	21	28	35	42	49	56	63	0
8	16	24	32	40	48	56	64	72	0
9	18	27	36	45	54	63	72	81	0

NOTE. Any figure multiplied by zero gives *zero* as the product.

21. To multiply one number by another:

Rule. Write the multiplier under the multiplicand so that the units of the same order may be under one another and draw a line underneath. If the multiplier be 9 or under, multiply with it the first right hand figure of the multiplicand. Set down the units' figure of the product under the units and carry the tens' figure, if any, to the product of the multiplier and the next (*i.e.* tens') figure of the multiplicand; set down, and carry, as in addition, and repeat the process until all the figures of the multiplicand are multiplied. If the multiplier be greater than 9, multiply successively as above by each figure of the multiplier, beginning at the units' figure, writing down each partial product so that its first figure from the right shall be under that figure of the multiplier with which the multiplicand is multiplied. Add together these partial products and the sum is the product required.

NOTE.—When a number is multiplied by another consisting of two or more digits, the product of the multiplicand by each digit in the multiplier taken separately is called *Partial Product.*

Ex. 1. Multiply 5743 by 5.

5743
5
28715

Write down the multiplier 5 under the units' figure of the multiplicand and draw a line underneath, and proceed as follows :

5 times 3 are 15 ; set down 5 below the line under the units' figure and carry 1. 5 times 4 are 20, and 1 carried is 21 ; set down 1 under the tens' figure and carry 2. 5 times 7 are 35 and 2 carried is 37 ; set down 7 under the hundreds' figure and carry 3. 5 times 5 are 25 and 3 carried is 28 ; set down 28 and the product is 28715.

Ex. 2. Multiply 3468 by 732.

3468
732
6936
10404
24276
2538576

Multiply the multiplicand first by 2 then by 3 and lastly by 7. Write the partial products so that the first figure from the right of the first partial product shall be under the digit 2 of the multiplier, the first figure of the second partial product shall be under the digit 3 of the multiplier, and the first figure of the third partial product under 7 in the multiplier. Add the partial products, and the sum, which is 2538576, is the product required.

Ex. 3. Multiply 7236 by 302.

7236
302
14472
21708
2185272

Since any number multiplied by 0 gives 0, the second partial product, if set down, will be simply a line of zeroes. As, however, the addition of zero and any number makes no change in that number, it is unnecessary to write such a line. But care must be taken to set down the next partial product in its proper place, *i.e.* with its first figure from the right under that digit in the multiplier with which the multiplicand is multiplied. Thus in the present example, 8 must be set down under 3 of the multiplier and not under 0.

22. When there is a cipher or ciphers *at the end* of the multiplicand, the multiplier, or both, multiply the significant figures only, and annex to the product thus obtained as many ciphers as there are at the end of the multiplicand and the multiplier taken together.

Ex. 4. Multiply 24300 by 36 ; 432 by 60 ; and 752000 by 4200.

243	432	752
36	6	42
1458	2592	1504
729		3008
8748		31584

In the first example the multiplicand has two ciphers at the end and the multiplier none. Therefore annex to the first product two ciphers. Similarly annex to the second product, one cipher, and to the third, five (=3+2) ciphers. The required products thus are 874800, 25920, and 3158400000.

23. When more than two numbers are to be multiplied together, any two of them may first be multiplied together, and the product multiplied by a third number and so on. This operation is called *continued* multiplication and the final product is called the *continued product.*

Ex. 5. Multiply 23, 17 and 29 together.

$$\begin{array}{r} 23 \\ \underline{17} \\ 161 \\ \underline{23} \\ 391 \\ \underline{29} \\ 3519 \\ \underline{782} \\ 11339 \end{array}$$

Multiply 23 by 17 and the product by 29. Therefore the required product is 11339.

24. The student will have noticed that the number of partial products in any operation of multiplication is usually the same as the number of significant digits in the multiplier. But sometimes it is convenient to reduce the number of partial products as in the following example. Thus,

Ex. 6. Multiply 53462 by 168.

$$\begin{array}{r} 53462 \\ \underline{168} \\ 427696 \\ \underline{855392} \\ 8981616 \end{array}$$

Since the multiplier is one hundred and sixty-eight or 16 tens and 8, and since 16 is twice 8, the work may be thus shortened :

Multiply the multiplicand by 8 and set down the partial product ; multiply this partial product by 2, and set down the new product; and we get the product of the multiplicand and 16. Set down this second product in its proper place and add up the two partial products. The sum is the required product.

25. When the multiplier itself is the product of two or more factors, the multiplication may be effected by multiplying the multiplicand by each of the factors in succession. Thus if it be required to multiply 1238 by 35, we may multiply 1238 first by 5 and the product by 7, since $35 = 5 \times 7$. It will be found that the final product is 43330.

26. When a number is multiplied by itself once, twice, thrice, four times, etc., the resulting products are respectively called the second, third, fourth, fifth power, etc., of the number.

The second power of a number is generally called the square of the number, and the third its cube. (There are no special names for the higher power.) Thus 3×3 or 9 is called the square of 3, and is expressed as 3^2 (read three squared or three raised to two); $4 \times 4 \times 4$ or 64 is called the cube of 4, and is expressed as 4^3 (read four cubed or four raised to three). Similarly $2 \times 2 \times 2 \times 2 \times 2$ or 32 is called the fifth power of 2, and is expressed as 2^5 (read two raised to the fifth power or two raised to five).

Ex. 7. Find the 4th power of 7 and the cube of 12.

$$\begin{aligned} 7^4 &= 7 \times 7 \times 7 \times 7 \\ &= 49 \times 7 \times 7 \\ &= 343 \times 7 \\ &= 2401. \end{aligned}$$

$$\begin{aligned} 12^3 &= 12 \times 12 \times 12 \\ &= 144 \times 12 \\ &= 1728. \end{aligned}$$

Ex. 8. Find the fifth power of 9 and the sixth power of **16.**

$$\begin{aligned} 9^5 &= 9 \times 9 \times 9 \times 9 \times 9 \\ &= 9^2 \times 9^3 \\ &= 81 \times 729 \\ &= 59049. \end{aligned}$$

$$\begin{aligned} 16^6 &= 16 \times 16 \times 16 \times 16 \times 16 \times 16 \\ &= 16^3 \times 16^3 \\ &= 4096 \times 4096 \\ &= 16777216. \end{aligned}$$

27. *Test for multiplication.* If by interchanging the multiplicand and the multiplier, the product remains the same, the work is correct.

EXERCISE V.

Multiply :

1. 342 by 2.
2. 1323 by 3.
3. 41221 by 4.
4. 546 by 3.
5. 23765 by 8.
6. 90723 by 5.

7. 4320 by 11. **8.** 60502 by 7. **9.** 54706 by 60.

10. 324792 by 24. **11.** 586429 by 16. **12.** 762538 by 35.

13. 162985 by 76. **14.** 3015120 by 67. **15.** 8702043 by 79.

16. Multiply 142857 by 2, 3, 4, 5, 6, and 7 separately.

17. Multiply 76923 by each one of the numbers 3, 4, 9, 12, and 13.

18. Multiply 152207 by each one of the numbers 73, 146, 219, 292, 365, 438, 511, 584, and 657.

19. Multiply 4023 by 24, 32, 56, and 63 separately by resolving each into two factors.

20. Multiply in two lines 2768 by each one of the numbers 123, 328, 567, 416, 535, and 648.

Multiply together :

21. 405 and 762. **22.** 720 and 938. **23.** 3804 and 480

24. 751284 and 1704. **25.** 30700 and 428795. **26.** 568930 and 4221.

27. 142842 and 573819. **28.** 8527396 and 3442342032.

29. Multiply together 128, 256, and 512.

30. Find the continued product of 37, 43, 59, and 67.

31. Which is greater, 7924 multiplied by 7926 or 7925 multiplied by itself, and by how much ?

32. Multiply together two 3's, three 4's, and four 5's.

33. Find the squares of 145, 342, 405, and 9009.

34. Find the cubes of 36, 85, 290, and 101.

35. Find the value of 4^3 multiplied by 5^2, and 8 times 12^6.

DIVISION.

28. *Division* is the operation of finding how many times one number can be taken from another number. It is therefore a shorter process for performing a number of subtractions. The number to be taken is called the *divisor*, that from which it is to be taken, the *dividend* ; and that which tells us how many times it can be taken, the *quotient*. If, after taking away the divisor from the dividend a certain number of times, a number remains which is less than the divisor, and from which, therefore, the divisor can not be taken away any more, that number is called the *remainder*.

29. To divide one number by another :

Rule. On either side of the dividend draw a curved line. Place the divisor on the left and the figures of the quotient, as they arise, on the right. From the left of the dividend mark off the least number of digits which form a number which is not less than the divisor. Find how many times the divisor is contained in these, and set down this number as the first digit in the quotient ; multiply the divisor by this digit and subtract the product from the number marked off from the dividend. To the remainder bring down the digit of the dividend which comes next after those first marked off. If the number thus formed be not less than the divisor, find how many times the divisor is contained in it ; write this number as the second digit of the quotient and proceed as before to find the remainder. If the number formed by bringing down the next digit in the dividend and the remainder be less than the divisor, bring down the next following digit of the dividend, and if necessary the next following digit and so on until the number thus formed becomes not less than the divisor ; for each digit of the dividend so brought down, set down a cipher in the quotient. Proceed in this way until all the figures of the dividend are brought down.

Ex. 1. Divide 69712 by 16.

```
16 ) 69712 ( 4357
     64
     ──
      57
      48
      ──
       91
       80
       ──
       112
       112
       ───
       ...
```

Draw curved lines on either side of the dividend and place the divisor on the left, reserving the right side for the quotient.

From the left of the dividend mark off the two digits which form 69, a number greater than 16, the divisor. Since 16 is contained in 69 four times, set down the number 4 as the first digit of quotient ; multiply the divisor by 4, and then write the product 64 under 69 ; subtract, and we have 5 as the remainder.

Bring down 7, the next digit of the dividend, to this remainder 5, and we have 57, in which 16 is contained three times ; put the number 3 as the second digit of the quotient ; multiply by it the divisor and subtract the product, 48, from 57. The remainder is 9.

Bring down the next figure 1, and we have 91, in which 16 is contained five times ; put 5 as the third figure of the quotient ; multiply by 5 the divisor and subtract the product, 80, from 91, leaving as remainder 11.

Bring down the next figure 2, and we have 112, in which 16 is exactly contained seven times. Set down 7 as the next digit of the quotient. Multiply the divisor by 7 and subtract the product, 112, from 112, which leaves no remainder. And as all the digits in the dividend have been brought down, the required quotient is 4357.

Ex. 2. Divide 920498 by 213.

```
213 ) 920498 ( 4321
      852
      ---
       684
       639
       ---
        459
        426
        ---
         338
         213
         ---
         125
```

Mark off 920 from the left of the dividend, and as 213 can be taken from 920 four times, 4 is the first digit of the quotient. Multiply 213 by 4 and subtract the product, 852, from 920 and we obtain as remainder 68. Bring down the next digit 4, and we have 684, in which 213 is contained three times. Put 3 as the second digit of the quotient; multiply 213 by 3 and subtract the product, 639, from 684; this gives 45 as the remainder. Bring down the next digit 9, and we have 459, in which 213 is contained twice. Write 2 as the third digit of the quotient; multiply 213 by 2 and subtract the product, 426, from 459, which leaves 33 as the remainder. Bring down the next digit 8, and we have 338, from which 213 can be taken only once. Set down 1 as the fourth digit of the quotient; multiply and subtract as before; and we obtain 125 as the final remainder. As all the digits in the dividend have been brought down, and as the final remainder 125 is less than the divisor 213, the answer is quotient 4321 and remainder 125.

NOTE. Division is said to be *exact* when no remainder is left after the last digit of the dividend has been brought down. If, on the other hand, a remainder is left, the division is called *inexact*.

Ex. 3. Divide 1733946 by 342.

```
342 ) 1733946 ( 5070
      1710
      ----
        2394
        2394
        ----
          06
```

Here the remainder after the first subtraction is 23; bringing down to this 9 from the dividend we have 239; since the divisor 342 is greater than 239, place a cipher as the next digit of the quotient, and bring down the next digit 4; we now obtain the number 2394, in which 342 is contained seven times; write 7 as the next digit of the quotient; multiply 342 by 7 and subtract the product, 2394, from 2394, which leaves no remainder. Bring down the last digit 6. Since 6 does not contain 342, put a zero as the last digit of the quotient. We thus obtain 5070 as the quotient, and 6 as the remainder.

30. When the divisor ends with a cipher or ciphers, the process may be shortened as follows:—Cut off from the divisor the ciphers at its end, and mark off an equal number of digits from the right of the dividend. Proceed with the division according to the usual rule and to the last remainder bring down the figures marked off from the dividend, the number thus formed being the final remainder.

Ex. 4. Divide 4732891 by 6300.

```
63|00 ) 47328|91 ( 751
        441
        ---
         322
         315
         ---
          78
          63
          --
          15
```

Dividing 47328 by 63 we obtain 751 as the quotient and 15 as the remainder. Bringing down 91 to 15 we obtain 1591 as the final remainder.

Test for division :—Multiply together the divisor and the quotient, add to the product the remainder if there be any ; if the result be equal to the dividend the quotient is correct.

31. When the divisor is not greater than 9, it is convenient to perform the operation of division mentally, only setting down the successive digits of the quotient and the final remainder.

Ex. 5. Divide 43915 by 7.

```
7 ) 43,915
    ------
     6,273
```

Write the divisor and dividend as usual, and proceed as follows :—7 into 43 is 6, with 1 over (this means 7 is contained in 43 6 times, leaving one as remainder) ; set down 6 under 3 and mentally bring down to the remainder 9. 7 into 19 is 2, with 5 over ; set down 2 under 9 and bring down the next digit. 7 into 51 is 7, with 2 over ; set down 7 under 1 and bring down 5. 7 into 25 is 3, with 4 over ; set down 3 under 5. Hence 6273 is the quotient and 4 the remainder.

This method of dividing is called *Short Division* as distinguished from the lengthy method given in Art. 29, which is called *Long Division.*

32. When the divisor consists of the product of two or more factors, each less than 9, the following process may be adopted :

Divide in succession by each one of factors of the divisor, setting aside the remainder, if any, at each step.

To find the real remainder, multiply each remainder by all the divisors preceding its own, add together all such products and to the sum add again the first remainder.

Ex. 6. Divide 13691 by 42.

Since $42 = 7 \times 6 = 7 \times 3 \times 2$, divide 13691 successively by 7, 3, and 2, setting aside the remainder at each step ; thus

$$\begin{array}{l} 7\ \underline{)\ 13691} \\ \quad 3\ \underline{)\ 1955}\ldots\ldots\ldots\ldots 6 \text{ remainder.} \\ \qquad 2\ \underline{)\ 651}\ldots\ldots\ldots\ldots 2 \quad ,, \\ \qquad\quad 325\ldots\ldots\ldots\ldots 1 \quad ,, \end{array}$$

∴ the quotient is 325,

$$\begin{aligned} \text{and the real remainder} &= 1 \times 3 \times 7 + 2 \times 7 + 6 \\ &= 21 + 14 + 6 \\ &= 41. \end{aligned}$$

Ex. 7. Divide 3853827 by 132.

Since $132 = 11 \times 12$, divide first by 11, and then the quotient by 12. Thus

$$\begin{array}{l} 11\ \underline{)\ 3853827} \\ \quad 12\ \underline{)\ 350347}\ldots\ldots\ldots\ldots 10 \\ \qquad\quad 29195\ldots\ldots\ldots\ldots 7 \end{array}$$

Hence the quotient $= 29195$.

And the remainder $= 11 \times 7 + 10 = 87$.

Ex. 8. Divide 37589246 by $15 \times 17 \times 23$.

$$\begin{array}{l} 15\ \underline{)\ 37589246} \\ \quad 17\ \underline{)\ 2505949}\ldots\ldots\ldots\ldots 11 \\ \qquad 23\ \underline{)\ 147408}\ldots\ldots\ldots\ldots 13 \\ \qquad\qquad 6409\ldots\ldots\ldots\ldots 1 \end{array}$$

Hence the quotient $= 6409$.

$$\begin{aligned} \text{And the remainder} &= 15 \times 17 \times 1 + 15 \times 13 + 11 \\ &= 255 + 195 + 11 \\ &= 461. \end{aligned}$$

33. The following test may be applied to ascertain the correctness of the answer : Deduct the real remainder from the dividend, and divide the remainder so obtained by the several factors successively. If the last quotient be the same as before, and if no remainder is left after division by each factor, the work is correct. Thus in Ex. 8 :

$$37589246 - 461 = 37588785.$$

$$\begin{array}{r} 15\,\underline{)\ 37588785} \\ 17\,\underline{)\ 2505919} \\ 23\,\underline{)\ 147407} \\ 6409, \end{array}$$

which is the same quotient as before.

34. We give below another test called ‘proof by casting out the nines’ that is frequently employed in testing a result in Multiplication and Division.

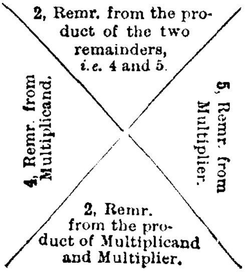
(1) Test for Multiplication: Add the digits of the multiplicand, divide the sum by 9, and note down the remainder; do the same with the multiplier. Multiply the remainders, divide the product by 9, and note down the remainder; if this remainder be equal to that obtained by dividing the sum of the digits of the product by nine, the work is probably correct.

(2) Test for Division: Add the digits of the divisor, divide the sum by 9, and note down the remainder; do the same with the quotient. Multiply the remainders, divide the result by 9, and note down the remainder; if this remainder be the same as that obtained by casting out nines from the dividend diminished by the remainder, the work is probably correct.

Ex. 9. Multiply and divide 482521 by 572, and show the correctness of the result by casting out the nines.

(i)

$$\begin{array}{r} 482521 \\ 572 \\ \hline 965042 \\ 3377647 \\ 2412605 \\ \hline 276002012 \end{array}$$

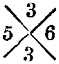

2, Remr. from the product of the two remainders, i.e. 4 and 5.
4, Remr. from Multiplicand.
5, Remr. from Multiplier.
2, Remr. from the product of Multiplicand and Multiplier.

Here the sum of the digits of the multiplicand is 22, which, on being divided by 9, leaves 4 as the remainder ; the sum of the digits of the multiplier is 14, which, on being divided by 9, leaves 5 as the remainder ; the product of these remainders, viz. of 4 and 5, is 20, which, being divided by 9, leaves the remainder 2.

Now the sum of the digits of the product is 20, which, on being divided by 9 leaves the same, viz. 2 as remainder. The product is therefore correct.

(ii)

```
572 ) 482521 ( 843
      4576
      ----
       2492
       2288
       ----
        041
       1716
       ----
        325
```



Dividing by 9 the sum of the digits of the divisor and of the quotient respectively, the remainders are 5 and 6 ; the product of 5 and 6, which is 30, divided by 9, leaves 3. Now subtract 325 from 482521, and we get as remainder 482196 ; the sum of whose digits is 30. This 30, divided by 9, also leaves 3. The work therefore is correct.

35. An exact division involves three quantities, viz. divisor, dividend and quotient, and of these, if any two be given, the third may be found by the following rule.

To find the *quotient*, divide the dividend by the divisor. To find the *divisor*, divide the dividend by the quotient. To find the *dividend*, multiply the divisor and the quotient.

36. An inexact division involves four quantities, viz. divisor, dividend, quotient and remainder; and if any three of these be given the fourth may be found by the following rule.

(1) To find the *divisor*, subtract the remainder from the dividend and divide the result by the quotient.

(2) To find the *dividend*, multiply the divisor by the quotient and to the product add the remainder.

(3) To find the *quotient*, subtract the remainder from the dividend and divide the result by the divisor.

(4) To find the *remainder*, multiply the divisor by the quotient and subtract the result from the dividend.

EXERCISE VI.

Divide

1. 1284 by 4. **2.** 18693 by 3. **3.** 482648 by 2.

4. 15625 by 5. **5.** 24672 by 6. **6.** 409680 by 8.

7. 40573 by 13. **8.** 39708 by 12. **9.** 712304 by 17.

10. 95318 by 19. **11.** 208173 by 23. **12.** 149003 by 29.

13. 23581297 by 31. **14.** 16183285 by 53. **15.** 628174298 by 59.

16. 187345 by 115. **17.** 1178527 by 217.

18. Divide the product of 382761 and 234 by 702.

Divide **19.** 795495 by 293. **20.** 537152 by 763.

21. 4855568 by 4173. **22.** 302129168 by 6704.

23. 34024375 by 4375. **24.** 218773386 by 6202.

25. What number multiplied by three thousand eight hundred and fifty will give five hundred and one million and five hundred ?

26. Divide the square of 409600 by the cube of 160.

27. If seven hundred and eighty-four thousand one hundred and thirty-five be divided by four thousand two hundred and five, what will be the remainder ?

Divide (use short division)

28. 15997 by 21. **29.** 570021 by 77.

30. 84381 by 84. **31.** 2360827 by 105.

32. Divide 784218965 by the continued product of 11, 13, and 17.

33. Divide 12643980 by 7, 9, and 23.

34. If in a division sum the dividend be 176111 and the quotient 437, find the divisor.

35. If the dividend be 239690, the quotient 329, and the remainder 178, find the divisor.

PROBLEMS IN MULTIPLICATION AND DIVISION.

37. These problems, strictly speaking, involve the notion of Proportion which is treated in chapter X. However, some easy problems which may be solved by the mere processes of Multiplication and Division without any definite knowledge of the principles of proportion, are given below.

Ex. 1. Fifty-seven boys have each thirty-nine mangoes; how many mangoes have they altogether?

To find the required number, 39 is to be repeated 57 times. Therefore we should multiply 39 by 57.

$$\begin{array}{r} 39 \\ \underline{57} \\ 273 \\ \underline{195} \\ 2223 \end{array}$$

Thus the total number of mangoes is 2223.

Ex. 2. A man spent 2952 rupees in buying horses at *Rs.*18 each; how many horses did he buy?

Out of the sum of 2952 rupees he gave 18 rupees for one horse, 18 for another, and 18 for a third, and so on. The required number of horses may therefore be determined by finding how many times 18 is contained in 2952. We should therefore divide 2952 by 18; thus:

$$18\left\{\begin{array}{l} 6\,)\,\underline{2952} \\ 3\,)\,\underline{492} \end{array}\right. \\ \phantom{18\{3)\,}164$$

Hence the number of horses is 164.

Ex. 3. A man bought 57 sheep at 8 rupees each; how many cows at 38 rupees each could he have bought for the same sum?

The cost of 57 sheep is 57 times 8 rupees, that is 456 rupees. To find the number of cows which he could have bought for the same sum, viz. 456, we should divide 456 by 38; thus:

$$\begin{array}{l} 38\,)\,456\,(\,12 \\ \underline{38} \\ 76 \\ \underline{76} \\ \cdot\,\cdot \end{array}$$

Hence the number of cows is 12.

Ex. 4. A man walks 184 miles in 23 days; find how many miles he walks in a day and also the number of days he will take to walk 248 miles.

Since the man walks 184 miles in 23 days, his rate of walking is $184 \div 23$ or 8 miles a day; and the number of days therefore which he will take to walk 248 miles is $248 \div 8$ or 31 days.

EXERCISE VII.

1. A fruiterer has 87 baskets each containing 156 oranges; how many oranges has he for sale?

2. In a garden there are 2669 trees arranged in 17 equal rows; how many are there in a row?

3. A farmer bought 26 oxen at 35 rupees each; find the cost of all the oxen he bought.

4. A book has 435 pages and each page has 34 lines ; how many lines are there in the book ?

5. Divide 407 mangoes equally among 11 persons.

6. Divide 559 oranges equally among 6 boys and 7 girls.

7. A railway train runs 28 miles an hour ; what distance will it travel in 25 hours ?

8. In going to school a boy takes 138 steps ; how many steps will he take in going to and returning from the school for 99 days ?

9. If 24 articles cost 264 rupees, what will 1 article cost ?

10. A baker sells two hundred and fifty-nine loaves in 7 days ; how many does he sell per day ?

11. A rupee is equal to 64 pice ; how many pice are 575 rupees equal to ?

12. A sovereign is equal to 15 rupees ; how many sovereigns are equal in value to 3405 rupees ?

13. A train runs 483 miles in a day ; how many miles will it travel in a month of 29 days ?

14. A fruiterer arranged 12019 guavas in a number of equal heaps, each containing 101 guavas ; how many heaps were there ?

15. An army consisted of 6237 men in all, each regiment contained 532 men and 35 officers ; how many regiments were there in the army ?

16. A man can remove 43 stones in an hour ; how many will he remove in 5 days of 12 hours each ?

17. A railway has 16 carriages ; each carriage has 6 compartments, each of which holds 8 persons ; how many persons does the train carry ?

18. Five trains start daily from a railway station ; each train takes 201 passengers at a time ; how many passengers would be taken away in a month of 30 days ?

19. An army contains 21 regiments ; each regiment has 12 companies and each company 264 men ; how many men are there in the army ?

20. A workman carries 125 bricks in an hour ; how many bricks will he carry in a month of 31 days, working 9 hours a day ?

21. Twelve men daily put the same sum each into a box ; at the end of a fortnight the box is found to contain twenty-seven hundred pice ; how much does each put in daily ?

22. Eleven hundred and fifty-seven rupees are to be distributed equally among 30 men, 29 women, 17 boys, and 13 girls ; how many will each get ?

23. In a street there are 143 houses ; each house contains 4 families, and each family 9 persons ; how many people live in the street ?

24. How many railway carriages are required to carry 799 persons if 47 persons be put into each carriage ?

25. A man bought 273 knives for 91 rupees; what would 111 knives cost at the same rate ?

26. If 8 mangoes can be bought for 24 pice, how many can be bought for 48 pice ?

27. If a bicyclist travel 47 miles a day, how far will he go in 23 days, and how long will he take to go 705 miles ?

28. A farmer gave 49 oxen for 35 buffaloes, valued at 35 rupees each ; what was the price of one ox ?

29. If a train travels from Poona to Bombay in 298 minutes and it goes 175 yards in each minute, find the distance in yards between Poona and Bombay.

30. How many sheep at 3 rupees each can be exchanged for 25 cows at 36 rupees each ?

SIGNS.

38. The following signs are employed in Arithmetic.

+ is read *plus*, and is the sign of addition. It indicates that of the two *quantities* or numbers, between which it is placed, the second is to be added to the first. Thus $8+5$ (which is read *eight plus five*) indicates that 5 is to be added to 8.

− is read *minus*, and is the sign of subtraction. It indicates that of the two quantities, between which it is placed, the second is to be subtracted from the first. Thus $8-5$ (which is read *eight minus five*) indicates that 5 is to be subtracted from 8.

× is read *multiplied by*, or *into*, and is the sign of multiplication. It indicates that of the two quantities, between which it is placed, the first is to be multiplied by the second. Thus 8×5 (which is read *eight multiplied by five*, or *eight into five*) indicates that 8 is to be multiplied by 5.

÷ is read *divided by*, or merely *by*, and is the sign of division. It indicates that of the two quantities, between which it is placed, the first is to be divided by the second. Thus $8\div 5$ (which is read *eight divided by five*, or *eight by five*) indicates that 8 is to be divided by 5.

When several numbers are to be taken collectively, *i.e.* are to be treated as a *single* number, they are enclosed within brackets as ( ), { }, [ ], or a bar or line, called *vinculum,* is drawn over them. Thus if we have to take $8+6-5$ collectively, we may write the numbers as $(8+6-5)$, or as $\overline{8+6-5}$.

Sometimes the sign of multiplication is omitted immediately before or after a bracket. Thus $4(5+6)$ or $(5+6)4$ means that the sum of 5 and 6 is to be multiplied by 4.

$=$ is read *equals,* or *is equal to,* and is the sign of equality. It indicates that of the two quantities, between which it is placed, the first is equal to the second. Thus $(8+6-5)=9$ indicates that the quantity enclosed in the brackets is equal to 9.

$\therefore$ is read *therefore,* and is used in stating a result.

$\because$ is read *because,* and is used in stating a reason.

39. When an expression consists of more than two numbers connected by the signs + and −, the numbers which are immediately preceded by the sign *plus* should be first added together; then the numbers which are similarly preceded by the sign *minus* should be added together; and then the second result should be subtracted from the first.

NOTE.—A number which is preceded by no sign may be regarded as preceded by the sign *plus.*

Ex. 1. Find the value of $3495+827-4852+2781-901$.

Here we have to subtract the sum of 4852 and 901 from the sum of the remaining three numbers. We thus have

$$\begin{array}{rr} 3495 & \\ 827 & 4852 \\ \underline{2781} & \underline{901} \\ 7103 & 5753 \end{array}$$

$\therefore$ the required value $=7103-5753=1350$.

40. When an expression consists of more than two numbers connected by the signs of addition or subtraction, as also of multiplication or division, the multiplication or division should be performed first and the addition or subtraction afterwards.

Ex. 2. Find the value of $13 - 725 \div 5 + 27 \times 12$.

$$\begin{aligned} 725 \div 5 &= 145. \\ 27 \times 12 &= 324. \end{aligned}$$

$$\begin{aligned} \therefore\ 13 - 725 \div 5 + 27 \times 12 &= 13 - 145 + 324 \\ &= 13 + 324 - 145 \\ &= 337 - 145 \\ &= 192. \end{aligned}$$

$\therefore$ 192 is the answer.

41. When an expression consists of more than two numbers connected by the signs of multiplication and division, the operations should be performed in order beginning from the left; or the operation of division should be first performed as may be more convenient, the result in both cases being the same.

Ex. 3. Simplify $18 \times 16 \div 12$, $232 \div 8 \times 12$, and $132 \times 21 \div 7$.

$$\begin{aligned} 18 \times 16 \div 12 &= 288 \div 12 \\ &= 24. \\ 232 \div 8 \times 12 &= 29 \times 12 \\ &= 348. \\ 132 \times 21 \div 7 &= 132 \times 3 \\ &= 396. \end{aligned}$$

42. In simplifying an expression in which brackets are used, the operations of Arithmetic indicated by the signs inside the brackets should be performed before the brackets are removed.

Ex. 4. Simplify $789 - (21 - 2 \times 6) + (5 \times 3 - 8) - 564$.

$$\begin{aligned} (21 - 2 \times 6) &= 21 - 12 = 9. \\ (5 \times 3 - 8) &= 15 - 8 = 7. \end{aligned}$$

$$\begin{aligned} \therefore\ \text{the given expression} &= 789 - 9 + 7 - 564 \\ &= 789 + 7 - 9 - 564 \\ &= 796 - 573 \\ &= 223. \end{aligned}$$

43. When one bracket is placed within another, the expression inside the former should be first simplified and then the expression inside the latter.

Ex. 5. Simplify $6 \div \{12 - (4 + 5)\}$.

$$\begin{aligned} 4 + 5 &= 9. \\ 12 - 9 &= 3. \\ 6 \div 3 &= 2. \end{aligned}$$

$\therefore$ 2 is the answer.

EXERCISE VIII.

Simplify the following expressions :

1. $235 - 438 + 329 - 13 + 509$. 2. $798 + 978 + 879 - 1529$.
3. $6217 - (123 + 213 + 321) - 1 \times 2 \times 3$. 4. $17 \times 13 - 16 \times 15 + 12 \times 19$.
5. $325 + 25 \times 72 - 357 \div 17$. 6. $11 \times 15 \times 19 - 19 \times 144$.
7. $37 \times 18 - 1296 \div 18 - 18 \times 7$. 8. $385 + 732 - 380 + 7 \times 9$.
9. $4679 - 7963 + 3571 - 1156 \div 68$.
10. $7 \times 9 - 9 \times 5 + 5 \times 8 - 8 \div 2 + 2 \times 6$.
11. $4^2 - 7 \times 6 + 5^3 - 2^5$. 12. $428 - (312 \div 13 + 9 \times 19)$.
13. $35 \times 36 + 36 \times 33 - (752 - 235)$. 14. $56 - (38 - 35 \div 5 + 2)$.
15. $3^4 + 4^3 - (18 + 25 - 7 \times 2)$. 16. $(81 \times 74 + 26) + \{81 \times (74 + 26)\}$.
17. $7 \times 13 - (18 \div 6 + 3) + (7 \times 2 + 1)$. 18. $5 + \{13 - (89 - 17 \times 5 + 6)\}$.
19. $61 - \{63 \div (30 - 7 \times 3)\}$. 20. $69 \div \{6 + (3 \times 8 - 7)\}$.

MISCELLANEOUS EXAMPLES ON THE FOUR SIMPLE RULES.

Ex. 1. Add together the greatest and least numbers that can be formed with all the digits of the number 317, taken together, and subtract the sum from the greatest number of four digits that you can form.

The given number consists of the digits 3, 1, and 7. Of these the greatest is 7, next to it in the descending order of magnitude is 3, and the least is 1. Therefore the greatest number that can be formed with these digits is 731. The number formed by reversing these digits will be the least. Therefore the sum of the required numbers is $731 + 137 = 868$.

The greatest of all digits is 9. Therefore the greatest number of four digits is 9999.

Hence the required difference is $9999 - 868 = 9131$.

Ex. 2. Find the sum of all the numbers that you can form with the digits 4, 5, and 7, any two being taken at a time.

Taking the digits 4 and 5 together, we form the numbers 45 and 54. Taking the digits 4 and 7 we form 47 and 74, and taking the digits 5 and 7 we form 57 and 75. No other numbers of two digits can be formed from the given digits. Hence the required sum

$$\begin{aligned} &= 45 + 54 + 47 + 74 + 57 + 75 \\ &= 352. \end{aligned}$$

Ex. 3. The sum of two numbers is 5798 and their difference is 952 ; find the numbers.

Since the difference is 952, it is clear that one of the numbers is greater than the other by 952. Therefore if from the given sum 952 be taken away, the remainder will be the sum of two numbers, each of which is equal to the smaller.

$$\begin{aligned} \therefore \text{ the smaller number} &= (5798 - 952) \div 2 \\ &= 4846 \div 2 \\ &= 2423. \\ \therefore \text{ the greater number} &= 2423 + 952 \\ &= 3375. \end{aligned}$$

Ex. 4. Find a number such that if it be divided by 478 it will give 324 as the quotient and 129 as the remainder.

To find the required number multiply 478 by 324 and add 129 to the product. Thus:

$$\begin{array}{r l} 478 & \\ \underline{324} & \\ 1912 & \\ \underline{15296} & \\ 154872 & = \text{the product.} \\ \underline{129} & \\ 155001 & = \text{the sum.} \end{array}$$

Here the multiplication is performed in two lines (see Art. 24).

$\therefore$ the required number is 155001.

Ex. 5. Find the smallest number which must be added to 7983741 in order to make it exactly divisible by twelve thousand and five hundred.

$$\begin{array}{r} 125|00\,)\,79837|41\,(\,638 \\ \underline{750} \\ 483 \\ \underline{375} \\ 1087 \\ \underline{1000} \\ 87 \end{array}$$

Dividing 7983741 by 12500 (as in Art. 30) we have 638 as the quotient and 8741 as the remainder.

The least number which is exactly divisible by 12500 is 12500 itself.

Therefore if the given number be increased by 12500 − 8741 = 3759, the remainder will become zero, the quotient, however, increasing by one.

$\therefore$ the least number required is 3759.

Ex. 6. A certain number is increased by 3 and then multiplied by 4; the product is diminished by 7 and the result is divided by 5; if the quotient be 21, find the number.

The quotient is 21 and the divisor 5.

$\therefore$ 21×5 or 105 is the result obtained after deducting 7.

$\therefore$ $105 + 7 = 112$ is the product mentioned in the question.

$\therefore$ $112 \div 4 = 28$ is the original number increased by 3.

$$\begin{aligned} \therefore \text{ the original number} &= 28 - 3 \\ &= 25. \end{aligned}$$

EXERCISE IX.

1. Express in figures ten million, ten thousand, ten hundred and ten.

2. Write down in words 4300702, 6070506, 341318, and the sum of these numbers.

3. What number is next higher than 1234 ?

4. Multiply four times 183 by 5.

5. What other number besides 137 will divide 24523 without remainder ?

6. Write in words 15006021, and state how many times the number is increased by annexing five ciphers to it.

7. What number immediately precedes 765 ?

8. Which is greater, twice the square of 16, or the square of twice 16 ?

9. Multiply the sum of two numbers 407823400 and 92176600 by their difference.

10. A plantation had 306225 trees in 75 rows ; how many were there in each ?

11. Express in words and in figures how much greater the value of one 5 is than that of the other in the number 658757.

12. A man was born in the year 1847 and lived 39 years. In what year did he die ?

13. The united ages of a father and his son amount to 63 years. If the father was 27 years old when his son was born, find the present age of the son.

14. If a number be divided by 39, the quotient is 135, and the remainder 2 ; what will the remainder be if the number be divided by 56 ?

15. If a number be diminished by 21 and the result be multiplied by 7, the product is 217 ; find the number.

16. What is expressed by each of the digits 6, 5, 2, 9 respectively in the number 615209 ?

17. What number must be taken from 35428 to leave one thousand twelve hundred and nine ? Express the answer in words.

18. By how much does 360 tens exceed nineteen hundred and twenty-one ?

19. How much will be left if 235 be taken four times from ten thousand ?

20. How many times can 101 be subtracted from 10201 ?

21. Write in figures the greatest number of 5 digits.

22. Express in words the least number of 7 digits.

23. How much must be subtracted from two thousand nineteen hundred and nine so that the remainder may consist of 3 nines ?

24. Subtract from the sum of 81459 and 54306 five times their difference.

25. Multiply four thousand and four by fifteen millions and fifteen.

26. Find the sum of all the numbers that you can form with the digits 2, 0, and 5, all of which being taken at each time.

27. A certain number, divided by 243, gives as quotient 1306 and as remainder 92. Find the number.

28. Multiply 129847 by 468. If in the process you shift all the figures resulting from the multiplication of the multiplicand by 4 two places farther to the left and then add, of what two numbers will the result be the product ?

29. Find the product of 2461 and 96124 in three lines of multiplication.

30. Divide 624351 successively by 3, 7, and 10.

31. Write the greatest and least numbers that can be formed from the digits 2, 5, 1, 3, all taken together.

32. Find the sum and difference of the numbers that can be formed out of the digits 2 and 9.

33. How many digits are cubes ? Find their sum.

34. A number is composed of three digits, each of which is a square. Find the sum of the digits.

35. There are two numbers the product of which is 229405. The greater number is 485 ; find the sum and difference of the numbers.

36. Divide the product of 2035674 and 396 by 198.

37. Multiply 13617 by 486 in two lines of multiplication.

38. Divide (use short division) 5963417, by each one of the numbers 84, 48, 132, 256.

39. If 4273 be divided by 43 it leaves 16 as the remainder, what other number will divide it leaving the same remainder ?

40. Express in figures one hundred thousand one hundred and one, and find the least number which must be added to it that the sum may be divisible by 7.

41. Find the number which, subtracted from eighty thousand, leaves fifty-seven thousand seven hundred and thirty-five ; and divide it by 365.

42. Find the product of the sum and difference of two numbers, one of which immediately precedes and the other follows a number of three nines.

43. The sum of three consecutive numbers is 12963. Find the numbers.

44. A man bought 43 horses at 79 rupees each, 73 cows at 47 rupees each, and 37 buffaloes at 53 rupees each. Find the total cost of the animals he bought.

45. Find the number, which, when divided by 7080, gives a quotient of 850967 and a remainder of 798.

46. When *A* received from *B* 78 rupees, the money in his possession became four times what he had at first. How many rupees had he at first ?

47. Simplify $(7 \times 18 - 0 \times 9 + 9 \times 11) \div (14 \times 15 - 15 \times 13)$.

48. How often must 67 be added to 590 so as to make the sum equal to 16000 ?

49. The last digit of the quotient obtained by dividing a certain number by 157 is 8 ; if the last digit of the remainder be 9, find the last digit of the dividend.

50. Find the least number which must be added to seven thousand and one million nine hundred and seven thousand and sixty-one in order that the sum may be divisible by seven hundred and nine thousand four hundred and eight.

51. What number nearest to 1590 is divisible by 37 ?

52. Simplify :

(1) $2463 \times 816 - 618 \times 2613$. (2) $224400 \div 132 - 12 \times 34$.
(3) $2691 \div 299 + 501 \div 167$. (4) $43 \times 43 - 43 \times 37 + 37 \times 37$.

53. Simplify :

(1) $1428 \times (275 - 25 \times 11)$. (2) $354 \div \{7 \times 6 + (5 + 4 \times 3)\}$.
(3) $132 \div \{95 - (6 + 7 \times 8)\}$. (4) $99 + \{384 - (3 \times 4 + 8 \times 4) - 3 \times 8 \times 4\}$.
(5) $125 - \{87 + (3 \times 2 - 1)\}$. (6) $23 \times \{38 - (17 \times 13 - 16 \times 12) + 42\}$.
(7) $432 \times 598 - 596 \times 1728 \div 48 - 32 \times 18$.

54. It is now 4 o'clock. What time was it 5 hours ago ?

55. What is the difference between ten thousand hundreds and ten hundred thousands ?

56. Find the sum of all the numbers that you can form with the digits 5, 3, and 7, all the digits being taken together.

57. If the product be 20736 when the multiplier is 48, what will be the product if the multiplicand remain the same, but the multiplier is changed to 84 ?

58. At a game of cricket *A*, *B*, and *C* together score 137 runs ; *A* and *B* together score 85 runs and *B* and *C* together 101 runs ; find the number of runs scored by each of them.

59. A farmer had a certain sum of money with which he can buy 42 sheep at a certain price per head ; but if the price rises by 5 rupees per head, he can buy only 27. Find how much money he had.

60. Four times a certain number is divided by 5 ; the result is increased by 2 and multiplied by 4 ; the product is diminished by 1 and divided by 7 ; if the quotient be 17, find the number.

CHAPTER III.

MEASURES AND MULTIPLES.

44. One number is said to be a *measure* of another when it *measures* or divides the other without a remainder. Thus 4 is a measure of 12.

A measure is also called a *factor*.

45. A *common measure* of two or more numbers is a number which divides each of these numbers without a remainder. Thus 3 is a common measure of 12, 18, and 48.

46. One number is said to be a *multiple* of another when it can be divided by the other without a remainder. Thus 12 is a multiple of 3.

47. A *common multiple* of two or more numbers is a number which can be divided by each of these numbers exactly, *i.e.* without a remainder. Thus 24 is a common multiple of 2, 3, 4, and 6.

48. A number which has no measure or factor except itself and unity is called a *prime* number. Thus 7 is a prime number.

49. Two or more numbers are said to be prime to one another when they have no common measure except unity. Thus 3, 5, 7 are prime to one another.

50. A number which can be divided exactly by some other number besides itself and unity is called a *composite* number.

51. Every composite number can be resolved into two or more factors.

52. The following tests of divisibility are of great use in resolving a composite number into factors.

Every composite number is divisible

by 2, if it be an even number, *i.e.* if it ends with 2, 4, 6, 8, or 0;

by 3, if the sum of its digits be divisible by 3;

by 4, if the number formed by its last two digits taken in order from the left be divisible by 4 ;

by 5, if it end with 5 or 0 ;

by 6, if it be an even number and have the sum of its digits divisible by 3 ;

by 8, if the number formed by the last three digits taken in order from the left be divisible by 8 ;

by 9, if the sum of its digits be divisible by 9 ;

by 10, if it end with a cipher ;

by 11, if the difference between the sum of the digits in the odd places and that of the digits in the even places be zero or be divisible by 11.

53. To resolve a composite number into prime factors :

Rule. Divide the given number by any small prime number by which it is exactly divisible and divide the quotient again by another similar small prime number ; proceed in this way till the quotient is a prime number. Then the last quotient and all the divisors are the factors required.

Ex. 1. Resolve 9072 into prime factors.

$$\begin{array}{r|l} 2 & 9072 \\ \hline 2 & 4536 \\ \hline 2 & 2268 \\ \hline 2 & 1134 \\ \hline 3 & 567 \\ \hline 3 & 189 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Therefore the required factors are 2, 2, 2, 2, 3, 3, 3, 3, and 7.

When the student acquires a little practice it will not be necessary for him to go on dividing as directed in the rule ; the given number may first be resolved by inspection into two factors, each or both of which are composite numbers, and the composite number or numbers may again be further resolved, till each factor is a prime number.

Ex. 2. Resolve 23040 into factors.

The student will easily perceive that 10 is a factor. Also, 9 is a factor, since the sum of the digits 2, 3, 4 is divisible by 9. Therefore we may divide 2304 mentally by 9 and proceed thus:

$$\begin{aligned} 23040 &= 10 \times 2304 \\ &= 10 \times 9 \times 256 \\ &= 10 \times 9 \times 8 \times 32 \\ &= 2 \times 5, \ \times 3 \times 3, \ \times 2 \times 2 \times 2, \ \times 2 \times 2 \times 2 \times 2 \times 2. \end{aligned}$$

Since in the above result 2 occurs 9 times and 3 twice, the answer may be put down as $2^9 \times 3^2 \times 5$.

EXERCISE X.

Resolve into prime factors:

1. 84. 2. 132. 3. 78. 4. 168. 5. 114.
6. 210. 7. 392. 8. 675. 9. 1024. 10. 1352.
11. 2160. 12. 504. 13. 565. 14. 476. 15. 1683.
16. 1485. 17. 2275. 18. 3179. 19. 4096. 20. 5832.
21. 13122. 22. 48510. 23. 12615. 24. 49686. 25. 64512.
26. 77616. 27. 82500. 28. 9360. 29. 34225.

30. Write down all the prime numbers between 60 and 75, between 150 and 225, and between 830 and 910.

GREATEST COMMON MEASURE.

54. The *Greatest Common Measure* of two or more numbers is the greatest number which divides each of these numbers an exact number of times, *i.e.* without a remainder. Thus 12 is the Greatest Common Measure of 36, 60 and 84.

The greatest common measure is also called the *Highest Common Factor*, the letters G.C.M. standing for the former and H.C.F. for the latter.

55. To find the greatest common measure of two numbers:

Rule. Divide the greater number by the less, divide the less by the remainder, divide that remainder by the next remainder, and so on until no remainder is left. The last divisor is the G.C.M. of the numbers.

Ex. 1. Find the G.C.M. of 493 and 667.

493) 667 (1
493
174) 493 (2
348
145) 174 (1
145
29) 145 (5
145
. . .

∴ 29 is the G.C.M. required.

Here we first divide 667 by 493 and obtain 174 as the first remainder; we then divide the first divisor 493 by 174, and we get 145 as the second remainder; we then divide 174 by 145 and obtain 29 as the next remainder. Lastly, we divide 145 by 29, and we see that there is no remainder left. 29 is therefore the G.C.M. of the numbers 493 and 667.

56. To find the greatest common measure of more than two numbers :

Rule. Find the G.C.M. of any two of these numbers, then the G.C.M. of this G.C.M. and another given number, and so on. The last G.C.M. will be the G.C.M. required.

Ex. 2. Find the G.C.M. of 528, 624, and 744.

528) 624 (1
528
96) 528 (5
480
48) 96 (2
96
. .

∴ 48 is the G.C.M. of 528 and 624.

Again

48) 744 (15
48
264
240
24) 48 (2
48
. .

∴ 24 is the G.C.M. required.

57. To find the G.C.M. of two or more numbers by resolving each into factors :

Rule. Resolve each of the given numbers into prime factors and then find the product of all the prime factors common to all the numbers. The product will be the G.C.M. required.

Ex. 3. Find the G.C.M. of 2310, 462, and 2730.

$$2310=10\times231=10\times3\times77=2\times5\times3\times11\times7\ ;$$
$$462=2\times231=2\times3\times77=2\times3\times7\times11\ ;$$
$$2730=10\times273=10\times3\times91=5\times2\times3\times7\times13.$$

The factors common to all the three given numbers are 2, 3, and 7. Therefore the required G.C.M. is $2\times3\times7=42$.

EXERCISE XI.

Find by inspection the G.C.M. of

1. 48 and 54 ; 63 and 35 ; 72 and 56.
2. 84, 132, and 204.
3. 289, 119, and 323.
4. 266, 126, 238, and 420.
5. 195, 104, 676, 208, and 546.

Find the H.C.F. of

6. 1682 and 1479.
7. 2312 and 3434.
8. 648, 396, and 936.
9. 6724, 902, and 3362.
10. 4332, 7220, 6498, and 5776.
11. 40457 and 420325.
12. 816, 4760, and 5780.
13. 855, 726, and 888.
14. 929181 and 1012891.
15. 14938 and 23474 and 27742.
16. 1058, 3128, 1794, and 2806.
17. 196×289 and 238×132.
18. 198×225 and 324×165.
19. Find the G.C.M. of the greatest and the least numbers that can be formed out of the digits 2, 4, and 3, all digits being taken together.
20. Find the greatest number which will measure each of the numbers 1019527 and 1231845 and 540810.

LEAST COMMON MULTIPLE.

58. The *least common multiple* of two or more numbers is the least number which contains each of these numbers an exact number of times, *i.e.* which can be divided by each of these numbers without a remainder. Thus 30 is the least common multiple of 2, 3, 5, and 6.

The least common multiple is sometimes called the *Lowest Common Multiple.* The letters L.C.M. are, for the sake of brevity, used to denote the expression Least or Lowest Common Multiple.

59. To find the least common multiple of two numbers:

Rule. Divide the product of the numbers by their G.C.M. The quotient will be the L.C.M. required. Or, what is the same thing, divide one of the numbers by their G.C.M. and multiply the quotient and the other number together. The product will be the L.C.M. of the numbers.

Ex. 1. Find the L.C.M. of 78 and 96.

The G.C.M. of 78 and 96 is 6;

$$\therefore \text{ the required L.C.M.} = 78 \times 96 \div 6 = 78 \times 16 = 1248.$$

60. To find the least common multiple of more than two numbers:

Rule. Find the L.C.M. of any two of the given numbers; then find the L.C.M. of this L.C.M. and another given number, and so on. The last L.C.M. will be the L.C.M. of the given numbers.

Ex. 2. Find the L.C.M. of 48, 72, and 84.

The G.C.M. of 48 and 72 is 24;

$$\begin{aligned}\therefore \text{ the L.C.M. of 48 and } 72 &= 48 \times 72 \div 24 \\ &= 48 \times 3 \\ &= 144.\end{aligned}$$

Again the G.C.M. of 144 and 84 is 12;

$$\begin{aligned}\therefore \text{ the L.C.M. of 144 and } 84 &= 144 \times 84 \div 12 \\ &= 144 \times 7 \\ &= 1008.\end{aligned}$$

$\therefore$ the required L.C.M. is 1008.

61. The method given above for finding the L.C.M. of more than two numbers may be shortened as follows:

Write down the given numbers in a line. Divide those numbers which have a common measure by that common measure and place the quotients and the undivided numbers in a line below the first. Divide those numbers in this second line that have a common measure by that common measure and then place the quotients and the undivided numbers in a new line as before. Repeat this process till a line is obtained in which no two numbers have a common measure. Then multiply all the numbers in this last line and all the common measures together. The product will be the required L.C.M.

Ex. 3. Find the L.C.M. of 12, 18, 27, 30, and 32.

$$\begin{array}{r|rrrrr}
2 & 12, & 18, & 27, & 30, & 32 \\ \hline
2 & 6, & 9, & 27, & 15, & 16 \\ \hline
3 & 3, & 9, & 27, & 15, & 8 \\ \hline
3 & 1, & 3, & 9, & 5, & 8 \\ \hline
 & 1, & 1, & 3, & 5, & 8
\end{array}$$

$$\therefore \text{ the required L.C.M.} = 8 \times 5 \times 3 \times 3 \times 3 \times 2 \times 2$$
$$= 4320.$$

NOTE.—It will be seen that in writing down any numbers in a line, if one number be a factor of another, the former may be struck out, as such omission simplifies the work without affecting in any way the principle on which the method of finding the L.C.M. is based. Thus,

Ex. 4. Find the L.C.M. of 16, 20, 24, 35, 56, 72, 96, and 105.

$$\begin{array}{r|rrrrrrrr}
2 & \cancel{16}, & 20, & \cancel{24}, & \cancel{35}, & 56, & 72, & 96, & 105 \\ \hline
2 & & 10, & & & 28, & 36, & 48, & 105 \\ \hline
2 & & \cancel{5}, & & & 14, & 18, & 24, & 105 \\ \hline
3 & & & & & \cancel{7}, & 9, & 12, & 105 \\ \hline
 & & & & & & 3, & 4, & 35
\end{array}$$

$$\therefore \text{ the required L.C.M.} = 35 \times 4 \times 3 \times 3 \times 2 \times 2 \times 2$$
$$= 10080.$$

16 is a factor of 96, 24 of 72, and 35 of 105. The numbers 16, 24, 5 are therefore crossed over for simplifying the work. For the same reason the numbers 5 and 7 are crossed over in lines 3 and 4.

62. When the numbers whose least common multiple is required are easily resolvable into factors, their least common multiple may be found out by resolving them into factors. For instance, if it be required to find the L.C.M. of 12, 16, 18, and 20, we may proceed as follows:

$$12 = 3 \times 4 = 3 \times 2 \times 2 = 3 \times 2^2;$$
$$16 = 2 \times 2 \times 2 \times 2 = 2^4;$$
$$18 = 2 \times 9 = 2 \times 3 \times 3 = 2 \times 3^2;$$
$$20 = 5 \times 4 = 5 \times 2 \times 2 = 5 \times 2^2.$$

Of the factors consisting of twos the greatest is 2^4, of threes, 3^2, and of fives, 5. Therefore $3^2 \times 2^4 \times 5$ or 720 is the L.C.M. required.

EXERCISE XII.

Find by inspection the L.C.M. of

1. 6 and 8. **2.** 9 and 12. **3.** 12 and 15.
4. 14 and 21. **5.** 24 and 36. **6.** 16 and 24.
7. 4, 6, and 8. **8.** 5, 15, and 25. **9.** 2, 4, 6, and 8.
10. 3, 6, 8, and 12.

Find the L.C.M. of

11. 325 and 625. **12.** 75 and 80. **13.** 132 and 156.
14. 112 and 168. **15.** 25, 35, and 45. **16.** 36, 42, and 48.
17. 32, 48, 40, and 14. **18.** 26, 5, 13, 15, and 40.
19. 3, 12, 28, 27, 4, and 6. **20.** 84, 96, 68, and 48.
21. 16, 30, 48, 60, 96, and 120. **22.** 132, 161, 168, and 253.
23. 259, 1443, and 273. **24.** 14938, 23474, and 32010.
25. Find the least number of which 33495 and 106260 are divisors.

MISCELLANEOUS EXAMPLES IN MEASURES AND MULTIPLES.

Ex. 1. Resolve 1575 into factors and find the least number by which it must be multiplied in order that the product may be a complete square.

$$\begin{aligned} 1575 &= 5 \times 315 \\ &= 5 \times 5 \times 63 \\ &= 5 \times 5 \times 9 \times 7 \\ &= 5 \times 5 \times 3 \times 3 \times 7 \\ &= 5^2 \times 3^2 \times 7. \end{aligned}$$

Each of the factors 5 and 3 is repeated twice in the product, while 7, only once. Therefore the number 1575 must be multiplied by 7 so as to make the product a complete square.

Ex. 2. Find the greatest number which divides 309 and 395, leaving as remainders 3 and 4 respectively.

Since the required number leaves 3 as remainder when it divides 309, it must exactly divide 309 − 3, *i.e.* 306 ; for the same reason, it must exactly divide 395 − 4, *i.e.* 391. Since the required number is the greatest, it must be equal to the G.C.M. of 306 and 391. To find the G.C.M. of 306 and 391 we have

$$
\begin{array}{llllll}
306\,)\ 391\ (\,1 \\
& \underline{306} \\
& 85\,)\ 306\ (\,3 \\
& & \underline{255} \\
& & 51\,)\ 85\ (\,1 \\
& & & \underline{51} \\
& & & 34\,)\ 51\ (\,1 \\
& & & & \underline{34} \\
& & & & 17\,)\ 51\ (\,3 \\
& & & & & \underline{51} \\
& & & & & \cdot\,\cdot
\end{array}
$$

∴ 17 is the number required.

Ex. 3. What is the least number which, when divided by 6, 8, 9, or 10, will in each case leave 3 as a remainder ?

The required number must be greater by 3 than the L.C.M. of 6, 8, 9, and 10. Therefore find the L.C.M. of 6, 8, 9, and 10, and add 3 to it.

$$
\begin{array}{r|l}
2 & 6,\ 8,\ 9,\ 10 \\ \hline
3 & 3,\ 4,\ 9,\ 5 \\ \hline
 & 1,\ 4,\ 3,\ 5
\end{array}
$$

$$
\begin{aligned}
\therefore \text{ the L.C.M.} &= 2 \times 3 \times 4 \times 3 \times 5 \\
&= 360. \\
\therefore \text{ the required number} &= 360 + 3 \\
&= 363.
\end{aligned}
$$

Ex. 4. **The G.C.M. of two numbers is 36 and their L.C.M. is 5184; if one of the numbers be 324, find the other.**

The product of the G.C.M. and the L.C.M. of two numbers is equal to their product.

$$
\begin{aligned}
\therefore \text{ the product of the two numbers} &= \text{G.C.M.} \times \text{L.C.M.} \\
&= 36 \times 5184 \\
&= 186624.
\end{aligned}
$$

As one number is 324, the other number is 186624 ÷ 324 = 576.

Ex. 5. Find the greatest number of 3 digits which is divisible by 12, 16, and 18.

The least number divisible by 12, 16, and 18 is their L.C.M. which is 144. Any multiple of this 144 will also be exactly divided by each of 12, 16, and 18. But since the number is not to exceed one thousand, the required number is 144 × 6 = 864.

EXERCISE XIII.

1. By what number less than 10 must 216 be multiplied in order that the product may be a complete square ?

2. Find all the numbers between 200 and 300 which have 53 for their G.C.M.

3. Find the greatest and the least of the numbers between 400 and 700 which are divisible by 79.

4. Divide the product of 361 and 437 by their least common multiple.

5. Find the product of the least common multiple and the greatest common measure of 12, 18, 24, and 36.

6. Find the least number by which 51 must be multiplied so that the product may be a multiple of 136.

7. Find the greatest common measure and the least common multiple of 936 and 2925.

8. Find the G.C.M. and L.C.M. of 4199, 6783, and 5187.

9. What is the least number which, when divided by 22, by 88, by 132, and by 198 gives in each case as remainder 7 ?

10. Find the least number which is divisible by each one of the first twelve numbers.

11. What is the smallest sum of money for which I can purchase an exact number of cows at 32 rupees, 25 rupees, 40 rupees, and 36 rupees each ?

12. Find the smallest sum of money for which I can buy horses at 63 rupees each, cows at 36 rupees each, or oxen at 56 rupees each.

13. Three farmers have respectively 396 rupees, 308 rupees, and 484 rupees, with which they agree to purchase a number of oxen at the highest price per head that will allow each man to invest all his money. How many oxen can each farmer buy ?

14. Two bells commence tolling at the same instant, and they toll at intervals of 45 minutes and 60 minutes respectively. When will they next toll together and how many times will each have tolled then ?

15. Find the greatest number which will divide each of the numbers 4406 and 3828, leaving as remainder 71 in each case.

16. *A*, *B*, and *C* can go round a circular course in 10 minutes, 12 minutes, and 15 minutes respectively ; if they start at the same time, when will they all next meet ?

17. Find the greatest number between five thousand and eight thousand which is divisible by each one of the odd numbers from 1 to 10.

18. Find all the numbers between 300 and 400 which are common multiples of 16 and 24.

19. Find the least number which, when diminished by 4, will contain any one of the numbers 48, 64, and 72 an exact number of times.

20. What is the greatest possible rate at which a man can travel 546 miles, 624 miles, and 702 miles in an exact number of hours ?

21. Find the greatest number that will divide 530 and 620 and will leave as remainders 13 and 9 respectively.

22. The G.C.M. of two numbers is 37, and their product is 442187 ; find their L.C.M.

23. The G.C.M. of two numbers is 18, and their L.C.M. is 270 ; if one of the numbers be 54, find the other.

24. When a number is divided by 3 it leaves as remainder 2, when divided by 5 it leaves a remainder of 4, and when divided by 7 it leaves as remainder 6 ; find the number.

25. Find the greatest number of 3 digits which is divisible by 2, 3, and 5.

26. Find the least common multiple of all the prime numbers from 1 to 10.

27. What number nearest to 200 is divisible by each one of the even digits ?

28. Find the least number of 4 digits which is divisible by 5, 7, and 9.

29. Find the least number of mangoes that can be equally distributed among 18, 16, 36, or 45 boys.

30. What sum of money between 5000 rupees and 6000 rupees will purchase an exact number of articles whether they be bought at 20, 30, 35, or 36 rupees each ?

31. A person wishes to distribute 851 plantains and 1073 mangoes equally among a number of beggars ; find the greatest number receiving the charity in this way.

32. Two men fire at a target at an interval of 16 minutes and 20 minutes respectively ; after what time will they again fire simultaneously, and how many shots will each have fired during the time ?

CHAPTER IV.

FRACTIONS.

63. A *whole number* or *integer* expresses how many times unity is repeated.

64. A *fraction* is a quantity which expresses a part or parts of anything considered as an integer or whole.

65. A fraction is denoted by two numbers, one of which is called the *numerator* and the other the *denominator.* The denominator shows into how many equal parts the whole is divided, and the numerator shows how many such parts are taken. The numerator is placed over the denominator separated from it by a line. Thus $\frac{3}{4}$ is a fraction of which the numerator is 3 and the denominator 4. $\frac{3}{4}$ of anything means that the thing is divided into 4 equal parts, 3 of which are taken in the present case. If the thing in the present case be supposed to be a rupee, $\frac{3}{4}$ of a rupee means that of its four equal parts, which are known as four-anna pieces, 3 are taken.

66. It will be seen from this that a fraction expresses what part the numerator is of the denominator, or in other words, it denotes the result of the division of the numerator by the denominator.

67. Fractions denoted in the manner described above are called *vulgar* or *common fractions.*

68. When the numerator of a fraction is less than the denominator, it is called a *proper fraction* ; as $\frac{5}{8}$, $\frac{7}{12}$.

69. An *improper fraction* is that in which the numerator *is equal to* or *greater than* the denominator ; as $\frac{7}{7}$, $\frac{15}{8}$.

70. A *mixed number* is composed of a whole number and a fraction ; as $3\frac{1}{4}$, $2\frac{3}{5}$.

71. A *simple fraction* is that in which the numerator and the denominator are simple integers ; as $\frac{2}{3}$, $\frac{13}{10}$.

72. A *complex fraction* is that in which either the numerator or the denominator or both are fractions or mixed numbers ; as $\frac{\frac{2}{3}}{7}$, $\frac{2\frac{1}{3}}{4}$, $\frac{\frac{5}{2}}{3}$, $\frac{6}{8\frac{1}{3}}$, $\frac{\frac{3}{4}}{\frac{5}{9}}$, $\frac{4\frac{3}{4}}{7\frac{1}{2}}$.

73. A *compound fraction* is a fraction of a fraction ; as $\frac{2}{5}$ of $\frac{3}{8}$.

74. The *reciprocal of a fraction* is the fraction formed by making the numerator the denominator, and the denominator the numerator. Thus $\frac{5}{9}$ is the reciprocal of $\frac{9}{5}$.

75. An integer may be expressed as a fraction by placing it as numerator and unity, *i.e.* the number 1 as denominator. Thus 6 may be written as $\frac{6}{1}$.

EXERCISE XIV.

State to what class of fractions the following belong :

1. $\frac{5}{8}$. 2. $3\frac{1}{2}$. 3. $\frac{4}{9}$. 4. $\frac{43}{34}$.

5. $\dfrac{13}{2\frac{3}{4}}$. 6. $8\frac{3}{5}$. 7. $\frac{2}{3}$ of $\frac{3}{4}$. 8. $\dfrac{6\frac{2}{3}}{9\frac{1}{4}}$.

Express in figures the following fractions

9. Three-sevenths. **10.** Eight and two thirds.

11. Nine-fifths. **12.** Twelve thirty-sevenths.

13. Nine-tenths. **14.** Seventy-five thirty-sixths.

15. Ten and a half. **16.** One hundred-and-twenty-fifth.

Read the following :

17. $\frac{3}{8}$. 18. $7\frac{4}{5}$. 19. $\frac{55}{9}$. **20.** $\frac{41}{50}$.

21. $5\frac{3}{4}$. 22. $\dfrac{93}{\frac{1}{2}}$. 23. $\frac{63}{108}$. **24.** $\frac{561}{1000}$.

25. If a unit be divided into eighty-three equal parts, how will you express in figures fifty-seven of these parts and a hundred and fifty of such parts ?

CONVERSION OF FRACTIONS FROM ONE FORM TO ANOTHER.

76. To express a mixed number as an improper fraction :

Rule. Multiply the integer by the denominator of the fraction, and add to the product the numerator. Place the result as the new numerator and retain the denominator.

Ex. 1. Express $17\frac{3}{4}$ as an improper fraction.

$$17\tfrac{3}{4}=\frac{17\times4+3}{4}=\frac{71}{4}.$$

EXERCISE XV.

Express as improper fractions :

1. $2\frac{3}{7}$. 2. $5\frac{7}{9}$. 3. $13\frac{2}{5}$. 4. $1\frac{131}{132}$.

5. $4\frac{27}{125}$. 6. $18\frac{3}{4}$. 7. $25\frac{15}{16}$. 8. $60\frac{53}{60}$.

9. Five and five-sixths. **10.** One hundred and a hundredth.

77. To express an improper fraction as a mixed number:

Rule. Divide the numerator by the denominator, put the quotient as the integer and the remainder as the numerator and the original denominator as the denominator of the fraction. If there be no remainder, the improper fraction can be reduced only to an integer or whole number, and not to a mixed number.

Ex. 1. Reduce $\frac{23}{6}$ to a mixed number.

Dividing 23 by 6 we get 3 as the quotient and 5 as the remainder.

$$\therefore \tfrac{23}{6} = 3\tfrac{5}{6}.$$

EXERCISE XVI.

Express as mixed numbers:

1. $\frac{25}{8}$.
2. $\frac{43}{5}$.
3. $\frac{29}{3}$.
4. $\frac{738}{7}$.
5. 23 elevenths.
6. 11 thirds.
7. 16 ninths.
8. Thirteen halves.
9. One thousand thirteenths.
10. Ninety-five sixths.
11. Five hundred and four fifths.

78. If the numerator and the denominator of a fraction be multiplied by the same integer the value of the fraction is not altered. Thus, if the numerator and the denominator of the fraction $\frac{3}{7}$ be multiplied by 4 we get $\frac{12}{28}$, a fraction having the same value as $\frac{3}{7}$.

79. To express an integer as a fraction having a given number for its denominator:

Rule. Multiply the integer by the given number; take this product for the new numerator and write the given number as its denominator.

Ex. Express 3 as a fraction having 7 for its denominator.

$$3 = \frac{3 \times 7}{7} = \frac{21}{7}.$$

The integer 3 may be written as $\frac{3}{1}$, an improper fraction. As the numerator and the denominator of a fraction may be multiplied by the same number without altering its value, we may multiply both 3 and 1 by 7 and the result, *i.e.* $\frac{21}{7}$, will have the same value as $\frac{3}{1}$, *i.e.* 3.

EXERCISE XVII.

1. Express 2, 5 and 7, each as a fraction having 9 for its denominator.

2. Express 29, 32 and 43, each as a fraction having 50 for its denominator.

3. Express 23, 6 and 32, each as a fraction having 9 for its denominator.

4. Express 125 as a fraction having 8 for its denominator.

5. Express 99 in the form of fractions having 8, 12, 63 and 101 for their denominators respectively.

80. If the numerator and the denominator of a fraction be divided by the same integer the value of the fraction is not altered.

Thus, if the numerator and the denominator of the fraction $\frac{18}{24}$ be divided by 6 we get $\frac{3}{4}$, a fraction having the same value as $\frac{18}{24}$.

81. A fraction is said to be in its simplest form, or its lowest terms, when its numerator and denominator have no common factor, *i.e.* when its numerator and denominator are prime to one another.

Thus $\frac{7}{11}$ and $\frac{3}{11}$ are fractions in their simplest forms or lowest terms.

82. To reduce a fraction to its simplest form, *i.e.* to its lowest terms :

Rule. Divide the numerator and the denominator of the fraction by all the factors common to them, *i.e.* by their G.C.M.

Ex. 1. Reduce $\frac{36}{84}$ to its lowest terms.

We see that 3 is a factor of both the numerator and the denominator ; dividing therefore both by 3, we get $\frac{12}{28}$.

Again we see that 4 is a common factor of 12 and 28 ; dividing therefore by 4, we obtain $\frac{3}{7}$.

As 3 and 7 are prime to each other, the fraction $\frac{36}{84}$, when reduced to its lowest terms or simplest form, becomes $\frac{3}{7}$.

It will be seen that instead of dividing the numerator and the denominator successively by 3 and 4, if we divide them by 12, the G.C.M. of the numerator and the denominator, the result will be the same.

Ex. 2. Reduce $\frac{391}{437}$ to its lowest terms.

The G.C.M. of 391 and 437 is 23.

$$\therefore \frac{391}{437} = \frac{391 \div 23}{437 \div 23} = \frac{17}{19}.$$

Ex. 3. Reduce $\frac{1365}{2205}$ to its lowest terms.

Here we see that 5 is a factor of both the numerator and the denominator. Therefore dividing by 5, we get $\frac{273}{441}$.

Again we see that 3 is a factor. Therefore dividing by 3, we get $\frac{91}{147}$.

Lastly we see that 7 is a common factor. Therefore dividing by 7, we get $\frac{13}{21}$.

As 13 and 21 are prime to each other, the given fraction in its lowest terms is $\frac{13}{21}$.

The whole process may be represented as follows :

$$\frac{\overset{\overset{\overset{13}{\cancel{91}}}{\cancel{273}}}{\cancel{1365}}}{\underset{\underset{\underset{21}{\cancel{147}}}{\cancel{441}}}{\cancel{2205}}} = \frac{13}{21}.$$

EXERCISE XVIII.

Reduce to their lowest terms the following fractions :

1. $\frac{24}{32}$. 2. $\frac{36}{48}$. 3. $\frac{45}{75}$. 4. $\frac{39}{57}$.

5. $\frac{108}{124}$. 6. $\frac{125}{175}$. 7. $\frac{68}{85}$. 8. $\frac{114}{145}$.

9. $\frac{224}{336}$. 10. $\frac{105}{112}$. 11. $\frac{153}{204}$. 12. $\frac{324}{720}$.

13. $\frac{357}{425}$. 14. $\frac{855}{888}$. 15. $\frac{459}{561}$. 16. $\frac{429}{715}$.

17. $\frac{2314}{3721}$. 18. $\frac{2793}{3591}$. 19. $\frac{7056}{7392}$. 20. $\frac{5929}{7007}$.

21. $\frac{5784}{6914}$. 22. $\frac{7241}{10907}$. 23. $\frac{2108}{3813}$. 24. $\frac{4844}{5556}$.

25. $\frac{7623}{8316}$. 26. $\frac{14701}{17593}$. 27. $\frac{2970}{22869}$. 28. $\frac{64176}{119184}$.

29. $\frac{39168}{329472}$. 30. $\frac{428571}{999999}$. 31. $\frac{769230}{999999}$.

83. To convert a compound fraction into a simple one :

Rule. Multiply together the several numerators for the numerator of the simple fraction and the several denominators for its denominator.

If any of the fractions forming part of the compound fraction be a mixed number, it must be reduced to an improper fraction before the numerators and the denominators are multiplied.

Ex. 1. Reduce $\frac{5}{19}$ of $3\frac{2}{3}$ to a simple fraction.

$$\tfrac{5}{19} \text{ of } 3\tfrac{2}{3} = \tfrac{5}{19} \text{ of } \tfrac{11}{3} = \tfrac{5 \times 11}{19 \times 3} = \tfrac{55}{57}.$$

84. If there be any factor common to the numerator and the denominator, that factor should be struck out.

Ex. 2. Reduce $\frac{6}{7}$ of $2\frac{4}{5}$ of $\frac{10}{9}$ to a simple fraction.

$$\tfrac{6}{7} \text{ of } 2\tfrac{4}{5} \text{ of } \tfrac{10}{9} = \tfrac{6}{7} \text{ of } \tfrac{14}{5} \text{ of } \tfrac{10}{9}$$

$$= \frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{14}} \times \overset{2}{\cancel{10}}}{\underset{1}{\cancel{7}} \times \underset{1}{\cancel{5}} \times \underset{3}{\cancel{9}}} = \frac{2 \times 2 \times 2}{3} = \frac{8}{3}.$$

Here the factors 7, 5 and 3 which are common to the numerator and the denominator have been struck out.

When the resulting simple fraction is an improper fraction, it is usual to state the result as a mixed number. Thus in the above example the resulting fraction $\frac{8}{3}$ should be written as $2\frac{2}{3}$.

EXERCISE XIX.

Reduce the following compound fractions to simple ones.

1. $\frac{2}{3}$ of $\frac{5}{6}$.
2. $\frac{4}{9}$ of $\frac{7}{8}$.
3. $\frac{3}{5}$ of $3\frac{3}{4}$.
4. $7\frac{1}{3}$ of $4\frac{4}{11}$.
5. $\frac{13}{15}$ of $\frac{105}{52}$.
6. $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{9}{10}$.
7. $\frac{5}{8}$ of $2\frac{2}{3}$ of $\frac{3}{4}$.
8. $\frac{4}{5}$ of $\frac{6}{7}$ of $11\frac{2}{3}$.
9. $\frac{3}{5}$ of $1\frac{1}{3}$ of $6\frac{1}{4}$.
10. $1\frac{1}{4}$ of $4\frac{4}{5}$ of $1\frac{1}{6}$.
11. $9\frac{5}{8}$ of $1\frac{71}{121}$ of $1\frac{1}{104}$ of $2\frac{45}{49}$.
12. $\frac{3}{4}$ of $6\frac{5}{18}$ of $\frac{4}{7}$ of $1\frac{2}{25}$ of $16\frac{2}{3}$.
13. $2\frac{18}{49}$ of $\frac{19}{255}$ of $1\frac{6}{29}$ of $1\frac{13}{38}$.
14. $\frac{5}{8}$ of $\frac{4}{11}$ of $\frac{7}{20}$ of $5\frac{1}{2}$ of $1\frac{1}{7}$.
15. $4\frac{3}{35}$ of $\frac{27}{58}$ of $\frac{15}{44}$ of $5\frac{8}{39}$.
16. $\frac{18}{25}$ of $4\frac{1}{4}$ of $\frac{37}{81}$ of $5\frac{1}{51}$ of $1\frac{10}{111}$ of $6\frac{21}{64}$.
17. $\frac{25}{117}$ of $14\frac{7}{16}$ of $\frac{72}{77}$ of $1\frac{29}{75}$.
18. $\frac{2}{5}$ of $\frac{4}{7}$ of $\frac{27}{56}$ of $1\frac{3}{4}$ of $\frac{14}{81}$ of $15\frac{1}{3}$.
19. $1\frac{67}{186}$ of $\frac{144}{161}$ of $2\frac{55}{112}$ of $1\frac{97}{99}$.
20. $\frac{1}{13}$ of $3\frac{2}{3}$ of $4\frac{1}{9}$ of $\frac{65}{495}$ of $2\frac{11}{37}$ of $13\frac{1}{2}$.

85. To reduce two or more fractions to equivalent fractions having a common denominator :

Rule. Take any common multiple of the denominators. Divide this multiple by the denominator of each fraction and multiply by the quotient the corresponding numerator.

Place these several products for new numerators, and the multiple for the common denominator.

Ex. 1. Reduce $\frac{5}{6}$ and $\frac{3}{8}$ to equivalent fractions having a common denominator. 24, 48, 72, etc., are common multiples of 6 and 8. Take 48 as the common multiple. Then $48 \div 6 = 8$, and $48 \div 8 = 6$.

Therefore $5 \times 8 = 40$ is the new numerator of the first fraction, and $3 \times 6 = 18$ is the new numerator of the second fraction. The given fractions, therefore, may be written as $\frac{40}{48}$ and $\frac{18}{48}$.

86. As it is obviously convenient to have the common denominator selected as small as possible, the student should, unless otherwise directed, take the least common multiple of the denominators of the given fractions for the required common denominator.

Ex. 2. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{9}$ and $\frac{8}{15}$ to equivalent fractions with the least common denominator.

The L.C.M. of 3, 5, 9 and 15 is 45.

$$\begin{array}{lrl} \text{Now} & 45 \div 3 = 15 \text{ and } \therefore & \frac{2}{3} = \frac{30}{45}. \\ & 45 \div 5 = 9 \text{ and } \therefore & \frac{4}{5} = \frac{36}{45}. \\ & 45 \div 9 = 5 \text{ and } \therefore & \frac{7}{9} = \frac{35}{45}. \\ & 45 \div 15 = 3 \text{ and } \therefore & \frac{8}{15} = \frac{24}{45}. \end{array}$$

Therefore the equivalent fractions with the least common denominator are $\frac{30}{45}$, $\frac{36}{45}$, $\frac{35}{45}$ and $\frac{24}{45}$.

87. Of two fractions having a common denominator, the *greater* is that which has the greater numerator.

Thus, of the two fractions $\frac{4}{7}$ and $\frac{3}{7}$, $\frac{4}{7}$ is obviously the greater.

Similarly, if there are more than two fractions with common denominator, the greatest is that which has the greatest numerator, the next greatest is that which has the next greatest numerator, and so on, the least being that which has the least numerator.

Thus, of the fractions $\frac{4}{17}$, $\frac{16}{17}$, $\frac{9}{17}$ and $\frac{3}{17}$, the greatest is $\frac{16}{17}$, the next greatest is $\frac{9}{17}$, the next after that in magnitude is $\frac{4}{17}$, and the least is $\frac{3}{17}$.

88. To compare the values of different fractions :

Rule. Reduce the given fractions to equivalent fractions having a common denominator. The values of the fractions then will depend upon the magnitudes of the new numerators.

Ex. 1. Which is the greater, $\frac{5}{8}$ or $\frac{7}{12}$?

Reducing the fractions to equivalent ones, with the least common denominator, we have $\frac{15}{24}$ in place of $\frac{5}{8}$ and $\frac{14}{24}$ in place of $\frac{7}{12}$ respectively.

As 15 is greater than 14, $\frac{5}{8}$ is greater than $\frac{7}{12}$.

Ex. 2. Arrange in order of magnitude the fractions $\frac{2}{5}$, $\frac{4}{7}$, $\frac{3}{8}$ and $\frac{5}{9}$.

The L.C.M. of 5, 7, 8 and 9 is 2520.

$$\frac{2520}{5}=504\,; \quad \therefore\ \frac{2}{5}=\frac{1008}{2520}.$$
$$\frac{2520}{7}=360\,; \quad \therefore\ \frac{4}{7}=\frac{1440}{2520}.$$
$$\frac{2520}{8}=315\,; \quad \therefore\ \frac{3}{8}=\frac{945}{2520}.$$
$$\frac{2520}{9}=280\,; \quad \therefore\ \frac{5}{9}=\frac{1400}{2520}.$$

It will be seen that the second of the given fractions is the greatest, the next highest is the fourth, next to that is the first, and the third is the least of all.

The given fractions when arranged in the order of magnitude stand thus :

$$\frac{4}{7},\ \frac{5}{9},\ \frac{2}{5} \text{ and } \frac{3}{8}.$$

Here the greatest comes first and the least last, and this is called the descending order of magnitude. If the least is placed first and the greatest last and the others correspondingly, the order would be called the ascending order. Thus in the above example the given fractions when arranged in ascending order would stand thus :

$$\frac{3}{8},\ \frac{2}{5},\ \frac{5}{9} \text{ and } \frac{4}{7}.$$

EXERCISE XX.

Express with a common denominator the following fractions :

1. $\frac{2}{3}$, $\frac{3}{4}$.
2. $\frac{4}{5}$, $\frac{3}{7}$.
3. $\frac{5}{12}$, $\frac{2}{9}$.
4. $\frac{12}{25}$, $\frac{17}{35}$.
5. $\frac{3}{16}$, $\frac{11}{60}$.
6. $\frac{1}{72}$, $\frac{1}{84}$.
7. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
8. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{5}{9}$.
9. $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{18}$.
10. $\frac{3}{7}$, 4, $\frac{5}{21}$.
11. $\frac{1}{9}$, $3\frac{1}{2}$, 2.
12. $\frac{1}{25}$, $\frac{2}{45}$, $\frac{3}{35}$.
13. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{1}{20}$, $\frac{5}{18}$.
14. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
15. $\frac{5}{7}$, 4, $6\frac{1}{3}$, $\frac{2}{21}$.
16. $\frac{3}{10}$, $\frac{1}{22}$, $\frac{9}{55}$, $\frac{7}{60}$.
17. Which is greater, $\frac{5}{12}$ or $\frac{7}{16}$?
18. Which is loss, $\frac{3}{4}$ of $\frac{8}{15}$, or $\frac{25}{32}$, or $\frac{18}{35}$?

19. Find the greatest and the least of the fractions

$$\frac{12}{25}, \frac{7}{16}, \frac{11}{24} \text{ and } \frac{3}{8}.$$

20. Arrange the fractions $\frac{5}{12}$, $\frac{7}{18}$, $\frac{9}{20}$ and $\frac{8}{15}$ of $1\frac{1}{4}$.

21. Compare the fractions $\frac{4+5}{5+7}$, $\frac{4}{5}$ and $\frac{5}{7}$.

ADDITION OF FRACTIONS.

89. To add together two or more fractions :

Rule. If all the fractions have a common denominator, add all the numerators together and write their sum for the new numerator, retaining the common denominator. If the fractions have not the same denominator reduce them to equivalent fractions having the least common denominator, and then add as above. Reduce the result, if the numerator and the denominator have a common factor, to its lowest terms, and reduce it to a mixed number if it be an improper fraction.

Ex. 1. Add together $\frac{2}{13}$, $\frac{5}{13}$, $\frac{3}{13}$.

$$\frac{2}{13}+\frac{5}{13}+\frac{3}{13}=\frac{2+5+3}{13}=\frac{10}{13}.$$

Ex. 2. Add together $\frac{1}{5}$, $\frac{2}{7}$ and $\frac{4}{15}$.

The L.C.M. of the denominators is 105, and the fractions with 105 as denominator, equivalent to the given ones, are $\frac{21}{105}$, $\frac{30}{105}$, $\frac{28}{105}$;

$$\therefore \text{ the required sum } = \frac{21+30+28}{105}=\frac{79}{105}.$$

Ex. 3. Add together $\frac{5}{16}$, $\frac{7}{36}$ and $\frac{2}{9}$.

The L.C.M. of the denominators is 144, and the fractions equivalent to the given ones are $\frac{45}{144}$, $\frac{28}{144}$, $\frac{32}{144}$;

$$\therefore \text{ the required sum } = \frac{45+28+32}{144}=\frac{105}{144}=\frac{35}{48}$$

(as 3 is a common factor of both 105 and 144).

Ex. 4. Add together $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{9}$.

Reducing the fractions to equivalent ones having a common denominator, we get $\frac{45}{90}$, $\frac{60}{90}$, $\frac{72}{90}$, $\frac{50}{90}$;

$$\therefore \text{ the required sum } = \frac{45+60+72+50}{90}=\frac{227}{90}=2\frac{47}{90}$$

(reducing the improper fraction to a mixed number).

90. If any of the given fractions be compound fractions, they should be reduced to simple ones before applying the rule.

Ex. 5. Add together $\frac{7}{9}$, $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{5}{8}$.

Since $\frac{2}{3}$ of $\frac{5}{8}=\frac{5}{12}$, we have

$$\frac{7}{9}+\frac{2}{3}+\frac{2}{3}\text{ of }\frac{5}{8}=\frac{7}{9}+\frac{2}{3}+\frac{5}{12}=\frac{28}{36}+\frac{24}{36}+\frac{15}{36}=\frac{67}{36}=1\frac{31}{36}.$$

If any of the given fractions be a mixed number, it should be reduced to an improper fraction before proceeding to add.

Ex. 6. Add together $\frac{3}{7}$, $2\frac{2}{3}$ and $\frac{5}{6}$.

$$\begin{aligned}\frac{3}{7}+2\frac{2}{3}+\frac{5}{6}&=\frac{3}{7}+\frac{8}{3}+\frac{5}{6}\\&=\frac{18}{42}+\frac{112}{42}+\frac{35}{42}\\&=\frac{165}{42}=\frac{55}{14}=3\frac{13}{14}.\end{aligned}$$

91. If any of the given fractions are mixed numbers, it is convenient to add together their integral parts, then add together their fractional parts and the remaining fractions, and finally add these two sums together.

Ex. 7. Add together $1\frac{2}{3}$, $2\frac{4}{5}$, $\frac{5}{6}$, $4\frac{3}{4}$ and $\frac{7}{12}$.

Here we first add together 1, 2 and 4. Their sum is 7. We next add $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{3}{4}$ and $\frac{7}{12}$. The sum of these is

$$\frac{40+48+50+45+35}{60}=\frac{218}{60}=\frac{109}{30}=3\frac{19}{30};$$

we then add together 7 and $3\frac{19}{30}$;

$$\therefore\ \text{the required sum}=7+3\frac{19}{30}=10\frac{19}{30}.$$

92. If any of the given fractions be not in its lowest terms the work is sometimes simplified by reducing it to its lowest terms. The work is also simplified by reducing improper fractions to whole or mixed numbers, as the case may be, before proceeding to add.

Ex. 8. Add together $\frac{5}{24}$, $\frac{16}{36}$ and $\frac{21}{56}$.

$$\frac{16}{36}=\frac{4}{9}\text{ and }\frac{21}{56}=\frac{3}{8}.$$

$$\begin{aligned}\therefore\ \frac{5}{24}+\frac{16}{36}+\frac{21}{56}&=\frac{5}{24}+\frac{4}{9}+\frac{3}{8}\\&=\frac{15}{72}+\frac{32}{72}+\frac{27}{72}\\&=\frac{15+32+27}{72}=\frac{74}{72}=\frac{37}{36}=1\frac{1}{36}.\end{aligned}$$

Ex. 9. Add together $1\frac{1}{4}$, $\frac{79}{12}$, 7, $\frac{4}{5}$ of $1\frac{7}{8}$ and $\frac{68}{144}$.

$$\frac{79}{12}=6\frac{7}{12}\,;\quad \frac{4}{5} \text{ of } 1\frac{7}{8}=\frac{4}{5} \text{ of } \frac{15}{8}=\frac{3}{2}=1\frac{1}{2}\,;\quad \frac{68}{144}=\frac{17}{36}.$$

$$\begin{aligned}\therefore\ 1\tfrac{1}{4}+\tfrac{79}{12}+7+\tfrac{4}{5} \text{ of } 1\tfrac{7}{8}+\tfrac{68}{144}&=1\tfrac{1}{4}+6\tfrac{7}{12}+7+1\tfrac{1}{2}+\tfrac{17}{36}\\&=1+6+7+1+\tfrac{1}{4}+\tfrac{7}{12}+\tfrac{1}{2}+\tfrac{17}{36}\\&=(1+6+7+1)+\left(\tfrac{9+21+18+17}{36}\right)\\&=15+\tfrac{65}{36}=15+1\tfrac{29}{36}=16\tfrac{29}{36}.\end{aligned}$$

EXERCISE XXI.

Find the sum of :

1. $\frac{1}{2}, \frac{1}{3}$.
2. $\frac{5}{6}, \frac{7}{9}$.
3. $\frac{4}{5}, \frac{5}{6}$.
4. $\frac{7}{12}, \frac{9}{16}$.
5. $\frac{10}{25}, \frac{3}{5}$.
6. $\frac{1}{32}, \frac{1}{48}$.
7. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$.
8. $\frac{1}{3}, \frac{2}{5}, \frac{4}{9}$.
9. $\frac{6}{12}, \frac{2}{3}, \frac{15}{25}$.
10. $\frac{15}{36}$, 2, $\frac{5}{16}, \frac{19}{20}$.
11. $\frac{1}{60}, \frac{1}{72}, \frac{1}{84}, \frac{1}{96}$.
12. $2\frac{3}{4}, 3\frac{7}{10}, \frac{1}{25}, 1\frac{7}{8}$.
13. $\frac{1}{9}, 3\frac{2}{15}, \frac{4}{45}, 2\frac{5}{12}$.
14. $\frac{1}{2}, \frac{3}{8}, \frac{4}{15}, \frac{7}{24}$.
15. $\frac{7}{15}, \frac{9}{17}, \frac{13}{25}, \frac{23}{24}$.
16. Add together 10, $7\frac{4}{9}$, $\frac{5}{27}$, $4\frac{2}{3}$ and $3\frac{1}{3}$ of $\frac{1}{9}$.
17. Add together $\frac{2}{3}$ of $\frac{1}{7}$ of $8\frac{3}{4}$ and $\frac{1}{4}$ of $\frac{2}{5}$.
18. Add together $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{1}{5}$ and $5\frac{1}{6}$.
19. Add $\frac{542}{813}$ to its reciprocal.
20. Add $\frac{153}{561}$ to the sum of $\frac{148}{407}$ and $\frac{172}{473}$.
21. Add together $2\frac{3}{4}$, $3\frac{2}{3}$ of $2\frac{7}{8}$ of $\frac{3}{4}$ and $\frac{1}{5}$ of $3\frac{1}{\text{[illegible]}}$.

Find the value of :

22. $\frac{17}{12}+2\frac{1}{5}+\frac{15}{9}+3\frac{1}{2}+4\frac{1}{6}+\frac{5}{3}$.
23. $4\frac{1}{3}+\frac{3}{4}$ of $\frac{7}{8}+\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{8}$.
24. $\frac{2}{3}$ of $\frac{5}{7}$ of $44\frac{1}{10}+\frac{3}{8}$ of $\frac{3}{11}$ of $9\frac{7}{9}+\frac{8}{23}$ of 1863.

SUBTRACTION OF FRACTIONS.

93. To subtract one fraction from another :

Rule. If the fractions have a common denominator, take the difference of the numerators for the new numerator and retain the common denominator. If the fractions have not the same denominator reduce them to equivalent ones having the least common denominator and then proceed as above. Reduce the result to its lowest terms or to a mixed number as may be necessary.

Compound fractions, mixed numbers and fractions not expressed in their lowest terms, should be treated as in addition before proceeding to subtract.

Ex. 1. Subtract $\frac{13}{24}$ from $\frac{17}{24}$.

$$\frac{17}{24}-\frac{13}{24}=\frac{17-13}{24}=\frac{4}{24}=\frac{1}{6}.$$

Ex. 2. Subtract $\frac{9}{32}$ from $\frac{7}{24}$.

L.C.M. of 24 and 32 is 96.

$$\therefore\ \frac{7}{24}-\frac{9}{32}=\frac{28}{96}-\frac{27}{96}=\frac{28-27}{96}=\frac{1}{96}.$$

Ex. 3. Subtract $\frac{2}{3}$ of $\frac{6}{7}$ from $\frac{68}{112}$.

$$\frac{2}{3}\text{ of }\frac{6}{7}=\frac{4}{7}\text{ and }\frac{68}{112}=\frac{17}{28}.$$

$$\therefore\ \frac{68}{112}-\frac{2}{3}\text{ of }\frac{6}{7}=\frac{17}{28}-\frac{4}{7}=\frac{17}{28}-\frac{16}{28}=\frac{17-16}{28}=\frac{1}{28}.$$

Ex. 4. Subtract $3\frac{5}{8}$ from $5\frac{2}{3}$.

$$3\tfrac{5}{8}=\frac{29}{8};\quad 5\tfrac{2}{3}=\frac{17}{3}.$$

$$\therefore\ 5\tfrac{2}{3}-3\tfrac{5}{8}=\frac{17}{3}-\frac{29}{8}$$

$$=\frac{136}{24}-\frac{87}{24}=\frac{136-87}{24}=\frac{49}{24}=2\tfrac{1}{24}.$$

94. When one mixed number is to be subtracted from another mixed number, if the fractional part of the former be less than the fractional part of the latter, it is convenient to subtract the fractional part from the fractional part and the integer from the integer. The remainders added together are the required result. Thus

$$\begin{aligned}5\tfrac{2}{3}-3\tfrac{5}{8}&=5-3+\tfrac{2}{3}-\tfrac{5}{8}\\&=(5-3)+\left(\tfrac{16-15}{24}\right)\\&=2+\tfrac{1}{24}\\&=2\tfrac{1}{24}.\end{aligned}$$

95. If the fractional part of the subtrahend be greater than the other fractional part, increase the latter by 1, taking that 1 from the integral portion of the minuend. Then proceed as above.

Ex. 5. Subtract $13\frac{5}{9}$ from $17\frac{2}{5}$.

$$\begin{aligned} 17\tfrac{2}{5} - 13\tfrac{5}{9} &= (16 + 1\tfrac{2}{5}) - 13\tfrac{5}{9} \\ &= 16 - 13 + 1\tfrac{2}{5} - \tfrac{5}{9} \\ &= 3 + \tfrac{7}{5} - \tfrac{5}{9} \\ &= 3 + \tfrac{63-25}{45} \\ &= 3 + \tfrac{38}{45} = 3\tfrac{38}{45}. \end{aligned}$$

Ex. 6. Subtract $\frac{6}{7}$ of $1\frac{4}{9}$ from $2\frac{11}{12}$ of $7\frac{1}{5}$.

$\frac{6}{7}$ of $1\frac{4}{9} = \frac{6}{7}$ of $\frac{13}{9} = \frac{26}{21} = 1\frac{5}{21}$.

$2\frac{11}{12}$ of $7\frac{1}{5} = \frac{35}{12}$ of $\frac{36}{5} = 21$.

$$\begin{aligned} \therefore\ 2\tfrac{11}{12} \text{ of } 7\tfrac{1}{5} - \tfrac{6}{7} \text{ of } 1\tfrac{4}{9} &= 21 - 1\tfrac{5}{21} \\ &= 20 + 1 - 1\tfrac{5}{21} \\ &= 20 - 1 + 1 - \tfrac{5}{21} \\ &= 19 + \tfrac{21-5}{21} \\ &= 19 + \tfrac{16}{21} \\ &= 19\tfrac{16}{21}. \end{aligned}$$

96. As in the case of integers, when an expression consists of several fractions, some of which are preceded by the sign + and the others by the sign −, those which are preceded by the sign + should be added together, then those which are preceded by the sign − should be added together, and then the latter sum should be deducted from the former.

Ex. 7. Find the value of $\frac{32}{35} - \frac{7}{25} - \frac{5}{21}$.

L.C.M. of 35, 25 and 21 is 525;

$$\begin{aligned} \therefore\ \tfrac{32}{35} - \tfrac{7}{25} - \tfrac{5}{21} &= \tfrac{32\times15}{525} - \tfrac{7\times21}{525} - \tfrac{5\times25}{525} \\ &= \tfrac{480}{525} - \tfrac{147}{525} - \tfrac{125}{525} \\ &= \tfrac{480-(147+125)}{525} \\ &= \tfrac{480-272}{525} = \tfrac{208}{525}. \end{aligned}$$

Ex. 8. Find the value of

$$1\tfrac{1}{2}-2\tfrac{3}{4}+7\tfrac{1}{5}-\tfrac{5}{6}\text{ of }3\tfrac{1}{5}+13-\tfrac{294}{315}.$$

$$\tfrac{5}{6}\text{ of }3\tfrac{1}{5}=\tfrac{5}{6}\text{ of }\tfrac{16}{5}=\tfrac{8}{3}=2\tfrac{2}{3};$$

$$\text{and}\quad \tfrac{294}{315}=\tfrac{98}{105}=\tfrac{14}{15};$$

$$\begin{aligned}\therefore\text{ the given expression}&=1\tfrac{1}{2}-2\tfrac{3}{4}+7\tfrac{1}{5}-2\tfrac{2}{3}+13-\tfrac{14}{15}\\&=1\tfrac{1}{2}+7\tfrac{1}{5}+13-2\tfrac{3}{4}-2\tfrac{2}{3}-\tfrac{14}{15}\\&=(21+\tfrac{1}{2}+\tfrac{1}{5})-(2\tfrac{3}{4}+2\tfrac{2}{3}+\tfrac{14}{15})\\&=(21+\tfrac{5}{10}+\tfrac{2}{10})-(4+\tfrac{3}{4}+\tfrac{2}{3}+\tfrac{14}{15})\\&=(21+\tfrac{7}{10})-(4+\tfrac{45}{60}+\tfrac{40}{60}+\tfrac{56}{60})\\&=21\tfrac{7}{10}-(4+\tfrac{141}{60})\\&=21\tfrac{7}{10}-(4+2\tfrac{7}{20})\\&=21\tfrac{7}{10}-6\tfrac{7}{20}\\&=21-6+\tfrac{7}{10}-\tfrac{7}{20}\\&=15+\tfrac{14-7}{20}=15\tfrac{7}{20}.\end{aligned}$$

EXERCISE XXII.

1. Subtract $\frac{3}{5}$ from $\frac{4}{5}$.
2. Subtract $\frac{2}{3}$ from $\frac{3}{4}$.
3. Subtract $\frac{5}{12}$ from $\frac{7}{16}$.
4. Subtract $\frac{7}{36}$ from $\frac{9}{32}$.
5. Subtract $\frac{16}{165}$ from $\frac{15}{132}$.
6. Subtract $\frac{1}{342}$ from $\frac{1}{306}$.
7. Subtract $4\frac{1}{4}$ from $5\frac{2}{3}$.
8. Subtract $2\frac{3}{4}$ from $3\frac{1}{5}$.
9. Subtract $2\frac{5}{6}$ from $9\frac{7}{8}$.
10. Subtract $1\frac{8}{9}$ from $3\frac{6}{7}$.
11. Take $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ from $\frac{2}{3}$ of $1\frac{1}{2}$.

Find the difference between

12. $\frac{17}{25}$ and $\frac{13}{27}$.
13. $\frac{5}{84}$ and $\frac{7}{96}$.
14. $\frac{43}{187}$ and $\frac{53}{238}$.
15. $\frac{413}{472}$ and $\frac{426}{497}$.

Find the value of

16. $\frac{4}{11}$ of $2\frac{5}{14}-\frac{2}{7}$ of $1\frac{1}{18}$.
17. $3\frac{1}{2}+2\frac{1}{12}+3\frac{3}{16}-7\frac{19}{24}$.
18. $1\frac{3}{8}+3\frac{1}{5}-5\frac{5}{6}+2\frac{1}{4}-1\frac{1}{12}$.
19. Which is greater, $\frac{6331}{15584}$ or $\frac{6821}{17232}$, and by how much ?
20. Subtract $3\frac{7}{15}+2\frac{11}{12}$ from $3\frac{4}{5}+1\frac{5}{16}+4\frac{3}{20}$.
21. Find the value of $\frac{20}{105}+\frac{4}{35}$ of $1\frac{3}{4}-\frac{7}{11}$ of $3\frac{2}{3}$ of $\frac{6}{49}$.
22. Subtract $4\frac{33}{35}$ from $\frac{4}{21}+13\frac{3}{10}+2\frac{1}{7}+\frac{1}{6}$
23. What must be added to $7\frac{3}{5}$ to make $24\frac{7}{12}$?

24. Find the difference between $\frac{3}{104}$ of $29\frac{1}{2}$ and $\frac{7}{26}$ of $\frac{5}{14}$ of $3\frac{1}{4}$.

25. Which is greater, $2\frac{1}{3}$ of $5\frac{1}{6}$ or $1\frac{7}{8}$ of $6\frac{1}{15}$, and by how much ?

26. Take $\frac{3}{4}$ of $\frac{3}{7}$ of 84 from $\frac{3}{5}$ of $\frac{2}{11}$ of $504\frac{1}{6}$.

27. Simplify $4\frac{4}{7}$ of $3\frac{1}{16} - 2\frac{2}{3}$ of $1\frac{5}{16} + \frac{8}{9} - 1\frac{3}{4}$ of $4\frac{4}{7}$ of $\frac{5}{14}$.

28. Find the difference of the reciprocals of $\frac{4}{5}$ and $5\frac{1}{3}$.

29. Subtract the difference of the greatest and the least of the fractions $\frac{171}{323}$, $\frac{12}{25}$ and $\frac{253}{561}$ from the remaining fraction.

30. What must be added to $\frac{1}{4} + \frac{3}{16} + \frac{2}{15} + \frac{7}{48}$ that the result may be the least possible integer ?

MULTIPLICATION OF FRACTIONS.

97. To find the product of two or more fractions:

Rule. Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator. Reduce the result to its lowest terms or to a mixed number, as may be necessary.

If any of the given fractions be a mixed number, it should be reduced to an improper fraction before multiplying.

If any numerator and any denominator have a common factor, the work is simplified by striking out that factor.

Ex. 1. Multiply $\frac{4}{5}$ by $\frac{7}{9}$.

$$\frac{4}{5} \times \frac{7}{9} = \frac{4 \times 7}{5 \times 9} = \frac{28}{45}.$$

Ex. 2. Multiply together $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{16}$.

$$\frac{1}{2} \times \frac{3}{4} \times \frac{5}{8} \times \frac{7}{16} = \frac{3 \times 5 \times 7}{2 \times 4 \times 8 \times 16} = \frac{105}{1024}.$$

Since an integer may be written as an improper fraction with unity or 1 as denominator, if it be required to multiply a fraction by an integer, the product may be obtained by multiplying the numerator of the fraction by the given integer ; also, where convenient, by dividing the denominator of the fraction by the integer.

Ex. 3. Multiply $\frac{3}{5}$ by 6.

$$\frac{3}{5} \times 6 = \frac{3 \times 6}{5} = \frac{18}{5} = 3\frac{3}{5}.$$

Ex. 4. Multiply $\frac{18}{35}$ by 7.

$$\frac{18}{35} \times 7 = \frac{18}{35 \div 7} = \frac{18}{5} = 3\frac{3}{5}.$$

Ex. 5. Find the product of

$$4\tfrac{1}{6},\ \tfrac{9}{10},\ 1\tfrac{13}{15},\text{ and }\tfrac{35}{48}.$$

$$4\tfrac{1}{6}\times\frac{9}{10}\times1\frac{13}{15}\times\frac{35}{48}=\frac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{6}}}\times\frac{\overset{1}{\cancel{\overset{3}{\cancel{9}}}}}{\underset{2}{\cancel{10}}}\times\frac{\overset{7}{\cancel{28}}}{\underset{1}{\cancel{\underset{3}{\cancel{15}}}}}\times\frac{\overset{7}{\cancel{35}}}{\underset{12}{\cancel{48}}}$$

$$=\frac{5\times7\times7}{2\times2\times12}$$

$$=\frac{245}{48}$$

$$=5\tfrac{5}{48}.$$

Ex. 6. Multiply $15\frac{2}{3}+6-\frac{1}{3}$ by $\frac{1}{2}+\frac{3}{8}$ of $\frac{16}{21}$.

$$15\tfrac{2}{3}+6-\tfrac{1}{3}=15+6+\tfrac{2}{3}-\tfrac{1}{3}=21+\tfrac{2-1}{3}=21+\tfrac{1}{3}=\tfrac{64}{3}.$$

$$\tfrac{1}{2}+\tfrac{3}{8}\text{ of }\tfrac{16}{21}=\tfrac{1}{2}+\tfrac{2}{7}=\tfrac{7}{14}+\tfrac{4}{14}=\tfrac{11}{14},$$

and
$$\frac{\overset{32}{\cancel{64}}}{3}\times\frac{11}{\underset{7}{\cancel{14}}}=\frac{32\times11}{3\times7}.$$

$$\therefore\ \text{the required product}=\frac{32\times11}{3\times7}=\frac{352}{21}=16\tfrac{16}{21}.$$

Ex. 7. Multiply $2\frac{1}{2}$ of $3\frac{1}{3}$ of $4\frac{1}{5}$ of 6 by $\frac{2}{9}$ of $\frac{12}{35}$.

$$2\tfrac{1}{2}\text{ of }3\tfrac{1}{3}\text{ of }4\tfrac{1}{5}\text{ of }6\times\tfrac{2}{9}\text{ of }\tfrac{12}{35}$$

$$=\tfrac{5}{2}\text{ of }\tfrac{10}{3}\text{ of }\tfrac{21}{5}\text{ of }\tfrac{6}{1}\times\tfrac{2}{9}\text{ of }\tfrac{12}{35}$$

$$=\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{\overset{2}{\cancel{10}}}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{\overset{3}{\cancel{21}}}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{2}{\cancel{6}}}{1}\times\frac{2}{\underset{1}{\cancel{\underset{3}{\cancel{9}}}}}\times\frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{\underset{5}{\cancel{35}}}}}$$

$$=2\times2\times4=16.$$

EXERCISE XXIII.

Multiply

1. $\frac{3}{8}$ by 4. **2.** $\frac{7}{9}$ by 6. **3.** $\frac{5}{12}$ by 8. **4.** $\frac{3}{7}$ by $\frac{14}{15}$.

5. $\frac{2}{5}$ by $\frac{15}{16}$. **6.** $\frac{24}{25}$ by $\frac{35}{36}$. **7.** $2\frac{3}{5}$ by $\frac{20}{39}$. **8.** $3\frac{4}{7}$ by $1\frac{3}{4}$.

9. $\frac{3}{4}$ by $1\frac{1}{2}$. **10.** $\frac{1}{8}$ of $1\frac{3}{4}$ of $3\frac{1}{5}$ by $4\frac{5}{7}$ of $6\frac{2}{3}$.

Multiply together

11. $\frac{4}{9}$, $\frac{3}{7}$, and $2\frac{1}{3}$. **12.** $\frac{50}{91}$, $\frac{22}{75}$, and $\frac{26}{55}$. **13.** $1\frac{11}{28}$, $\frac{35}{36}$, $2\frac{10}{15}$.

14. $1\frac{15}{22}$, $\frac{19}{51}$, and $1\frac{76}{111}$. **15.** $\frac{11}{42}$, $1\frac{11}{24}$, $\frac{28}{55}$, and $1\frac{13}{35}$.

16. $1\frac{25}{63}$, $2\frac{6}{25}$, $\frac{175}{648}$, $1\frac{5}{121}$, and $6\frac{30}{49}$.

17. Multiply the sum of $\frac{186}{279}$ and $\frac{237}{316}$ by their difference.

18. Multiply $\frac{7}{90}$ of $\frac{13}{16}$ of $\frac{15}{17}$ by $\frac{51}{77}$ of $\frac{99}{100}$ of $\frac{256}{585}$.

19. Multiply the sum of the greatest and the least of the fractions $\frac{138}{276}$, $\frac{194}{291}$, $\frac{201}{268}$, and $\frac{164}{205}$ by the difference of the remaining two.

20. Reduce to a simple fraction $\frac{5}{21} \times \frac{13}{15} \times \frac{48}{109} \times \frac{105}{208} \times 21\frac{4}{5}$.

21. Simplify $1\frac{2}{3} \times 3\frac{1}{5} \times \frac{11}{20} \times 8\frac{4}{7} \times 9\frac{1}{3}$.

22. Simplify $3\frac{1}{7} \times 3\frac{3}{10} \times 2\frac{3}{11} \times 2\frac{5}{22} \times \frac{2}{35}$.

23. Simplify $1\frac{1}{207} \times 5\frac{11}{13} \times 10\frac{7}{20} \times \frac{5}{19} \times \frac{1}{16}$.

24. Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $4\frac{1}{2}$ of $\frac{8}{27}$ of $4\frac{5}{6}$ by the reciprocal of the product of the sum and the difference of $\frac{45}{72}$ and $\frac{56}{96}$.

25. Multiply $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ by $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 1\frac{1}{5} \times 1\frac{1}{6}$.

DIVISION OF FRACTIONS.

98. To divide one fraction by another :

Rule. Multiply the dividend by the reciprocal of the divisor. The product will be the required quotient.

Mixed numbers should be reduced to improper fractions and compound fractions to simple ones before applying the rule.

When a fraction is to be divided by an integer, the result may be obtained either by dividing the numerator or by multiplying the denominator by the given integer.

Ex. 1. Divide $\frac{5}{9}$ by $\frac{8}{13}$.

$$\frac{5}{9} \div \frac{8}{13} = \frac{5}{9} \times \frac{13}{8} = \frac{65}{72}.$$

Ex. 2. Divide $7\frac{3}{8}$ by $2\frac{2}{3}$.

$$7\frac{3}{8} = \frac{59}{8} \text{ and } 2\frac{2}{3} = \frac{8}{3}.$$

$$\therefore\ 7\frac{3}{8} \div 2\frac{2}{3} = \frac{59}{8} \div \frac{8}{3} = \frac{59}{8} \times \frac{3}{8} = \frac{177}{64} = 2\frac{49}{64}.$$

Ex. 3. Divide $\frac{12}{25}$ by 4.

$$\frac{12}{25} \div 4 = \frac{12}{25} \times \frac{1}{4} = \frac{3}{25},$$

or

$$\frac{12}{25} \div 4 = \frac{12 \div 4}{25} = \frac{3}{25}.$$

Ex. 4. Divide $\frac{4}{13}$ of $5\frac{1}{5}$ by $\frac{4}{7}$ of $\frac{14}{15}$.

$$\frac{4}{13} \text{ of } 5\frac{1}{5} = \frac{4}{13} \text{ of } \frac{26}{5} = \frac{8}{5},$$

and

$$\frac{4}{7} \text{ of } \frac{14}{15} = \frac{8}{15}.$$

$$\therefore\ \frac{4}{13} \text{ of } 5\frac{1}{5} \div \frac{4}{7} \text{ of } \frac{14}{15} = \frac{8}{5} \div \frac{8}{15} = \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{8}}} = 3.$$

Ex. 5. Simplify $\dfrac{\frac{4}{5}}{1\frac{2}{3}}$.

$$\frac{\frac{4}{5}}{1\frac{2}{3}}=\tfrac{4}{5}\div 1\tfrac{2}{3}=\tfrac{4}{5}\div\tfrac{5}{3}=\tfrac{4}{5}\times\tfrac{3}{5}=\tfrac{12}{25}.$$

Ex. 6. Simplify $\dfrac{3\frac{4}{9}}{1+\dfrac{4}{5+\frac{1}{6}}}$.

$$3\tfrac{4}{9}=\tfrac{31}{9},$$

and
$$1+\frac{4}{5+\frac{1}{6}}=1+\frac{4}{\frac{31}{6}}=1+4\div\tfrac{31}{6}$$
$$=1+\tfrac{4}{1}\times\tfrac{6}{31}$$
$$=1+\tfrac{24}{31}=\tfrac{55}{31}.$$

$\therefore$ the given fraction $=\tfrac{31}{9}\div\tfrac{55}{31}$
$$=\tfrac{31}{9}\times\tfrac{31}{55}=\tfrac{961}{495}=1\tfrac{466}{495}.$$

Ex. 7. Divide the sum of $6\frac{2}{3}$ and $5\frac{1}{4}$ of $4\frac{4}{5}$ by their difference.

$$5\tfrac{1}{4}\text{ of }4\tfrac{4}{5}=\tfrac{21}{4}\text{ of }\tfrac{24}{5}=\tfrac{126}{5}=25\tfrac{1}{5}.$$

$\therefore$ the required sum $=6\frac{2}{3}+25\frac{1}{5}$
$$=6+25+\tfrac{2}{3}+\tfrac{1}{5}$$
$$=31+\tfrac{13}{15}=\tfrac{478}{15},$$

and $5\frac{1}{4}$ of $4\frac{4}{5}-6\frac{2}{3}=25\frac{1}{5}-6\frac{2}{3}$.

$\therefore$ the required difference $=24+1\frac{1}{5}-6\frac{2}{3}$
$$=24+\tfrac{6}{5}-6\tfrac{2}{3}$$
$$=24-6+\tfrac{6}{5}-\tfrac{2}{3}$$
$$=18+\tfrac{8}{15}=\tfrac{278}{15}.$$

$\therefore$ the required quotient $=\frac{478}{15}\div\frac{278}{15}$
$$=\tfrac{478}{15}\times\tfrac{15}{278}=\tfrac{478}{278}$$
$$=1\tfrac{200}{278}$$
$$=1\tfrac{100}{139}.$$

The above example may be given in either of the following terms:

$$\frac{6\frac{2}{3}+5\frac{1}{4}\text{ of }4\frac{4}{5}}{5\frac{1}{4}\text{ of }4\frac{4}{5}-6\frac{2}{3}},$$

or
$$(6\tfrac{2}{3}+5\tfrac{1}{4}\text{ of }4\tfrac{4}{5})\div(5\tfrac{1}{4}\text{ of }4\tfrac{4}{5}-6\tfrac{2}{3}).$$

Similarly Example 6 may be given as follows:

$$3\tfrac{4}{9}\div\left\{1+4\div(5+\tfrac{1}{6})\right\}.$$

Ex. 8. Simplify $\dfrac{\frac{7}{8}+\frac{2}{3}-\frac{29}{48}}{\frac{7}{8}\text{ of }\frac{2}{3}-\frac{2}{3}\text{ of }\frac{1}{4}}$.

The Numerator $=\frac{42+32-29}{48}=\frac{45}{48}=\frac{15}{16}$.

The Denominator $=\frac{7\times 1}{4\times 3}-\frac{1\times 1}{3\times 2}=\frac{7}{12}-\frac{1}{6}=\frac{7-2}{12}=\frac{5}{12}$.

$\therefore$ the given fraction $=\frac{15}{16}\div\frac{5}{12}=\dfrac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{16}}}\times\dfrac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{5}}}=\frac{3\times 3}{4}=\frac{9}{4}=2\frac{1}{4}$.

Ex. 9. Simplify $\dfrac{\frac{1}{4}+\frac{1}{3}\text{ of }\frac{7}{8}}{\frac{1}{2}+\frac{1}{3}}\times\dfrac{2-\frac{3}{4}}{1+\frac{3}{2}}\div\left(\dfrac{\frac{1}{3}+\frac{3}{4}\text{ of }\frac{2}{9}}{\frac{7}{3}+\frac{11}{12}}\times\dfrac{4+\frac{1}{3}}{\frac{1}{4}+\frac{5}{12}}\right)$.

$$\dfrac{\frac{1}{4}+\frac{2}{3}\text{ of }\frac{7}{8}}{\frac{1}{2}+\frac{1}{3}}=\dfrac{\frac{1}{4}+\frac{2\times 7}{3\times 8}}{\frac{3+2}{6}}=\dfrac{\frac{1}{4}+\frac{7}{12}}{\frac{5}{6}}=\dfrac{\frac{12}{10}}{\frac{5}{6}}=\frac{10}{12}\div\frac{5}{6}=\dfrac{\overset{1}{\overset{\cancel{2}}{\cancel{10}}}}{\underset{1}{\underset{\cancel{2}}{\cancel{12}}}}\times\dfrac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{5}}}=1.$$

$$\dfrac{2-\frac{3}{4}}{1+\frac{3}{2}}=\dfrac{\frac{8-3}{4}}{\frac{2+3}{2}}=\dfrac{\frac{5}{4}}{\frac{5}{2}}=\frac{5}{4}\div\frac{5}{2}=\dfrac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{4}}}\times\dfrac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}}=\frac{1}{2}.$$

$$\dfrac{\frac{1}{3}+\frac{3}{4}\text{ of }\frac{2}{9}}{\frac{7}{3}+\frac{11}{12}}=\dfrac{\frac{1}{3}+\frac{1}{6}}{\frac{7}{3}+\frac{11}{12}}=\dfrac{\frac{2+1}{6}}{\frac{28+11}{12}}=\dfrac{\frac{3}{6}}{\frac{39}{12}}=\frac{3}{6}\div\frac{39}{12}=\dfrac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{6}}}\times\dfrac{\overset{2}{\cancel{12}}}{\underset{13}{\cancel{39}}}=\frac{2}{13}.$$

$$\dfrac{4+\frac{1}{3}}{\frac{1}{4}+\frac{5}{12}}=\dfrac{\frac{12+1}{3}}{\frac{3+5}{12}}=\dfrac{\frac{13}{3}}{\frac{8}{12}}=\frac{13}{3}\div\frac{8}{12}=\dfrac{13}{\underset{1}{\cancel{3}}}\times\dfrac{\overset{1}{\overset{\cancel{4}}{\cancel{12}}}}{\underset{2}{\cancel{8}}}=\frac{13}{2}.$$

$\therefore$ the given expression $=1\times\frac{1}{2}\div\left(\frac{2}{13}\times\frac{13}{2}\right)$

$=\frac{1}{2}\div 1=\frac{1}{2}$.

EXERCISE XXIV.

Divide

1. $\frac{3}{7}$ by 6.
2. $\frac{6}{11}$ by 9.
3. $\frac{9}{13}$ by 12.
4. $\frac{4}{5}$ by $\frac{8}{15}$
5. $\frac{6}{7}$ by $\frac{24}{35}$.
6. $\frac{48}{85}$ by $\frac{64}{153}$.
7. $2\frac{3}{5}$ by $\frac{39}{40}$.
8. $4\frac{1}{2}$ by $6\frac{3}{4}$.
9. $13\frac{1}{3}$ by $16\frac{2}{3}$.
10. Divide $\frac{2}{3}$ of $3\frac{1}{2}$ of $3\frac{3}{4}$ by $\frac{5}{6}$ of $\frac{7}{12}$ of $1\frac{1}{8}$.

Find the value of

11. $\frac{13}{96}\div\frac{65}{84}$.
12. $10\frac{5}{16}\div 1\frac{13}{64}$.
13. $1\frac{29}{55}\div\frac{108}{121}$.
14. $3\frac{7}{27}\div\frac{16}{81}$.
15. $1\frac{79}{245}\div 1\frac{853}{875}$.
16. $30\frac{5}{64}\div 21\frac{7}{128}$.
17. Divide the difference between $4\frac{25}{36}$ and $1\frac{57}{64}$ by the sum of $2\frac{1}{6}$ and $1\frac{3}{8}$.
18. Divide the product of $\frac{17}{44}$ and $1\frac{14}{51}$ by that of $\frac{15}{22}$ and $1\frac{1}{12}$.

19. Divide the product of the sum and the difference of $3\frac{1}{3}$ and $7\frac{1}{2}$ by $40\frac{5}{8}$.

20. The product of two fractions is equal to the difference between $\frac{276}{345}$ and $\frac{295}{354}$, and one of them is equal to the sum of $\frac{249}{332}$ and $\frac{372}{465}$; find the other.

Simplify :

21. $\dfrac{17}{1\frac{2}{3}+\frac{3}{5}}$; $\dfrac{1\frac{1}{2} \text{ of } 2\frac{2}{3}}{3\frac{3}{4} \text{ of } 4\frac{4}{5}}$; $\dfrac{2\frac{2}{3}-1\frac{1}{2}}{2\frac{2}{3}+1\frac{1}{2}}$.

22. $\dfrac{\frac{5}{7} \text{ of } \frac{2}{9} \times 13\frac{1}{2}}{\frac{1}{9} \text{ of } \frac{3}{7}+40}$; $2\frac{1}{2}$ of $\dfrac{1}{3\frac{1}{3}+\dfrac{1}{4\frac{1}{4}}}$.

23. $\dfrac{\frac{7}{5} \text{ of } 3\frac{1}{3}+\frac{5}{7} \text{ of } 1\frac{10}{11}+\frac{341}{363}}{\frac{9}{11} \text{ of } \frac{10}{11}}$

24. $\dfrac{3\frac{1}{6}+\dfrac{1}{3\frac{1}{6}}}{3-\frac{1}{2} \text{ of } \frac{5}{9}} \div \dfrac{\frac{2}{9} \text{ of } 1\frac{2}{3} \text{ of } 2\frac{2}{5}}{\frac{7}{9}+\frac{5}{12}-\frac{3}{4}}$.

25. $\dfrac{1+\frac{1}{6}-\frac{1}{4}}{\frac{11}{16}+\frac{1}{4}-\frac{1}{48}} \div (\frac{889}{999}-\frac{88}{99})$

26. $\dfrac{4}{3+\frac{1}{3}}$; $\dfrac{1}{1+\dfrac{1}{2\frac{1}{2}}}$; $\dfrac{\frac{6}{7}}{2+\dfrac{1}{1+\frac{1}{3}}}$; $\dfrac{4\frac{5}{11}}{4+\dfrac{5}{6+\frac{7}{8}}}$.

27. $5 \div \left\{7-9 \div \left(3-\frac{3}{4}\right)\right\}$.

28. $3 \div \left[2+2 \div \left\{3+2 \div \left(3+\frac{2}{3}\right)\right\}\right]$.

29. $(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}) \div (\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16})$.

30. $\left\{\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\dfrac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}\right\} \div \left\{\dfrac{\frac{1}{4}-\frac{1}{6}}{\frac{1}{4}+\frac{1}{6}}-\dfrac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}\right\}$.

G.C.M. AND L.C.M. OF FRACTIONS.

99. To find the G.C.M. of two or more fractions:

Rule. Express the fractions in their lowest terms if they be not already so. Then take the G.C.M. of the numerators for the numerator and the L.C.M. of the denominators for the denominator. The fraction so formed is the G.C.M. of the given fractions.

Mixed numbers should be reduced to improper fractions and compound and complex fractions to simple ones before applying the above rule.

Ex. 1. Find the G.C.M. of $\frac{35}{48}$ and $\frac{28}{51}$.

Here the G.C.M. of 35 and 28 is 7.

And the L.C.M. of 48 and 51 is 816.

$\therefore$ the required G.C.M. is $\frac{7}{816}$.

Ex. 2. Find the G.C.M. of $3\frac{1}{8}$ and $5\frac{5}{6}$.

$$3\tfrac{1}{8}=\tfrac{25}{8} \text{ and } 5\tfrac{5}{6}=\tfrac{35}{6}.$$

G.C.M. of 25 and 35 is 5.

And L.C.M. of 6 and 8 is 24.

$\therefore$ the required G.C.M. is $\frac{5}{24}$.

Ex. 3. Find the G.C.M. of $\frac{27}{28}$, $\frac{42}{49}$, $\frac{39}{76}$.

Since $\frac{42}{49}=\frac{6}{7}$, we must find the G.C.M. of $\frac{27}{28}$, $\frac{6}{7}$, and $\frac{39}{76}$.

G.C.M. of 27, 6, and 39 is 3.

And the L.C.M. of 28, 7, and 76 is 532.

$\therefore$ the required G.C.M. $=\frac{3}{532}$.

100. To find the L.C.M. of two or more fractions:

Rule. Express the fractions in their lowest terms if they be not already so. Then take the L.C.M. of the numerators as numerator and the G.C.M. of the denominators as denominator. The fraction so formed is the L.C.M. of the given fractions.

Mixed numbers should be reduced to improper fractions and compound and complex fractions to simple ones before applying the above rule.

Ex. 1. Find the L.C.M. of $\frac{16}{33}$ and $\frac{20}{27}$.

The L.C.M. of 16 and 20 is 80.

And the G.C.M. of 33 and 27 is 3.

$\therefore$ the required L.C.M. $=\frac{80}{3}=26\frac{2}{3}$.

Ex. 2. Find the L.C.M. of $15\frac{1}{6}$ and $7\frac{2}{9}$.

$$15\tfrac{1}{6}=\tfrac{91}{6} \text{ and } 7\tfrac{2}{9}=\tfrac{65}{9}.$$

The L.C.M. of 91 and 65 is 455.

And the G.C.M. of 6 and 9 is 3.

$\therefore$ the required L.C.M. $=\frac{455}{3}=151\frac{2}{3}$.

Ex. 3. Find the L.C.M. of $\frac{72}{130}$, $\frac{120}{455}$, and $\frac{160}{390}$.

Reducing the given fractions to their lowest terms, we have

$$\tfrac{36}{65},\ \tfrac{24}{91},\ \tfrac{16}{39}.$$

The L.C.M. of 36, 24, and 16 is 144.

And the G.C.M. of 65, 91, and 39 is 13.

$\therefore$ the required L.C.M. $= \frac{144}{13} = 11\frac{1}{13}$.

EXERCISE XXV.

Find the G.C.M. of

1. $\frac{4}{9}$ and $\frac{6}{8}$.
2. $\frac{15}{69}$ and $\frac{24}{161}$.
3. $\frac{10}{33}$ and $\frac{15}{22}$.
4. $2\frac{2}{3}$ and $4\frac{2}{9}$.
5. $\frac{39}{78}$ and $\frac{3}{8}$.
6. $\frac{85}{108}$ and $\frac{187}{204}$.
7. $\frac{16}{24}$, $\frac{8}{9}$, $\frac{70}{87}$, and $\frac{36}{42}$.
8. $\frac{105}{124}$, $1\frac{1}{4}$, $3\frac{1}{8}$, and $\frac{165}{192}$.

Find the L.C.M. of

9. $\frac{6}{25}$ and $\frac{9}{35}$.
10. $4\frac{1}{6}$ and $2\frac{2}{9}$.
11. $\frac{28}{66}$ and $\frac{119}{132}$.
12. $26\frac{2}{3}$ and $\frac{32}{27}$.
13. $\frac{18}{216}$ and $\frac{134}{201}$.
14. $\frac{1}{16}$ and $\frac{1}{24}$.
15. $6\frac{27}{32}$, $8\frac{5}{8}$, $12\frac{3}{4}$, $6\frac{15}{28}$, $5\frac{13}{16}$, and $\frac{237}{316}$.

Find the G.C.M. and the L.C.M. of

16. $2\frac{7}{24}$, $10\frac{5}{6}$, $7\frac{1}{12}$, $11\frac{1}{9}$, and $6\frac{17}{18}$.

17. Find the greatest number which will divide each of the numbers $4\frac{1}{2}$, $3\frac{3}{8}$, $5\frac{1}{4}$, and $2\frac{1}{16}$ an exact number of times.

18. Find the least number which will be divided exactly by any of the fractions $\frac{12}{35}$, $2\frac{2}{7}$, $\frac{24}{77}$, $\frac{32}{63}$, and $\frac{36}{91}$.

19. Find the least number which, when divided by $4\frac{1}{6}$ and $3\frac{8}{9}$, will give in each case an integer as quotient.

20. The G.C.M. of two fractions is $\frac{4}{175}$ and their L.C.M. is $\frac{48}{5}$. If one of the fractions is $\frac{12}{25}$, find the other.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

Ex. 1. Find the value of $18\frac{2}{3} + \frac{4}{9} \times 1\frac{1}{8} - 3\frac{1}{3}$ of $7\frac{1}{2} + 15\frac{3}{4} \div 2\frac{1}{3}$.

The first term is $18\frac{2}{3}$

The second term is $\frac{4}{9} \times 1\frac{1}{8} = \frac{4}{9} \times \frac{9}{8} = \frac{1}{2}$.

The third term is $3\frac{1}{3}$ of $7\frac{1}{2} = \frac{10}{3} \times \frac{15}{2} = 25$.

The fourth term is $15\frac{3}{4} \div 2\frac{1}{3} = \frac{63}{4} \times \frac{3}{7} = \frac{27}{4} = 6\frac{3}{4}$.

$\therefore$ the given expression $= 18\frac{2}{3} + \frac{1}{2} - 25 + 6\frac{3}{4}$

$$
\begin{aligned}
&= 18\tfrac{2}{3} + \tfrac{1}{2} + 6\tfrac{3}{4} - 25\\
&= 18 + 6 + \tfrac{2}{3} + \tfrac{1}{2} + \tfrac{3}{4} - 25\\
&= 24 + \frac{8+6+9}{12} - 25\\
&= 24 + 1\tfrac{11}{12} - 25\\
&= 25 - 25 + \tfrac{11}{12}\\
&= \tfrac{11}{12}.
\end{aligned}
$$

Ex. 2. Simplify $\frac{3}{5} \div \frac{2}{7}$ of $\frac{4}{9}$.

$$\frac{3}{5} \div \frac{2}{7} \text{ of } \frac{4}{9} = \frac{3}{5} \div \frac{8}{63} = \frac{3}{5} \times \frac{63}{8} = \frac{189}{40} = 4\frac{29}{40}.$$

Ex. 3. Simplify $\frac{3}{5} \div \frac{2}{7} \times \frac{4}{9}$.

$$\frac{3}{5} \div \frac{2}{7} \times \frac{4}{9} = \frac{3}{5} \times \frac{7}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{9}$$
$$= \frac{7 \times 2}{5 \times 3} = \frac{14}{15}.$$

Ex. 4. *Simplify*

$$2\frac{6}{7} \text{ of } 4\frac{1}{5} \div \frac{2}{3} + \frac{3}{4} \times 2\frac{2}{3} \div \frac{5}{9} \times 1\frac{1}{5} - 3\frac{5}{6} \div \frac{2}{7} \text{ of } 2\frac{1}{3}.$$

$$2\frac{6}{7} \text{ of } 4\frac{1}{5} \div \frac{2}{3} = \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{28}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{5}}} \times \frac{3}{\underset{1}{\cancel{2}}} = 18.$$

$$\frac{3}{4} \times 2\frac{2}{3} \div \frac{5}{9} \times 1\frac{1}{5} = \frac{3}{4} \times \frac{8}{3} \times \frac{9}{5} \times \frac{6}{5} = \frac{2 \times 9 \times 6}{5 \times 5} = \frac{108}{25}.$$

$$3\frac{5}{6} \div \frac{2}{7} \text{ of } 2\frac{1}{3} = \frac{23}{6} \div \left(\frac{2}{7} \times \frac{7}{3}\right) = \frac{23}{6} \div \frac{2}{3}$$
$$= \frac{23}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{2} = \frac{23}{2 \times 2} = \frac{23}{4}.$$

$\therefore$ the given expression $= 18 + \frac{108}{25} - \frac{23}{4}$

$$= \frac{1800 + 432 - 575}{100}$$
$$= \frac{1657}{100} = 16\frac{57}{100}.$$

Ex. 5. Simplify $1 \div \left[2 + 3 \div \left\{4 - 5 \div \left(6 + \frac{1}{7}\right)\right\}\right]$.

We commence with the innermost bracket.

$$6 + \frac{1}{7} = \frac{43}{7}.$$
$$5 \div \frac{43}{7} = \frac{5}{1} \times \frac{7}{43} = \frac{35}{43}.$$
$$4 - \frac{35}{43} = \frac{172 - 35}{43} = \frac{137}{43}.$$
$$3 \div \frac{137}{43} = \frac{3}{1} \times \frac{43}{137} = \frac{129}{137}.$$
$$2 + \frac{129}{137} = \frac{274 + 129}{137} = \frac{403}{137}.$$
$$1 \div \frac{403}{137} = \frac{1}{1} \times \frac{137}{403} = \frac{137}{403}.$$

The above example may be set down as follows :

$$\cfrac{1}{2 + \cfrac{3}{4 - \cfrac{5}{6 + \frac{1}{7}}}}.$$

Such a fraction is called a continued fraction, in simplifying which, it will be seen from what has been given above, that we must commence with the lowest line.

Ex. 6. Simplify $\dfrac{4}{9-\dfrac{3}{1-\frac{2}{5}}}$.

$$1-\tfrac{2}{5}=\tfrac{5-2}{5}=\tfrac{3}{5}.$$
$$3\div\tfrac{3}{5}=\tfrac{\cancel{3}}{1}\times\tfrac{5}{\cancel{3}}=5.$$
$$9-5=4.$$
$$4\div 4=1.$$

Ex. 7. Simplify $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\frac{5}{13}}}}$.

$$1-\tfrac{5}{13}=\tfrac{13-5}{13}=\tfrac{8}{13}.$$
$$1\div\tfrac{8}{13}=\tfrac{1}{1}\times\tfrac{13}{8}=\tfrac{13}{8}.$$
$$2-\tfrac{13}{8}=\tfrac{16-13}{8}=\tfrac{3}{8}.$$
$$1\div\tfrac{3}{8}=\tfrac{1}{1}\times\tfrac{8}{3}=\tfrac{8}{3}.$$
$$4-\tfrac{8}{3}=\tfrac{12-8}{3}=\tfrac{4}{3}.$$
$$1\div\tfrac{4}{3}=\tfrac{1}{1}\times\tfrac{3}{4}=\tfrac{3}{4}.$$

Ex. 8. Simplify

$$\frac{\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}-4\frac{13}{18}\times3\frac{33}{34}\div3\frac{37}{96}}{8\frac{17}{19}\times5\frac{14}{39}\div4\frac{15}{32}-7\frac{19}{20}\times5\frac{11}{65}\div14\frac{21}{25}}\times4\frac{8}{23}.$$

$$\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}=\frac{1}{\underset{1}{\cancel{23}}}\times\frac{\overset{5}{\cancel{115}}}{\underset{1}{\cancel{17}}}\times\frac{\overset{19}{\cancel{323}}}{13}=\frac{95}{13}.$$

$$4\frac{13}{18}\times3\frac{33}{34}\div3\frac{37}{96}=\frac{85}{18}\times\frac{135}{34}\div\frac{325}{96}$$

$$=\frac{\overset{\overset{1}{\cancel{5}}}{\cancel{85}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{18}}}\times\frac{\overset{\overset{3}{\cancel{15}}}{\cancel{135}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{34}}}\times\frac{\overset{\overset{24}{\cancel{48}}}{\cancel{96}}}{\underset{\underset{13}{\cancel{65}}}{\cancel{325}}}=\frac{3\times24}{13}=\frac{72}{13}.$$

$\therefore$ the numerator $=\frac{95}{13}-\frac{72}{13}=\frac{23}{13}.$

$$8\frac{17}{19}\times5\frac{14}{39}\div4\frac{15}{32}=\frac{169}{19}\times\frac{209}{39}\div\frac{143}{32}$$

$$=\frac{\overset{\overset{1}{\cancel{13}}}{\cancel{169}}}{\underset{1}{\cancel{19}}}\times\frac{\overset{\overset{1}{\cancel{11}}}{\cancel{209}}}{\underset{3}{\cancel{39}}}\times\frac{32}{\underset{\underset{1}{\cancel{11}}}{\cancel{143}}}=\frac{32}{3}.$$

$$7\tfrac{19}{20}\times 5\tfrac{11}{65}\div 14\tfrac{21}{25}=\frac{159}{20}\times\frac{336}{65}\div\frac{371}{25}$$

$$=\frac{\overset{3}{\cancel{159}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{20}}}\times\frac{\overset{\overset{12}{\cancel{48}}}{\cancel{336}}}{\underset{13}{\cancel{65}}}\times\frac{\overset{\overset{1}{\cancel{5}}}{\cancel{25}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{371}}}=\frac{12\times 3}{13}=\frac{36}{13}.$$

$$\therefore \text{ the denominator}=\frac{32}{3}-\frac{36}{13}$$

$$=\frac{32\times 13-36\times 3}{39}=\frac{308}{39}.$$

$$\therefore \text{ the given fraction}=\frac{\;\frac{23}{13}\;}{\frac{308}{39}}\times 4\tfrac{8}{23}$$

$$=\frac{\overset{1}{\cancel{23}}}{\underset{1}{\cancel{13}}}\times\frac{\overset{3}{\cancel{39}}}{\underset{77}{\cancel{308}}}\times\frac{\overset{25}{\cancel{100}}}{\underset{1}{\cancel{23}}}=\frac{75}{77}.$$

Ex. 9. From a number is taken the sum of its third, fifth, and sixth parts, and the remainder is 63 ; what is the number ?

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{6}=\frac{10+6+5}{30}=\frac{21}{30}.$$

But the remainder by the problem is $63=\left(1-\frac{21}{30}\right)$ or $\frac{9}{30}$ of the required number.

$\therefore$ 9 times the thirtieth part of the number is 63.

$\therefore$ the thirtieth part of the number is $63\div 9=7$.

$\therefore$ the required number $=7\times 30=210$.

Ex. 10. A boy had a certain number of counters ; he gave 2 more than half of the number to *A* and then 3 more than half of the remainder to *B* and then had nothing left. How many counters had he at first ?

If the boy had given only half of the remainder to *B*, the other half would have been left with him ; therefore by the problem one-half of the remainder $=3$; therefore the remainder $=2\times 3=6$; this 6 remained with him after he had given one-half the original number of counters and 2 more to *A* ; if he had given *A* only one-half the original number and retained the other half with himself, he would have had $6+2=8$. Therefore half the original number $=8$. Therefore the original number of counters $=8\times 2=16$.

EXERCISE XXVI.

1. Three persons having *Rs.*$4\frac{3}{8}$, *Rs.*$8\frac{7}{16}$, and *Rs.*$13\frac{1}{3}$ subscribe respectively $\frac{4}{5}$, $\frac{2}{9}$, and $\frac{9}{16}$ of their money. What is their total subscription ?

2. Four men together paid a bill : the first paid $\frac{1}{5}$, the second $\frac{1}{4}$, and the third $\frac{1}{3}$. What fraction did the fourth pay ?

3. A fruiterer having sold two-thirds of his fruits to one person, and three-fourths of the remainder to another, had 60 left. How many fruits had he at first ?

4. A carpenter cuts off $\frac{5}{8}$ of a plank and then $\frac{7}{9}$ of the remainder what portion of the whole will be left ?

5. In a garden $\frac{1}{2}$ of the trees are mango trees, one-third guava trees, one-tenth palm trees, and the remainder, 30 in number, are plantain trees. How many are there in all ?

6. Reduce $\frac{3}{7}$, $\frac{5}{9}$, $\frac{2}{5}$, and $\frac{1}{3}$ to equivalent fractions having the least common denominator.

7. What must be added to $\frac{13}{18}$ of $1\frac{3}{5}$ of $3\frac{3}{4}$ to make it equal to $8\frac{13}{17}$?

8. The sum of two fractions is $26\frac{3}{5}$ and one of them is $9\frac{7}{8}$; find the other.

9. The product of two fractions is $\frac{2}{3}$, and one of them is $1\frac{7}{9}$; find the other.

10. Find the sum, the difference, and the product of $3\frac{7}{12}$ and $\frac{5}{8}$.

11. Four-ninths of an income goes for housekeeping, four-ninths of the remainder for education, and four-ninths of what is still left for rent, and *Rs.*375 remains over ; find the income.

12. The sum of two fractions is 1 and their numerators are 37 and 41 respectively ; find the fractions.

13. Divide $2\frac{2}{5}$ of $2\frac{5}{8}$ by $2\frac{3}{5} - 2\frac{1}{4}$.

14. What number divided by $\frac{16}{45}$ will give $\frac{25}{32}$?

15. What number multiplied by $\frac{78}{85}$ will give $\frac{18}{25}$?

16. How often can $7\frac{1}{5}$ be taken from $28\frac{8}{10}$?

17. Add $\frac{2}{5}$ of $\frac{3}{7}$ to $\frac{3}{7}$ of $2\frac{1}{3}$, and multiply the result by

$$\left(\tfrac{2}{3} \text{ of } \tfrac{5}{6}\right) \div \left(\tfrac{5}{4} + \tfrac{4}{5}\right).$$

18. Divide the product of $2\frac{1}{4} - 1\frac{3}{8}$ and $4\frac{1}{2} + 3\frac{1}{16}$ by the difference between $2\frac{3}{16}$ and $1\frac{1}{24}$.

19. Simplify $\dfrac{\frac{3}{4}}{\frac{5}{4}} + \dfrac{2\frac{1}{4} - \frac{3}{7}}{5\frac{1}{4} + \frac{1}{14}} - \dfrac{2}{3\frac{4}{7}}$.

20. Divide $2\frac{3}{4} + \frac{5}{2}$ of $\dfrac{7}{3\frac{4}{5}} + \dfrac{1\frac{2}{3}}{2\frac{1}{2}}$ by $1\frac{71}{228}$.

21. Divide $\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \times 3\frac{1}{3} + \frac{13}{36}} - \dfrac{1}{2\frac{1}{2}}$ by $\dfrac{1}{1\frac{2}{3}}$.

22. If A be $\frac{1}{2}$ of $2\frac{2}{3}$ of B, and C be $1\frac{1}{5}$ of B, what fraction is A of C ?

23. Simplify $\dfrac{1-\frac{4}{9}}{3\frac{1}{4}+1\frac{1}{2}+\dfrac{1\frac{2}{3}}{6\frac{2}{3}}}$ of $\left(\dfrac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)^2$.

24. $\dfrac{2\frac{1}{4}+3\frac{1}{28}}{3\frac{16}{35}+12\frac{2}{5}}+\dfrac{6\frac{2}{3}+3\frac{19}{39}}{12\frac{19}{26}+2\frac{1}{2}}$.

25. Arrange in the ascending order of magnitude $\frac{8}{33}$, $\frac{39}{161}$, and $\frac{47}{194}$, and express the difference of the first two as a fraction of the difference of the last two.

26. Reduce to its simplest form $2\frac{1}{2}+\dfrac{1}{3\frac{1}{3}+\dfrac{1}{4\frac{1}{4}}}$.

27. Find the value of

$$\frac{1}{3\frac{1}{5}}-\frac{2\frac{1}{4}}{9}+\frac{3\frac{5}{8}}{2}+\frac{\frac{4}{7}}{4\frac{4}{7}}.$$

28. To $3\frac{1}{2}$ of $5\frac{1}{6}$ add $\frac{2}{3}$ of $\left(6\frac{1}{4}-1\frac{1}{2}\right)$ and from their sum subtract $\dfrac{2\frac{1}{6}}{\frac{2}{3}}$.

29. Divide $\left(4\frac{1}{5}-3\frac{1}{4}\right)\times\left(3\frac{4}{5}+5\frac{1}{9}-\frac{1}{45}\right)$ by

$$1\tfrac{5}{11}+2\tfrac{1}{8}-\left(2\tfrac{9}{16}-\tfrac{1}{8}-\tfrac{1}{22}\right).$$

30. Out of a cistern five-eighths full, 25 gallons are drawn ; 80 gallons are then added, and it is found to be seven-ninths full. How much does it contain ?

Reduce to their simplest forms the following :

31. $2\frac{3}{4}\div5\frac{1}{4}$ of $3\frac{1}{7}$.

32. $2\frac{3}{4}\div5\frac{1}{4}\times3\frac{1}{7}$.

33. $1\frac{5}{9}$ of $2\frac{1}{7}\div1\frac{1}{2}$ of $\frac{4}{5}$.

34. $1\frac{5}{9}$ of $2\frac{1}{7}\div1\frac{1}{2}\times\frac{4}{5}$.

35. $\frac{7}{8}\div\frac{5}{6}\div\frac{3}{4}$.

36. $6\frac{3}{7}\div1\frac{7}{8}\times2\frac{1}{3}$.

37. $\frac{4}{9}\times1\frac{1}{5}\div\frac{3}{4}$ of $\frac{8}{15}$.

38. $6\frac{2}{3}\times2\frac{1}{10}\div7\frac{1}{2}\times1\frac{2}{3}$.

39. $8\frac{1}{3}\times1\frac{4}{5}\div2\frac{6}{7}$.

40. $12\frac{3}{4}\div2\frac{2}{3}\times\frac{3}{4}\div\frac{5}{8}$ of $2\frac{2}{5}$.

41. Add together $3\frac{5}{11}$, $3\frac{2}{9}$, and $2\frac{5}{27}$ and divide the sum by $19\frac{31}{33}$.

42. Reduce to its simplest form the following :

$$\frac{443}{4216}+\frac{3\frac{7}{8}\times1\frac{1}{17}+4\frac{1}{15}-3\frac{9}{16}}{5\frac{1}{9}-7\frac{7}{8}\div28\frac{7}{20}+\frac{1}{3}}.$$

43. Find which is the greater of $\frac{56}{143}$ and $\frac{72}{187}$ and divide $\frac{16}{11}$ by their difference.

44. Find the value of $\frac{32}{1287}+\frac{37}{2145}-\frac{47}{1170}$ and multiply the result by $514\frac{4}{5}$

45. Express as a single fraction $\frac{84}{85}\times\frac{102}{117}\times\frac{91}{132}\times\frac{105}{392}$.

46. Multiply $2\frac{37}{55}$ by $\frac{43089}{71500}$.

47. Divide 15824 by $\dfrac{5\frac{5}{8}\text{ of }13\frac{5}{7}}{6\frac{6}{7}\text{ of }3\frac{3}{4}}$ of $\dfrac{5\frac{5}{6}\text{ of }2\frac{7}{9}}{9\frac{3}{8}\text{ of }2\frac{11}{12}}$.

48. Reduce to its lowest terms $\frac{6324}{11439}$; $\frac{2265120}{2909907}$.

49. Simplify $\frac{215}{440}$ and $\frac{5}{12}$ of $\frac{9}{16}$ of $2\frac{2}{11}$ and add together the results.

Find the value of

50. $\dfrac{\frac{97}{194}+\frac{291}{388}}{\frac{485}{582}-\frac{291}{388}}$.

51. $\dfrac{\frac{153}{187}+\frac{91}{117}}{\frac{5}{8}+\frac{3}{11}}-\frac{133}{171}$.

52. $\dfrac{2\frac{1}{9}}{5\frac{3}{7}}$ of $1\frac{1}{8}-\dfrac{15\frac{3}{4}-4\frac{3}{7}\text{ of }3\frac{1}{2}}{3\frac{5}{9}\text{ of }\frac{3}{8}+2\frac{2}{3}}$.

53. $\dfrac{8\frac{3}{5}-7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}}\times\frac{2}{11}$ of 365.

54. $\dfrac{\frac{5}{14}-\frac{3}{7}\text{ of }\frac{1}{2}}{\frac{5}{16}+\frac{7}{12}\text{ of }3\frac{1}{4}-\left(\frac{7}{8}\text{ of }\frac{37}{21}-\frac{1}{3}\right)}\div\dfrac{\frac{1}{3}\text{ of }\frac{1}{2}+\frac{3}{2}\text{ of }5}{9\frac{1}{3}-1\frac{2}{3}}$.

5 . $\dfrac{\frac{1}{3}+\frac{2}{11}+\frac{7}{24}-\frac{1}{3}\text{ of }\frac{2}{11}\text{ of }\frac{7}{24}}{1-\frac{1}{3}\text{ of }\frac{2}{11}-\frac{2}{11}\text{ of }\frac{7}{24}-\frac{7}{24}\text{ of }\frac{1}{3}}$.

56. $\dfrac{5}{7-\dfrac{9}{3-\frac{3}{4}}}+\dfrac{5}{16-\dfrac{11}{2-\frac{1}{6}}}$.

57. $\dfrac{1}{2+\dfrac{1}{3+\frac{1}{4}}}\times\dfrac{3}{4-\dfrac{6}{9+\frac{3}{8}}}$.

58. $8-8\times\dfrac{2\frac{1}{5}-1\frac{2}{7}}{2-\dfrac{1}{6-\frac{1}{6}}}$.

59. $\dfrac{39}{39-\dfrac{1}{4\frac{1}{3}}}-\dfrac{40}{40-\dfrac{1}{4\frac{3}{4}}}$.

60. $\dfrac{17}{7+\dfrac{3}{4-2\frac{3}{4}}}\times\dfrac{2021}{2193}\div\left(1\frac{27}{48}-\frac{15}{16}\right)$.

61. $\left\{\left(5\frac{3}{8}\right)^2-\left(3\frac{5}{8}\right)^2\right\}\div\left(5\frac{3}{8}-3\frac{5}{8}\right)$.

62. $\left\{1+\frac{1}{2}+\left(\frac{1}{2}^2\right\}\div\left\{\left(1-\left(\frac{1}{2}\right\}^3\right)\right.\right.$.

63. $\dfrac{\left(1\frac{5}{12}\right)^3-\left(\frac{2}{3}\right)^3-\left(\frac{3}{4}\right)^3}{3\times1\frac{5}{12}\times\frac{2}{3}\times\frac{3}{4}}$.

64. Divide the product of $\frac{1}{8}$ and $\frac{1}{4}$ by the sum of the square of the former and the cube of the latter.

65. Multiply the difference of the squares of $\frac{2}{3}$ and $\frac{1}{3}$ by the sum of their cubes.

66. Divide the G.C.M. of $26\frac{2}{3}$ and $\frac{105}{126}$ by their L.C.M.

67. By what fraction will $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$ differ from unity ?

68. A man owned $\frac{16}{25}$ of an estate ; he sold $\frac{5}{8}$ of his share for *Rs.*342 ; what was the value of the whole estate ?

69. The sum of four fractions is 2 ; their numerators are respectively 3, 5, 7, and 11. Find the fractions.

70. A certain fraction, when increased by $\frac{1}{8}$, amounts to $\frac{2}{3}$; if the numerator of the fraction be $19\frac{1}{2}$, find the denominator.

71. By what fraction of itself does $\frac{1}{13}$ exceed $\frac{3}{41}$?

72. $\frac{3}{7}$ of $\frac{1}{5}$ of a certain fraction lies between $\frac{12}{25}$ and $\frac{9}{14}$; find between what numbers the fraction itself lies.

73. A farmer had a number of sheep. He sold half of the number and one more to one person, and two-thirds of the remainder to another, and then had 3 sheep left. Find the whole number of sheep which he had for sale.

74. *A* and *B* play at a game. *A* had 16 rupees to begin with. When *A* gained 5 rupees, *B* had $\frac{2}{3}$ of what *A* then had. How many rupees had *B* at first ?

75. A boy had a number of mangoes ; he gave one more than half of what he had to his father, one more than half of the remainder to his mother, one more than half of what he then had to his sister, and lastly, one more than half of what was still left to his brother, when he found that he had nothing left. How many mangoes had he at first ?

CHAPTER V.

DECIMALS.

101. A *decimal fraction* or a *decimal* is a fraction which has ten or any power of ten for its denominator, and which is written in a peculiar notation.

102. The notation of decimals is *simply an extension of the denary notation.* In the denary notation the value of a digit *decreases* in a ten-fold degree for each place that it advances to the right, and by extending this, we adopt the following convention for the notation of decimals.

103. As we proceed after the units' place to the right, the local value of each successive figure decreases ten-fold. Just as the places to the left of the units' place are called *tens, hundreds, thousands,* etc., those to the right of the units' place are called *tenths, hundredths, thousandths,* etc. We have, in this manner, a decimal scale of notation as follows :

Etc.	Tens of Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten-Thousandths	Etc.

104. When a decimal consists of an integer and a fraction, the point (·) is placed after the digit in the units' place and before the one in the tenths' place to separate the integer from the fraction, and is called the decimal point. When there is no integer, the decimal point is placed before the digit in the tenths' place, with zero sometimes written in the units' place. Thus in 56·713 (read, *fifty*-six and seven hundred and thirteen thousandths or 56 decimal point seven, one, and three) the 7 stands in the tenths' place and represents $\frac{7}{10}$, the 1 stands in the hundredths' place and represents $\frac{1}{100}$, and the 3 stands in the thousandths' place and represents $\frac{3}{1000}$.

Similarly ·73 or 0·73 denotes that the decimal contains no integer and that it consists of 7 tenths and 3 hundredths.

105. The term decimal places is used to denote the number of digits that follow the decimal point. Thus ·713 has 3 decimal places, 7·32 has 2 decimal places, and ·07392 has 5 decimal places.

106. As decimals can have only 10 or any power of 10 for their denominator, it is clear that if we want to express a vulgar fraction which has 10 or any power of 10 for its denominator as a decimal, we can do so readily. Thus $\frac{3}{10}$

may be written ·3, $\frac{3}{100}$ may be written ·03, because there is no digit here in the tenths' place.

To express therefore this class of vulgar fractions as decimals, we have only to place the decimal point before the numerator. If, however, we have to express a vulgar fraction which has not 10 or any power of 10 for its denominator, as a decimal fraction, the operation is not so easy. For example, let it be required to express $\frac{3}{4}$ as a decimal fraction. Now here we have to give the fraction a form in which the denominator will be some power of 10, and this is done by multiplying the numerator and the denominator by 25. Thus $\frac{3}{4}=\frac{3\times25}{4\times25}=\frac{75}{100}$. Now we can express the fraction in the new form as a decimal fraction. Thus $\frac{3}{4}=\cdot75$. The conversion of vulgar fractions into decimals will, however, be dealt with later on.

We add here a few more examples.

Ex. 1. Express in words the decimal ·3579.

This is read as

(1) Three-tenths, five-hundredths, seven-thousandths, and nine ten-thousandths.

(2) Three thousand five hundred and seventy-nine ten-thousandths.

(3) Decimal point (or point) three, five, seven, nine.

Ex. 2. Express 16·048 in words.

Sixteen and forty-eight thousandths, or

Sixteen, decimal point, cipher (dot or zero), four, eight.

Ex. 3. Express as a decimal, four and twenty-one thousandths.

Here there are no tenths ; therefore the number is written as **4·021.**

EXERCISE XXVII.

Express as decimals the following:

1. $\frac{453}{10}$. **2.** $\frac{9}{100}$. **3.** $\frac{190}{100}$. **4.** $\frac{753}{1000}$.
5. $\frac{53}{10000}$. **6.** $\frac{1}{10000}$. **7.** $\frac{85779}{100000}$. **8.** $\frac{26}{1000}$.
9. $\frac{20600}{10}$. **10.** $5\frac{17}{1000}$. **11.** $189\frac{223}{1000}$. **12.** $\frac{111}{10000}$.

13. Four hundred and twenty-three thousandths.

14. Nine hundred and ninety-nine ten-thousandths.

15. Sixteen and five hundredths.

16. Forty-nine millionths.

17. One hundred and twenty-three thousand four hundred and fifty-six ten-millionths.

18. 4 tenths, 7 hundredths and 9 thousandths.

19. 25 and 3 tenths 5 thousandths and 7 millionths.

20. What is the value of the digits 2, 5 and 6, in the number 726·058604 ?

107. *To express a decimal as a vulgar fraction.*

Write the given decimal as a whole number (leaving out, of course, the decimal point) for the numerator of the vulgar fraction, and write for the denominator, 1 followed by as many ciphers as there are decimal places in the given decimal. Then reduce the fraction, where necessary, to its lowest terms.

Ex. 1. Express ·47, ·435, 15·625, as vulgar fractions.

$$\cdot 47 = \frac{47}{100}.$$

$$\cdot 435 = \frac{435}{1000} = \frac{87}{200}.$$

$$15\cdot 625 = \frac{15625}{1000} = \frac{625}{40} = 15\tfrac{5}{8};$$

or

$$= 15\tfrac{625}{1000} = 15\tfrac{5}{8}.$$

EXERCISE XXVIII.

Express as vulgar fractions :

1. ·2. **2.** 2·5. **3.** ·08. **4.** 4·05.

5. 1·75. **6.** ·004. **7.** ·3125. **8.** 6·1875.

9. ·0875. **10.** 40·0125. **11.** ·078125. **12.** ·0004096.

13. 2·1024. **14.** 25·0576. **15.** 28·256. **16.** ·15625.

17. ·010205. **18.** ·05625. **19.** 3·01296. **20.** ·0096875.

108. As the denominator in a decimal fraction is 10 or any power of 10, the multiplication and division of decimals by 10 or any power of 10 are very easy operations. Thus

$$\cdot 63 \times 10 = \frac{63}{100} \times 10 = \frac{63}{10} = 6\tfrac{3}{10} = 6\cdot 3\ ;$$

$$4\cdot 5681 \times 1000 = \frac{45681}{10000} \times 1000 = \frac{45681}{10} = 4568\cdot 1\ ;$$

also

$$\cdot 0057 \times 10 = \frac{57}{10000} \times 10 = \frac{57}{1000} = \cdot 057.$$

Hence the multiplication of a decimal fraction by 10 or any power of 10 is performed by simply shifting the decimal point to the right as many places as there are ciphers in the multiplier.

Similarly

$$\cdot 63 \div 10 = \tfrac{63}{100} \times \tfrac{1}{10} = \tfrac{63}{1000} = \cdot 063\,;$$
$$4\cdot 5681 \div 1000 = \tfrac{45681}{10000} \times \tfrac{1}{1000} = \tfrac{45681}{10000000} = \cdot 0045681\,;$$
$$\cdot 0057 \div 10 = \tfrac{57}{10000} \times \tfrac{1}{10} = \tfrac{57}{100000} = \cdot 00057.$$

Hence to divide a decimal by 10 or any power of 10, we have only to shift the decimal point to the left as many places as there are ciphers in the divisor.

109. No change is made in the value of an integer if a decimal point be placed after it and any number of ciphers placed after the decimal point. Thus the value of 6 is not altered by writing it as 6·00, because $6\cdot 00 = \tfrac{600}{100} = 6$.

Similarly the value of a decimal is not altered by annexing to it or taking off from it any number of ciphers at the end. Thus ·57 is the same as ·5700. For

$$\cdot 57 = \tfrac{57}{100} = \tfrac{5700}{10000} = \cdot 5700.$$

Similarly ·73000 will be found to be the same as ·7300, ·730, and ·73.

Ex. 1. Multiply ·7145 by 10 and 2·45 by 1000.

Since the first number is to be multiplied by 10 and the second by 1000 we should remove decimal point one place to the right in the first and three places to the right in the second. Thus

$$\cdot 7145 \times 10 = 7\cdot 145,$$

and

$$2\cdot 45 \times 1000 = 2450\cdot = 2450.$$

Ex. 2. Divide 243·784 by 100 and ·83 by 10.

Since the first number is to be divided by 100 and the second by 10, we should shift the decimal point two places to the left in the first and one place to the left in the second. Thus

$$243\cdot 784 \div 100 = 2\cdot 43784\,;$$
$$\cdot 83 \div 10 = \cdot 083.$$

Ex. 3. Express 7·5, ·324, 95, and ·240000, each as having *four* decimal places.

Since each is to have four decimal places we should annex three ciphers to the first decimal, one to the second, and four to the third (after first placing the decimal point to the right of the units' digit); in the case of the fourth we must cut off two ciphers at the end.

Thus the required decimals are 7·5000, ·3240, 95·0000, and ·2400.

EXERCISE XXIX.

1. Multiply 6·5218 by 10 and 1000 separately.

2. Multiply 45·032 and ·085 each by 10 and 100 separately.

3. Divide 24·3812 by 10, 100, and 1000 separately.

4. Divide 37·921 and ·008453 each by 10, 100, and 1000 separately.

5. Multiply ·16 by 100000.

6. Divide ·001 by 1000.

7. Reduce 5·6, ·075, 12·456 to equivalent decimals each containing five decimal places.

8. Express 24 and 2·4 as decimals each containing three decimal places.

9. Write ·13 so that it should have the same number of decimal places as ·14159.

10. How many ciphers must be annexed to 71·37 so as to have the same number of decimal places as there are in 5·2318 ?

ADDITION OF DECIMALS.

110. To add together two or more decimals:

Rule. Have the same number of decimal places in all the given numbers, by affixing ciphers if necessary—by which, as has been shown, no change is made in the value of a decimal. Then add up as in whole numbers and mark off in the result as many decimal places as there are in each decimal (after the necessary addition of ciphers has been made).

Ex. 1. Add together 4·5, ·026, 31·25, and ·00452.

$$\begin{array}{r} 4{\cdot}50000 \\ {\cdot}02600 \\ 31{\cdot}25000 \\ {\cdot}00452 \\ \hline 35{\cdot}78052 \end{array}$$

The operation may be performed in practice without affixing ciphers, provided the decimal points be placed under one another.

$$\begin{array}{l} 4{\cdot}5 \\ {\cdot}026 \\ 31{\cdot}25 \\ {\cdot}00452 \\ \hline 35{\cdot}78052 \end{array}$$

Ex. 2. Add together ·85, ·106, and ·24375.

$$\begin{array}{r} ·85 \\ ·106 \\ ·24375 \\ \hline 1·19975 \end{array}$$

EXERCISE XXX.

Find the sum of the following decimals:

1. 131·5675, 5·2, 38·555, 8·40, ·005.

2. 25·0001, 2·12345, ·0003, 4·100, 125.

3. 23·119, 1·09, 2·005, 37, 12·148, 24·715.

4. 444·4, ·0551, 323·612, 231·0008, ·932.

5. ·9, ·99, ·999, ·9999, ·99999, 9.

6. ·9, ·09, ·009, ·0009, ·0001, 100·01.

7. 5·25, 25·125, 125·625, 625·3125, ·15625.

8. 2·4, 4·8, ·816, 163·2, ·4096, ·016384.

9. ·16, 2·56, 4·096, 16·0003, ·384, 5.

10. 523·05, 365·14, 73·009, 39·8, ·0010.

Add together:

11. 11·01, 2·202, 330·3, 44·04, ·5505, ·009.

12. Sixty-one and one hundredth, a hundred and four thousandths, ten and four hundredths, one and four tenths.

13. Five tenths, four hundred and fifty and five tenths, ninety-nine hundredths, ninety-nine thousandths.

Find the value of:

14. 11·5 + 88·4 + ·054 + ·0054 + ·04061.

15. 323·528 + 8·501 + 23·0500 + ·008.

16. ·001234 + ·012345 + ·123456 + 1·234567.

17. 98765·4 + 9876·54 + 987·654 + 98·7654.

18. 37·74111 + 74148·222 + 63·126189.

19. 24 + 24·48 + 2448·72 + 244872·96 + ·120.

20. 23·023161 + 1006·0023 + ·0161 + 1·138.

SUBTRACTION OF DECIMALS.

111. To subtract one decimal from another:

Rule. Have the same number of decimal places in the two fractions, by affixing ciphers if necessary. Then subtract as in whole numbers and mark off as many decimal places in the result as there are in each decimal (after the necessary addition of ciphers has been made).

Ex. 1. Subtract 12·542 from 25·7.

$$\begin{array}{r}25\cdot700\\12\cdot542\\\hline 13\cdot158\end{array}$$

Ex. 2. Subtract 6·75 from 7·8421.

$$\begin{array}{r}7\cdot8421\\6\cdot7500\\\hline 1\cdot0921\end{array}$$

EXERCISE XXXI.

Subtract

1. 32·83 from 43·94.

2. 53·0932 from 54·0181.

3. 101·0101 from 202.

4. 111·11000 from 148·001.

5. 323·019 from 423·018.

6. ·00005 from ·05.

7. 123·456 from 234·567.

8. 123·456 from 2345·67.

9. 88·9999 from 99·8888.

10. 11·22223 from 88.

Find the difference between

11. 119·10285 and 112·09680.

12. 80·00987 and 81·01230.

13. Three hundred and thirty-three ten thousandths and three hundred and thirty-three millionths.

14. Five and a tenth and one and five tenths.

15. The sum of 5 and ·0005 and the sum of 4 and ·0009.

16. Subtract 4·87936 from 89·5 and add 8462·064 to the remainder.

17. Subtract the sum of ·412 and ·083 from ·51.

Find the value of

18. 114·0034 + 41·0168 − 238·45 + 99·98070.

19. 999·0099 − 1010·0001 − 23·0023 + 202·202.

20. 12·34 + 5·678 − 9·101112 − 13·1415 + 16·17.

MULTIPLICATION OF DECIMALS.

112. To multiply one decimal by another :

Rule. Multiply the decimals together as in whole numbers, and mark off in the product as many decimal places as there are in the multiplier and the multiplicand together (prefixing ciphers, if necessary, to the product to bring up the number of digits in the product to the total number of decimal places).

Ex. 1. Multiply 4·63 by 5·2.

$$\begin{array}{r} 4\cdot63 \\ 5\cdot2 \\ \hline 9\,26 \\ 231\,5 \\ \hline 24\cdot076 \end{array}$$

Here we have two decimal places in the multiplicand and one in the multiplier. Therefore we must mark off three decimal places in the product.

The above process and the result may be stated as follows:

$$4\cdot63 \times 5\cdot2 = \tfrac{463}{100} \times \tfrac{52}{10} = \tfrac{24076}{1000} = 24\cdot076.$$

Ex. 2. Multiply ·3487 by ·034.

$$\begin{array}{r} \cdot3487 \\ \cdot034 \\ \hline 13948 \\ 10461 \\ \hline \cdot0118558 \end{array}$$

Here we have four decimal places in the multiplicand and three in the multiplier. Therefore we must mark off seven decimal places in the product. But in the product there are only six digits. Therefore we prefix one cipher to the product, and then place behind it the decimal point.

Ex. 3. Multiply 1·25 by 1·6.

$$\begin{array}{r} 1\cdot25 \\ 1\cdot6 \\ \hline 750 \\ 125 \\ \hline 2\cdot000 \end{array}$$

Here we have two decimal places in the multiplicand and one in the multiplier. Therefore we must mark off three decimal places in the product. As the three digits thus marked off are all ciphers, the given product may be written as simply 2.

113. To multiply a decimal by an integer:

Rule. Multiply as in whole numbers and in the product mark off as many decimal places as there are in the given decimal.

Ex. 4. Multiply 3·045 by 18.

$$\begin{array}{r} 3{\cdot}045 \\ \underline{18} \\ 54{\cdot}810 \end{array}$$

The result may be written as already explained in Art. **112.**

∴ 54·81 is the required product.

EXERCISE XXXII.

Multiply :

1. 3·215 by 7·24.
2. 40·0625 by 3·204.
3. 542·1875 by 26·8.
4. 28·395 by ·00114.
5. 13·02 by ·6982.
6. ·576 by 83·4.
7. 6·012 by ·00345.
8. 72·14 by ·0376.
9. 41·038 by 3·94.
10. 86·5 by ·00164.
11. 15·625 by ·0064.
12. 76·035 by ·0580079.
13. 5·61023 by ·597001.
14. ·020476 by 2·406.
15. 43·7246 by ·24805.
16. 40·637 by ·028403.
17. ·0435075 by 3·40604.
18. 98·003535 by ·0042074.
19. ·053076 by 9·8003.
20. 4·807309 by 12·086.

Find the continued product of :

21. 1·2, 3·1, and 2·3.
22. 43·5, ·504, and 1875.
23. 475, ·08, and ·125.
24. ·25, 1·6, 12·5, and 3·2.
25. ·3, ·04, 56·1, and 1·50.
26. Find the square of ·1 and the cube of ·2.
27. Multiply $(1{\cdot}25)^2$ by 10^3.
28. Multiply the cube of ·01 by 10^4.

Find the value of :

29. $(9{\cdot}5 + 1{\cdot}95) \times (9{\cdot}5 - 1{\cdot}95)$.
30. $\cdot375 \times \cdot375 + \cdot375 \times 2 \times \cdot625 + \cdot625 \times \cdot625$.

DIVISION OF DECIMALS.

114. To divide one decimal by another:

Rule. Make the divisor an integer by shifting the decimal point as many places to the right as may be necessary; and remove the decimal point in the dividend also the same number of places to the right. Then perform the operation of division as in whole numbers and mark off in the quotient as many decimal places as there are in the dividend (after the decimal point has been shifted).

Ex. 1. Divide 783·5436 by 3·57.

By shifting the decimal point two places to the right in the divisor and the same number of places in the dividend, we have

```
357 ) 78354·36 ( 219·48
      714
      ---
       695
       357
       ----
       3384
       3213
       ----
        1713
        1428
        ----
         2856
         2856
         ....
```

Since there are two decimal places in the dividend (after the shifting of the decimal point), we mark off two decimal places in the quotient. Therefore the required quotient is 219·48.

The above process and the result may also be stated as follows:

$$\frac{783\cdot5436}{3\cdot57} = \frac{783\cdot5436\times100}{3\cdot57\times100} = \frac{78354\cdot36}{357}$$

$$= \frac{7835436}{100}\times\frac{1}{357} = \frac{21948}{100} = 219\cdot48.$$

Ex. 2. Divide 912·64 by 7·36.

```
736 ) 91264 ( 124
      736
      ----
      1766
      1472
      ----
       2944
       2944
       ....
```

It will be seen here that as the given dividend and the divisor had both two decimal places, when the decimal point was shifted two places to the right in each, both became integers. The quotient therefore is also an integer.

Ex. 3. Divide 2821·0051 by ·0029332.

29332) 28210051000 (961750
263988
181125
175992
51331
29332
219990
205324
146660
146660
......0

∴ 961750 is the required quotient.

Ex. 4. Divide ·06321782 by 512·3.

5123) ·6321782 (1234
5123
11987
10246
17418
15369
20492
20492
.....

Since there are seven decimal places in the dividend there must be seven decimal places in the quotient. Therefore the required quotient is ·0001234.

Ex. 5. Divide ·005868 by 36.

36) ·005868 (·000163
36
226
216
108
108
...

Ex. 6. Divide 75 by 312·5.

3125) 750·00 (·24
625 0
125 00
125 00
....

∴ ·24 is the required quotient.

115. When all the digits of the dividend as formed after the shifting of the decimal point have been brought down, if a remainder be left, the division may be carried on by affixing ciphers to the dividend till either the division terminates or the required number of decimal places in the quotient is obtained.

If the dividend be an integer, place a decimal point after the units' figure before ciphers are annexed, by which, as explained in Art. 109, no change is made in its value.

Ex. 7. Divide 8·72 by 2·5.

```
25 ) 87·200 ( 3·488
     75
     ---
     122
     100
     ---
      220
      200
      ---
       200
       200
       ---
       ...
```

∴ 3·488 is the required quotient.

Ex. 8. Divide ·26 by 32·5.

```
325 ) 2·600 ( ·008
      2 600
      -----
      ....
```

∴ ·008 is the required quotient.

Ex. 9. Divide ·0000371 by ·0128 to five decimal places.

```
128 ) ·37100 ( ·00289
       256
       ----
       1150
       1024
       ----
        1260
        1152
        ----
         108
```

∴ ·00289 is the required quotient.

EXERCISE XXXIII.

Division of Decimals.

Divide

1. 72·5026 by 7·85.
2. ·282892 by 3·94.
3. 7 by 1·75.
4. 39·13 by 1·3.

5. ·375 by 1·5. **6.** ·00148 by 5·92.
7. 5·92 by ·01480. **8.** ·2332728 by 32·58.
9. ·1 by ·15625. **10.** ·001 by ·0256.
11. 78·009 by 40·96. **12.** 9·4 by 1638·4.
13. ·0056 by 70. **14.** ·056 by ·00007.
15. 5·6 by 700. **16.** ·003 of ·004 by ·006. **17.** 384·006 by ·123.

Find the value of

18. (2·1 − ·00105) ÷ ·0175. **19.** (4·255 + ·0064) ÷ ·00032.
20. 4·215 × 2·9 ÷ 3·2 × ·04. **21.** 11·3 ÷ 28·25.
22. 1524·6 ÷ 32·00. **23.** 3·75 ÷ 24.
24. 3·1415 ÷ ·0016. **25.** 564 ÷ ·3125.
26. 222·027 ÷ ·0013. **27.** 144 ÷ ·00012.
28. ·05265 ÷ 13·5. **29.** ·0079968 ÷ 2·24. **30.** 27124·64 ÷ 72·14.

Divide to seven places of decimals where necessary

31. 4·7362 by ·0435. **32.** ·064375 by ·009573.
33. 2·873 by ·48607. **34.** 4·0975 by ·2070. **35.** 1 by 3·14159.

G.C.M. AND L.C.M. OF DECIMALS.

116. To find the G.C.M. of decimals:

Rule. Make the number of decimal places in all the given decimals the same, by annexing ciphers, where necessary; find the G.C.M. as if the decimals were whole numbers, and mark off in the result as many decimal places as there are in any of these decimals (after ciphers have been annexed, where necessary).

Ex. 1. Find the G.C.M. of ·4, ·06, and ·008.

·4, ·06, ·008 may be written as ·400, ·060, and ·008 respectively, so as to have three decimal places in each.

The G.C.M. of 400, 60, and 8 is 4.

As there must be three decimal places in the present G.C.M., the required G.C.M. is ·004.

117. To find the L.C.M. of decimals:

Rule. Make the number of decimal places in all the given decimals the same (by annexing ciphers where necessary); find the L.C.M. as if the decimals were whole numbers; and mark off in the result as many decimal places as there are in any of these decimals (after ciphers have been annexed, where necessary).

Ex. 2. Find the L.C.M. of 1·5, 12, and ·36.

1·5, 12, and ·36 may be written as 1·50, 12·00, and ·36 respectively.
The L.C.M. of 150, 1200, and 36 is 3600.
Marking off two decimal places in 3600, we have 36·00 = 36.
Therefore the required L.C.M. is 36.

EXERCISE XXXIV.

Find the G.C.M. of

1. ·24 and ·32.
2. 4·2 and ·28.
3. 1·6 and ·72.
4. ·52 and 19·5.
5. 4·5, 3·6, and 26·1.
6. 14·3, 1·87, and ·594.
7. 8·4, 24, and ·96.
8. 1·8, ·63, 6, and ·081.

Find the L.C.M. of

9. 12·6 and 15·4.
10. ·85 and 6·8.
11. ·625 and 12·5.
12. ·09 and ·36.
13. 10·8, 1·44, and ·9.
14. 1·2, 3, and 2·1.
15. 1·05, ·49, 21, and 3·5.
16. 3·9, 15·6, 13·00, and ·091.

Find the G.C.M. and the L.C.M. of

17. ·64, 3·2, and ·056.
18. ·098, 1·12, and 7·00.
19. ·04, ·1, and 2.
20. ·1, 1, ·05, and 1·5.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS; RECURRING DECIMALS; AND APPROXIMATIONS.

118. To reduce a vulgar fraction to a decimal:

Rule. Take the numerator as the dividend; place the decimal point after the units' digit; and annex after the decimal point as many ciphers as may be found necessary. Divide by the denominator, as in decimals; the quotient will be the required decimal.

Ex. 1. Reduce $\frac{21}{25}$ to a decimal.

25) 21·00 (·84
200
100
100
...

Therefore the decimal equivalent to $\frac{21}{25}$ is ·84.

119. It sometimes happens, however, that we are not able to get an *exact* or terminating decimal to represent a vulgar fraction. When this is the case, certain digits in the decimal are found to *recur* or appear over and over again in the same order. Such decimal fractions are called *recurring*, *circulating* or *repeating* decimals; the portion recurring is called a *period*. Thus:

Ex. 2. Reduce $\frac{8}{15}$ to equivalent decimals.

15) 8·000 (·533
7 5
50
45
50
45
5

Here we find that after getting the second digit in the decimal the remainder is the same as that after getting the first digit and as we have only to annex ciphers to the remainder to carry on the division, it is evident that the third digit of the quotient and similarly all the succeeding digits will be the same as the second digit. And as there is no limit to the number of ciphers we may annex, there is also no limit to the number of times the second digit will repeat itself in the quotient. The decimal equivalent to $\frac{8}{15}$ therefore is ·53333 etc., which it is usual to write as ·5$\dot{3}$ and to read as decimal point, five, three, *three recurring.*

Ex. 3. Reduce $\frac{21}{22}$ and $\frac{23}{27}$ to equivalent decimals.

22) 21·000 (·954
19 8
1 20
1 10
100
88
12

27) 23·000 (·851
21 6
1 40
1 35
50
27
23

In the case of $\frac{21}{22}$ we find that after three digits in the quotient have been obtained the remainder left is 12, which is the same as what was left after the first digit had been obtained; therefore the second and the third digits of the quotient, *i.e.* 54, will be the recurring period.

Similarly in the case of $\frac{23}{27}$ it will be seen that the digits 8, 5, and 1, *i.e.* 851, will recur. Therefore the decimal equivalent to $\frac{23}{27}$ is ·851851... .

When the recurring period consists of more digits than one, the first and the last digits only of that period are marked with a dot placed above them. Thus ·95454... is written as $\cdot 9\dot{5}\dot{4}$ and ·851851... as $\cdot\dot{8}5\dot{1}$.

120. Recurring decimals are called *pure* when the same series of figures occurs from the beginning. They are called *mixed* when some of the figures in the quotient do not recur and these are followed by others which do. Thus in Ex. 3 given above, $\cdot\dot{8}5\dot{1}$ is a pure circulating decimal, while $\cdot 9\dot{5}\dot{4}$ is a mixed circulating decimal.

121. To convert a recurring decimal into its equivalent vulgar fraction:

Rule. If the given decimal be a pure recurring one, write the recurring figures for the numerator, and for the denominator write as many nines as there are figures in the numerator.

If the given decimal be a mixed circulating decimal, subtract the figures that do not recur from the figures of the decimal taken to the end of the first period, as if both were whole numbers. Take the result for the numerator and for the denominator write down as many nines as there are recurring figures followed by as many ciphers as there are non-recurring figures. The fraction thus obtained should, where necessary, be reduced to its lowest terms.

Ex. 4. Reduce $\cdot\dot{4}\dot{3}$, $\cdot\dot{5}\dot{4}$, and $\cdot\dot{4}8\dot{1}$ to equivalent vulgar fractions.

$$\cdot\dot{4}\dot{3} = \frac{43}{99}.$$
$$\cdot\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}.$$
$$\cdot\dot{4}8\dot{1} = \frac{481}{999} = \frac{13}{27}.$$

Ex. 5. Reduce $\cdot 58\dot{3}$, $\cdot 064\dot{8}1\dot{4}$, and $3\cdot 1\dot{2}\dot{7}$ to their equivalent vulgar fractions.

$$\cdot 58\dot{3} = \frac{583-58}{900} = \frac{525}{900} = \frac{21}{36} = \frac{7}{12}.$$
$$\cdot 064\dot{8}1\dot{4} = \frac{64814-64}{999000} = \frac{64750}{999000} = \frac{7}{108}.$$
$$3\cdot 1\dot{2}\dot{7} = 3 + \frac{127-1}{990} = 3 + \frac{126}{990} = 3 + \frac{7}{55} = 3\tfrac{7}{55}$$
or
$$= \frac{3127-31}{990} = \frac{3096}{990} = \frac{172}{55} = 3\tfrac{7}{55}.$$

122. The following points in connection with circulating decimals should be carefully noted by the student.

1. A vulgar fraction converted into a decimal must either terminate or circulate.

2. If the denominator of a vulgar fraction *reduced to its lowest terms be wholly made up of powers of* 2 *and* 5 either alone or multiplied together, the fraction is reducible to a terminating decimal.

3. If the denominator of a fraction *in its lowest terms be solely composed of factors other than powers of* 2 *and* 5, the fraction can be expressed as a *pure circulating decimal.*

4. If the denominator of a fraction in its *lowest terms* be composed *partly of powers of* 2 *and* 5 *and partly of other factors*, the fraction can be expressed only as a *mixed circulating decimal.*

123. Approximations :—When a decimal contains or is expected to contain a large number of digits after the decimal point, a small error in denoting the fraction is sometimes considered to be of no consequence, and when that is the case, it is thought sufficient to give the decimal correct only up to a required number of decimal places. Thus for example, in dealing with the decimal ·596314251 if it be considered that an error of less than $\frac{1}{1000}$ would not matter, the decimal in question may be simply given as ·596. The amount of error here as represented by the portion omitted is ·000314251, which is less than $\frac{1}{3000}$. Here the decimal is said to be given correct only to three places of decimals.

If in the above instance, it be required that the error should be less than $\frac{1}{10000}$, the first four digits after the decimal point should be taken, and so on.

Ex. 1. Reduce $\frac{63}{64}$ to its equivalent decimal correct to three decimal places.

$$
\begin{array}{l}
64\,)\,630000\,(\,\cdot 9843 \\
\underline{576} \\
540 \\
\underline{512} \\
280 \\
\underline{256} \\
240 \\
\underline{192} \\
48
\end{array}
$$

$\therefore$ ·984 is the decimal required.

Ex. 2. Express $\frac{4}{7}$ as a decimal with the least number of digits so that the error committed may be less than $\frac{1}{10000}$.

7) 400000 (·57142
35
50
49
10
7
30
28
20
14
6

·5714 is the decimal required.

124. When the portion omitted begins with a digit which is 5 or greater than 5, it is usual to increase by 1 the immediately preceding digit (*i.e.* the last one of those that are retained) as by so doing the error committed in omitting the portion is made smaller. Thus if the decimal ·142875 be required to be given correct only to three decimal places the error is smaller if we write it as ·143 than as ·142. As in the former case the error is

$$\cdot 143 - \cdot 142875 = \cdot 000125,$$

while in the latter case the error is

$$\cdot 142875 - \cdot 142 = \cdot 000875.$$

Therefore the above decimal when stated correct to three decimal places only should be written as ·143.

Ex. 3. Express $\frac{19}{21}$ as a decimal correct to four decimal places.

21) 1900000 (·90476
189
100
84
160
147
130
126
4

∴ ·9048 is the required decimal.

EXERCISE XXXV.

Convert the following vulgar fractions into decimals :

1. $\frac{3}{5}$, $\frac{5}{32}$. **2.** $\frac{17}{50}$, $\frac{6}{15}$. **3.** $\frac{7}{12}$, $\frac{179}{256}$. **4.** $\frac{179}{125}$.

5. $\frac{371}{640}$. **6.** $\frac{713}{512}$. **7.** $\frac{1}{3}$, $\frac{1}{7}$. **8.** $\frac{1}{8}$, $\frac{1}{13}$.

9. $6\frac{571}{625}$. **10.** $\frac{77}{120}$. **11.** $\frac{1}{768}$. **12.** $3\frac{7}{44}$.

13. State whether the following fractions give terminating decimals or recurring decimals :

$\frac{5}{18}$, $\frac{7}{25}$, $\frac{13}{24}$, $\frac{1}{3125}$, $\frac{249}{250}$ and $\frac{96}{105}$.

14. Find the decimals equivalent to the vulgar fractions given in Ex. 13 above.

Express the following decimals as vulgar fractions in their lowest terms :

15. $\cdot 35$, $\cdot 75$. **16.** $\cdot 125$, $1\cdot 25$. **17.** $\cdot\dot{3}$, $\cdot\dot{0}6\dot{3}$. **18.** $\cdot 1625$.

19. $\cdot 0875$. **20.** $6\cdot 00496$. **21.** $\cdot 3\dot{6}$. **22.** $\cdot\dot{2}\dot{7}$, $\cdot 2\dot{7}$.

23. $\cdot 1\dot{7}$, $\cdot 01\dot{3}$. **24.** $\cdot\dot{4}$, $\cdot\dot{0}\dot{4}$. **25.** $\cdot\dot{0}\dot{9}$, $\cdot\dot{6}\dot{0}$.

26. $\cdot\dot{3}\dot{9}$, $\cdot 3\dot{9}$. **27.** $\cdot 2\dot{2}0\dot{7}$. **28.** $\cdot 09\dot{2}\dot{7}$.

29. $\cdot 00\dot{0}86\dot{4}$. **30.** $\cdot 1\dot{2}\dot{7}$. **31.** $\cdot 236\dot{1}$.

32. $\cdot 53\dot{6}$. **33.** $\cdot 03\dot{2}\dot{8}$. **34.** $\cdot 87\dot{1}\dot{2}$.

35. $\cdot 991\dot{6}$. **36.** $\cdot 31\dot{4}36\dot{5}$. **37.** $\cdot 7\dot{7}9365\dot{0}$.

38. $\cdot\dot{0}136986\dot{3}$. **39.** $\cdot 00356\dot{4}8\dot{1}$. **40.** $\cdot 10\dot{7}1428\dot{5}$.

Express the following decimals each correct to five decimal places :

41. $\cdot 04512868$. **42.** $13\cdot 14159265$. **43.** $1\cdot 76429813$.

44. Which of the two decimals $\cdot 428$ and $\cdot 429$ is nearer to $\frac{3}{7}$?

45. If $\frac{15}{16}$ be expressed as $\cdot 93$, by how much does the decimal differ from the true fraction ?

46. Express $\frac{8}{27}$ as a decimal correct to four decimal places.

47. Find decimals corresponding to $\frac{5}{7}$ and $\frac{3}{13}$ and $\frac{16}{17}$ which shall not differ from the correct fractions by more than a thousandth.

Find the value correct to seven decimal places of

48. $\frac{355}{113}$. **49.** $\frac{1}{3\cdot 8}$. **50.** $\frac{22\cdot 5}{8\cdot 5}$.

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF RECURRING DECIMALS.

125. The Addition, Subtraction, Multiplication and Division of recurring decimals may be performed by first reducing the recurring decimals to vulgar fractions.

Ex. 1. Add together $\cdot 6\dot{7}6190\dot{4}$ and $\cdot 0\dot{4}2857\dot{1}$.

$$\cdot 6\dot{7}6190\dot{4} = \frac{6761904-6}{9999990} = \frac{6761898}{9999990} = \frac{71}{105},$$

and $$\cdot 0\dot{4}2857\dot{1} = \frac{428571}{9999990} = \frac{3}{70};$$

$$\therefore\ \cdot 6\dot{7}6190\dot{4} + \cdot 0\dot{4}2857\dot{1} = \frac{71}{105} + \frac{3}{70} = \frac{151}{210}.$$

Ex. 2. Find the difference between $\cdot 6\dot{4}2857\dot{1}$ and $\cdot\dot{8}5714\dot{2}$.

$$\cdot 6\dot{4}2857\dot{1} = \frac{6428571-6}{9999990} = \frac{6428565}{9999990} = \frac{9}{14},$$

and $$\cdot\dot{8}5714\dot{2} = \frac{857142}{999999} = \frac{6}{7};$$

$$\therefore\ \cdot 857142 - \cdot 6\dot{4}2857\dot{1} = \frac{6}{7} - \frac{9}{14} = \frac{3}{14}.$$

Ex. 3. Find the product of $1{\cdot}\dot{5}7142\dot{8}$ and $\cdot\dot{4}\dot{5}$.

$$1{\cdot}\dot{5}7142\dot{8} = \frac{1571428-1}{999999} = \frac{1571427}{999999} = \frac{11}{7},$$

and $$\cdot\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11};$$

$$\therefore\ 1{\cdot}\dot{5}7142\dot{8} \times \cdot\dot{4}\dot{5} = \frac{11}{7} \times \frac{5}{11} = \frac{5}{7}.$$

Ex. 4. Divide $\cdot\dot{2}5\dot{9}$ by $\cdot 541\dot{6}$.

$$\cdot\dot{2}5\dot{9} = \frac{259}{999} = \frac{7}{27},$$

and $$\cdot 541\dot{6} = \frac{5416-541}{9000} = \frac{4875}{9000} = \frac{13}{24};$$

$$\therefore\ \cdot\dot{2}5\dot{9} \div \cdot 541\dot{6} = \frac{7}{27} \div \frac{13}{24} = \frac{7}{27} \times \frac{24}{13} = \frac{56}{117}.$$

126. In the case of Addition and Subtraction when the result is required to be correct up to a certain number of decimal places only, the form of the given fractions may be retained and the operation performed after carrying the circulating part in each decimal to two or three more places than the required number. In the result thus obtained the superfluous digits should be struck off, increasing the immediately preceding digit by 1 if the portion omitted begin with 5 or with a digit greater than 5.

Ex. 5. Find the sum and the difference of 5·83 and $2{\cdot}72\dot{9}$, each correct to four decimal places.

$$\begin{array}{r} 5{\cdot}833333 \\ 2{\cdot}729729 \\ \hline 8{\cdot}563062 \end{array}$$

∴ 8·5631 is the sum required.

Again,

$$\begin{array}{r} 5{\cdot}833333 \\ 2{\cdot}729729 \\ \hline 3{\cdot}103604 \end{array}$$

∴ 3·1036 is the difference required.

EXERCISE XXXVI.

1. Add together $2\cdot8\dot{3}$, $5\cdot\dot{4}\dot{5}$, $\cdot\dot{2}\dot{7}$, and $1\cdot6\dot{8}\dot{1}$.
2. Subtract $5\cdot7\dot{6}$ from $8\cdot1\dot{3}$.

Find the sum and difference of :

3. $1\cdot7\dot{2}$ and $3\cdot\dot{1}8\dot{0}$. 4. $2\cdot3\dot{5}7142\dot{8}$ and $5\cdot\dot{2}8571\dot{4}$. 5. $\cdot3\dot{2}5\dot{1}$ and $\cdot76\dot{3}$.

Multiply :

6. $\cdot\dot{2}\dot{4}$ by $\cdot0\dot{6}$. 7. $\cdot1\dot{6}$ by $1\cdot\dot{3}$. 8. $\cdot\dot{2}\dot{7}$ by $\cdot91\dot{6}$.
9. $1\cdot\dot{0}45\dot{9}$ by $32\cdot8$. 10. $\cdot9\dot{5}\dot{4}$ by $\cdot\dot{0}3\dot{7}$. 11. $\cdot641\dot{6}$ by $\cdot3\dot{0}\dot{9}$.

Divide :

12. $2\cdot791\dot{6}$ by $1\cdot86\dot{1}$. 13. $\cdot809\dot{3}\dot{4}$ by $3\cdot2\dot{3}\dot{7}$.
14. $\cdot05\dot{0}\dot{9}$ by $\cdot\dot{5}\dot{5}$. 15. $17\cdot9\dot{8}6\dot{4}$ by $16\cdot\dot{3}5\dot{1}$.

Find the value correct to four decimal places of the following expressions :

16. $43\cdot3\dot{8}+6\cdot\dot{7}\dot{2}+13\cdot\dot{6}+54\cdot041\dot{6}+\cdot\dot{0}8\dot{1}$.
17. $7\cdot35\dot{2}+1\cdot23\dot{4}7\dot{1}+4\cdot\dot{0}\dot{5}+\cdot873\dot{6}$. 18. $\cdot3\dot{2}4\dot{7}+\cdot42781\dot{3}+3\cdot\dot{7}\dot{1}+2\cdot\dot{3}5\dot{9}$.
19. $1\cdot37\dot{6}+\cdot001\dot{4}+2\cdot01\dot{3}+2\cdot18\dot{9}$. 20. $\cdot74\dot{3}+\cdot15\dot{2}8\dot{6}-\cdot2\dot{4}\dot{1}+1\cdot4\dot{3}5\dot{8}$.

Express as vulgar fractions in their lowest terms :

21. $\dfrac{\cdot\dot{3}+\cdot1\dot{6}}{\cdot72\dot{9}}$. 22. $\dfrac{4\cdot\dot{4}-2\cdot8\dot{3}}{1\cdot\dot{6}+2\cdot\dot{6}2\dot{9}}$. 23. $\dfrac{\cdot\dot{0}\dot{9}\text{ of }\cdot0625}{\cdot1\dot{3}\dot{6}\text{ of }\cdot375}$.

24. $\cdot01\dot{7}4\dot{7}\div\cdot002\dot{4}7\dot{7}$. 25. $\dfrac{1\cdot\dot{5}47\dot{6}\times10\cdot618}{2\cdot\dot{6}547}$. 26. $\dfrac{0\cdot\dot{9}\dot{0}\times0\cdot\dot{8}4615\dot{3}}{0\cdot\dot{4}6153\dot{8}\times0\cdot8\dot{3}}$.

27. $\dfrac{0\cdot\dot{4}2857\dot{1}\times0\cdot\dot{7}1428\dot{5}}{0\cdot\dot{2}8571\dot{4}\times0\cdot0\dot{5}7142\dot{8}}$. 28. $\dfrac{6\cdot\dot{0}7692\dot{3}-3\cdot\dot{1}5384\dot{6}}{6\cdot\dot{0}7692\dot{3}+3\cdot\dot{1}5384\dot{6}}$.

29. $\dfrac{3\cdot4\dot{4}\dot{5}\times3\cdot4\dot{4}\dot{5}-1\cdot5\dot{5}\dot{4}\times1\cdot5\dot{5}\dot{4}}{4\cdot\dot{1}\times\cdot\dot{4}0\dot{5}}$. 30. $8\cdot1$ of $\dfrac{7\cdot\dot{0}3\dot{7}-5\cdot\dot{1}3\dot{5}}{15\cdot4+3\cdot5\dot{9}}\times1\cdot2\dot{3}$.

MISCELLANEOUS EXAMPLES IN DECIMALS.

Ex. 1. Simplify $\dfrac{1\cdot8\dot{3}+2\cdot041\dot{6}+\cdot\dot{3}-3\frac{1}{3}}{1\cdot0025+\cdot0625-1\frac{1}{16}}$.

$$3\tfrac{1}{3}=3\cdot\dot{3}\text{ and }1\tfrac{1}{16}=1\cdot0625\;;$$

$$\begin{aligned}\therefore\; 1\cdot8\dot{3}+2\cdot041\dot{6}+\cdot\dot{3}-3\cdot\dot{3}&=1+\cdot8\dot{3}+2+\cdot041\dot{6}+\cdot\dot{3}-3\cdot\dot{3}\\&=3+\cdot\dot{3}-3\cdot\dot{3}+\cdot8\dot{3}+\cdot041\dot{6}\\&=\cdot8\dot{3}+\cdot041\dot{6}\\&=\tfrac{75}{90}+\tfrac{375}{9000}\\&=\tfrac{5}{6}+\tfrac{1}{24}=\tfrac{7}{8}.\end{aligned}$$

And $1{\cdot}0025+{\cdot}0625-1\frac{1}{16}=1+{\cdot}0025+{\cdot}0625-1{\cdot}0625$

$$=1+{\cdot}0625-1{\cdot}0625+{\cdot}0025$$

$$={\cdot}0025=\frac{25}{10000}=\frac{1}{400};$$

$\therefore$ the given expression $=\frac{7}{8}\div\frac{1}{400}$

$$=\frac{7}{8}\times\frac{400}{1}=350.$$

Ex. 2. Reduce $\frac{1}{17}$ to a decimal.

This may be done by the ordinary method or we may proceed as follows:

Let us divide 1 by 17 till we obtain a smaller number than 9 for a remainder.

$$17\,)\,\underline{1{\cdot}00000}$$
$$\cdot 05882\tfrac{6}{17}$$

$$\therefore\ \frac{1}{17}={\cdot}05882\tfrac{6}{17}.$$

Multiply each of these fractions by 6.

Then

$$\frac{6}{17}={\cdot}35292\tfrac{36}{17}={\cdot}35294\tfrac{2}{17};$$

$$\therefore\ \frac{1}{17}={\cdot}0588235294\tfrac{2}{17}.$$

Multiply each of these by 2.

Then

$$\frac{2}{17}={\cdot}1176470588\tfrac{4}{17};$$

$$\therefore\ \frac{1}{17}={\cdot}05882352941176470588\tfrac{4}{17}.$$

Here we see that the last four figures, viz., 0, 5, 8, and 8 are the same as the first four. We thus have discovered the recurring period of the decimal. The result therefore is

$$\frac{1}{17}={\cdot}\dot{0}58823529411764\dot{7}.$$

Ex. 3. Reduce to its simplest form

$$\frac{12{\cdot}8\times{\cdot}15}{3{\cdot}9\times1{\cdot}15}\text{ of }\frac{92\times{\cdot}065}{3{\cdot}2\times{\cdot}008}.$$

Multiplying by 1000 the numerator and the denominator of

$$\frac{12{\cdot}8\times{\cdot}15}{3{\cdot}9\times1{\cdot}15}\text{ we obtain }\frac{128\times15}{39\times115}.$$

Multiplying by 100000 the numerator and the denominator of

$$\frac{{\cdot}92\times{\cdot}065}{3{\cdot}2\times{\cdot}008}\text{ we obtain }\frac{92\times65}{32\times8\times10};$$

$$\therefore\ \frac{12{\cdot}8\times{\cdot}15}{3{\cdot}9\times1{\cdot}15}\text{ of }\frac{{\cdot}92\times{\cdot}065}{3{\cdot}2\times{\cdot}008}=\frac{128\times15\times92\times65}{39\times115\times32\times8\times10}=\mathbf{1}.$$

Ex. 4. Find the value of

$$\frac{2{\cdot}791\dot{6}\times 3{\cdot}2\dot{3}\dot{7}}{1{\cdot}86\dot{1}\times {\cdot}809\dot{3}\dot{4}}.$$

$$2{\cdot}791\dot{6}=\frac{27916-2791}{9000}=\frac{25125}{9000},$$

$$3{\cdot}2\dot{3}\dot{7}=\frac{3237-32}{990}=\frac{3205}{990},$$

$$1{\cdot}86\dot{1}=\frac{1861-186}{900}=\frac{1675}{900},$$

$${\cdot}809\dot{3}\dot{4}=\frac{80934-809}{99000}=\frac{80125}{99000};$$

$$\therefore\ \frac{2{\cdot}791\dot{6}\times 3{\cdot}2\dot{3}\dot{7}}{1{\cdot}86\dot{1}\times {\cdot}809\dot{3}\dot{4}}=\frac{25125}{9000}\times\frac{3205}{990}\div\left(\frac{1675}{900}\times\frac{80125}{99000}\right)$$

$$=\frac{25125}{9000}\times\frac{3205}{990}\times\frac{900}{1675}\times\frac{99000}{80125}$$

$$=\frac{25125\times 3205\times 10}{1675\times 80125}=6.$$

Ex. 5. Find the value of

$$\frac{1}{2}+\frac{1}{2\times 3}+\frac{1}{2\times 3\times 4}+\frac{1}{2\times 3\times 4\times 5}+\frac{1}{2\times 3\times 4\times 5\times 6}+\ldots$$

correct to five decimal places.

$$\begin{array}{r|l}
2 & 1 \\
3 & {\cdot}5 = {\cdot}5 = \text{1st term.} \\
4 & {\cdot}1666666 = {\cdot}1666666 = \text{2nd term.} \\
5 & {\cdot}0416666 = {\cdot}0416666 = \text{3rd term.} \\
6 & {\cdot}0083333 = {\cdot}0083333 = \text{4th term.} \\
7 & {\cdot}0013888 = {\cdot}0013888 = \text{5th term.} \\
8 & {\cdot}0001984 = {\cdot}0001984 = \text{6th term.} \\
9 & {\cdot}0000248 = {\cdot}0000248 = \text{7th term.} \\
10 & {\cdot}0000027 = {\cdot}0000027 = \text{8th term.} \\
 & {\cdot}0000002 = {\cdot}0000002 = \text{9th term.}
\end{array}$$

$$\text{Sum} = {\cdot}7182817.$$

∴ the required sum $= {\cdot}71828$.

Here each term is reduced to a decimal; the second term **is** obtained by dividing the decimal equivalent to the first by 3; the third by dividing the decimal equivalent to the second by 4, and so on. Each decimal is carried, where possible, to seven places (*i.e.* two places more than the required number). There are taken in all nine terms, all the succeeding ones being rejected, as in the case of any of these there is no possibility of finding a significant figure within the first seven places.

EXERCISE XXXVII.

1. Express as a decimal $\frac{552}{2^5 \times 5^5} + \frac{923}{2^3 \times 5^3}$.

2. Add together 265 millionths, 98 ten-thousandths, and 46 hundredths, and subtract their sum from 7 tenths and 4 hundred-thousandths.

3. Add together $23\cdot07\dot{6}$, $19\cdot\dot{2}4\dot{5}$ and $31\cdot20\dot{3}$.

4. Divide the product of ·000015625 and 120 by the difference between ·5817 and ·4892.

5. Reduce $\cdot0\dot{3}\dot{6}$, ·001875, $\cdot\dot{3}90\dot{9}$ and $\cdot\dot{9}2\dot{5}$ to equivalent vulgar fractions in their lowest terms.

6. Reduce to its lowest terms the fraction $\frac{\frac{3}{4} \text{ of } 0\cdot0603}{\frac{5}{4} \text{ of } 0\cdot00594}$.

7. Multiply $3\cdot\dot{2}8571\dot{4}$ by $\cdot\dot{3}8461\dot{5}$ and express the result as a circulating decimal.

8. Divide 2 by ·2, ·002 by ·02, and 2·2 by 2·1.

9. Find the G.C.M. of 9·8, ·021 and 10·5.

10. Find the value of $\frac{2\cdot15 \times \cdot0202 \times 1\cdot5}{1\cdot01 \times \cdot025 \times 8\cdot6}$.

11. Simplify $\cdot1 \times \cdot01 \div \cdot0002 - \cdot6375 \times \frac{1}{1\cdot7} \div \cdot125$.

12. Perform the operations indicated below:

(1) $47\cdot03 - 2\cdot876843$. (2) $5\cdot776 \times 2\cdot003$.
(3) $62\cdot5 \div 125\cdot125$. (4) $6\cdot25 \div \cdot000125$.

13. Add together $\cdot\dot{4}1\dot{4}$, $\cdot0\dot{3}5\dot{2}$ and $6\cdot\dot{1}0\dot{1}$.

14. Find the first five decimal places in the quotient of 3·065 by ·0291.

15. Multiply ·0001234 by 512·3 and by ·05123.

16. Divide 3833336 by $\cdot031 \times 2\cdot05$ and by $8\cdot99 \times 20\cdot8$.

17. Find the L.C.M. of 1·95, 22·5 and 1·5.

18. Simplify $\frac{\cdot\dot{7}\dot{8} \times \cdot00\dot{0}\dot{4}}{\cdot0\dot{1}\dot{3} \times \cdot\dot{0}1\dot{5}}$.

19. Find the value of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ correct to three places of decimals.

20. Find the number which must be added to the sum of $\cdot06$, $\cdot4217$, $3\cdot1$, $\cdot0005$ and $\cdot010203$ in order to make the result the least possible integer.

21. Find the value of $\cdot\dot{0}1\dot{2}$ of $\cdot\dot{3}9\dot{6}$ of $16\cdot65$.

22. Reduce $4\frac{173}{3125}$ and $5\frac{43}{540}$ to decimals; and $\cdot9375$ and $\cdot4\dot{9}2\dot{5}$ to vulgar fractions.

23. Multiply $28\cdot8$ by $\cdot0595$ and divide the product by 9520.

24. Reduce to a single decimal $\frac{\cdot04275}{3\cdot0\dot{5}}\times\frac{4\cdot21\dot{6}}{\cdot\dot{3}4\dot{2}}\times\frac{2\cdot\dot{7}}{1\cdot5318}$.

25. Simplify $\frac{3\cdot125}{2\cdot16}$ of $\frac{\cdot2\dot{4}}{\cdot125}\div\frac{2\cdot\dot{2}}{1\cdot5}$ of $\frac{187\cdot5}{3\cdot42}$.

26. Find the value to six places of decimals of

$$1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}+\frac{1}{3^6}.$$

27. Multiply the sum of $\cdot3$ and $\cdot04$ by their difference.

28. Reduce $\frac{2}{5}$ of $\frac{7}{12}$ of $\frac{11}{21}$ of $2\frac{1}{2}$ to a decimal.

29. Find the G.C.M. and the L.C.M. of $1\cdot2$, $\cdot36$, $\cdot08$ and 6.

30. Simplify $\frac{1\cdot\dot{1}8\dot{5}-\cdot6\dot{1}}{19\cdot\dot{5}-12\cdot\dot{8}}\times\frac{2\cdot41}{2\cdot13}\div\frac{1\cdot55\times\cdot\dot{0}3\dot{7}}{4\cdot\dot{3}\dot{0}}$.

31. Find the value correct to six places of decimals of

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)^2+\left(\frac{1}{2}-\frac{1}{3}\right)^3+\dots.$$

32. Multiply $43\cdot\dot{2}9\dot{1}$ by $6\cdot2\dot{4}$ correct to five decimal places.

33. Divide $2\cdot\dot{9}80\dot{1}$ by $1\cdot\dot{4}5\dot{0}$ correct to five decimal places.

34. Reduce $3\cdot175+3\cdot\dot{3}\div57\cdot\dot{7}\dot{2}$ to its simplest form.

35. Divide $91\cdot\dot{8}6\dot{3}$ by $87\cdot5\dot{6}$ correct to five decimal places.

36. Express the difference between $2\cdot\dot{5}3\dot{5}$ and $2\cdot535$; (1) as a vulgar fraction and (2) as a circulating decimal.

37. Find a decimal equivalent to $\frac{17}{42}$ which shall be correct within $\frac{1}{10000}$.

38. If $\cdot\dot{6}$ be the dividend and $\cdot\dot{2}8571\dot{4}$ be the quotient, what is the product of the dividend, the quotient and the divisor?

39. Multiply $1\cdot\dot{0}99\dot{0}$ by $2\cdot\dot{7}2\dot{9}$ and divide $37\cdot4\dot{3}\dot{6}$ by $\cdot13\dot{9}$.

40. Convert into decimals $\frac{17}{37}$ and $\frac{3}{64}$.

41. Multiply 7642 by $\cdot12\dot{1}$.

42. Divide 1761 by $\cdot\dot{5}28\dot{3}$.

43. Simplify $\left\{\frac{\cdot0019}{3\cdot1\dot{6}}\text{ of }\frac{4\cdot\dot{4}}{\cdot0005}\right\}\div\left\{\frac{8\cdot\dot{8}}{7}\text{ of }\frac{4}{5\cdot625}\right\}$ and give the result as a decimal.

Simplify the following expressions :

44. $\dfrac{2\cdot 791\dot{6} \times 3\cdot 2\dot{3}\dot{7}}{1\cdot 86\dot{1} \times \cdot 809\dot{3}\dot{4}}$.

45. $\dfrac{\cdot 000\dot{9} \times 2\cdot 1}{\cdot 000035} \div \dfrac{\cdot \dot{0}7692\dot{3}}{2\cdot 3 \times 5\cdot \dot{6}}$.

46. $\dfrac{2\cdot 8 \text{ of } 2\cdot \dot{2}\dot{7}}{1\cdot \dot{3}\dot{6}} + \left(\dfrac{4\cdot \dot{4} - 2\cdot 8\dot{3}}{1\cdot \dot{3} + 2\cdot 62\dot{9}}\right)$.

47. $\cdot 0011 \times \cdot 091 \div \cdot 0035 + \cdot 015 \times \cdot 507 \div \cdot 39$.

48. $\cdot 0057 \times 2\cdot 09 \div \cdot 361 - \cdot 00165 \times \cdot 077 \div \cdot 0105$.

49. $\cdot 002 \times 36\cdot 25 \div \cdot 029 - 102\cdot 85 \times \cdot 04 \div 1\cdot 7$.

50. $3\tfrac{1}{3} \text{ of } \cdot \dot{6} + \dfrac{1\cdot 75}{2\cdot 625} \text{ of } 17 + 5\cdot 75 \text{ of } 4 - \dfrac{\cdot \dot{7}1428\dot{5}}{2\cdot \dot{0}9523\dot{8}}$.

51. $\dfrac{5\cdot 7}{1\cdot 52} + \dfrac{\cdot 012\dot{3}}{\cdot 74}$.

52. $\dfrac{\cdot 608 \times \cdot 0014 \times 3\cdot 6}{\cdot 0063 \times \cdot 152}$.

53. $\dfrac{22\cdot 4}{\cdot 25} + \dfrac{250}{\cdot 8} + \dfrac{1\cdot 2}{\cdot 0075}$.

54. $\left(\dfrac{12\cdot 5}{\cdot 0025} + \dfrac{4\cdot 2\dot{6}}{\cdot 7\dot{3}}\right) - \dfrac{610\cdot 61}{\cdot 305} \times \dfrac{4\cdot 05}{1\cdot 62} + \cdot \dot{1}\dot{8}$.

55. $\dfrac{\cdot 12 \text{ of } (\cdot 0104 - \cdot 002) + \cdot 36 \times \cdot 002}{\cdot 12 \times \cdot 12}$.

56. $\dfrac{3\cdot 125}{2\cdot 1\dot{6}} \text{ of } \dfrac{\cdot 2\dot{1}}{\cdot 125} \div \dfrac{2\cdot \dot{2}}{1\cdot 5} \times \dfrac{188\cdot 5}{3\cdot 42}$.

57. $\dfrac{\cdot 044 \times 2\cdot 1}{\cdot 000035} \div \dfrac{3\cdot \dot{0}7692\dot{3}}{2\cdot \dot{3} \times 5\cdot \dot{6}}$.

58. $\dfrac{\cdot 02 \times \cdot 9 \times \cdot 15 - \cdot 14 \times \cdot 06 \times \cdot 03 + \cdot 13 \times \cdot 01 \times \cdot 04}{\cdot 05 \times \cdot 04 \times \cdot 03}$.

59. $\left\{24\cdot \dot{4}2857\dot{1} - \dfrac{24\cdot \dot{4}2857\dot{1} + 38\cdot \dot{5}7142\dot{8}}{3}\right\} \times \cdot 2430\dot{5}$.

60. $\dfrac{\cdot \dot{0}7692\dot{3}}{\cdot \dot{0}3\dot{7}} \times \dfrac{999}{\dot{0}2\dot{7}} \times \dfrac{\cdot 001}{111} \times \dfrac{13}{\cdot \dot{0}0\dot{9}}$.

CHAPTER VI.

ROOTS.

SQUARE ROOT.

127. The *square root* of a number is that number which when multiplied by itself produces the given number. Thus 4 is the square root of 16 because 4 multiplied by 4 produces 16.

128. The sign $\surd$ is used to denote square root. It is placed before the number of which the root is required. Thus $\sqrt{25}$ denotes the square root of 25 so that $\sqrt{25}=5$.

129. To find the square root of a number:

Rule. Point every second figure beginning with that in the units' place, proceeding to the left (and also to the right if there be a decimal), and thus divide the whole number into periods. Find the greatest number whose square is contained in the first period; this is the first figure in the root; subtract its square from the first period and to the remainder bring down the next period. Divide this quantity omitting the last figure by twice the part of the root already obtained and annex the result to the root as its second figure and also annex it to the divisor; then multiply the divisor, as it now stands, by the part of the root last obtained, *i.e.* the second figure, and subtract the product from the number formed, as above mentioned, by the first remainder and the second period. If there be more periods to be brought down, the operation must be repeated. The number of figures in the root will be the same as the number of periods in the given number.

It sometimes happens, especially in the earlier stages of the work, that the trial quotient figure selected gives a product too large for subtraction. In such cases, that figure should be abandoned and the next lower one should be tried, till a product small enough is obtained.

In the case of a decimal, the decimal point is to be placed immediately before that figure of the root which is obtained by using the first period of the decimal part.

If there be a remainder at the last step, add ciphers to the remainder for another period and proceed as before, repeating the process as often as may be necessary to get a required number of decimal places in the quotient.

Ex. 1. Extract the square root of 4624.

$$\begin{array}{r|l} 6 & 4\dot{6}2\dot{4}\ (\ 68 \\ 6 & 36 \\ \hline 128 & 1024 \\ & 1024 \\ \hline & \cdots\cdots \end{array}$$

Setting down the given number and pointing off according to the rule we proceed as follows:

The first period is 46; the greatest number whose square is contained in 46 is 6; put down 6 to the right and also to the left of the given number; write 36, the square of 6, below 46, and subtract; the remainder is 10; bring down 24, the figures of the next period; the whole number thus formed is 1024; then to get twice the part of the root already obtained write 6 below 6 on the left and add; the sum is 12; since 12 is contained more than 8 times but less than 9 times in 102, 8 seems to be the second figure of the root; put it to the right of the first figure and also to the right of 12. Multiply 128 by 8 and write down the product below 1024 and subtract; the remainder is zero. Hence 68 is the square root required.

Ex. 2. Extract the square root of 75625.

Here the first period is 7; the greatest number whose square can be subtracted from 7 is 2. Write 2 to the right and to the left of the given number; below 7 write 4, the square of 2, and subtract: the remainder is 3; bring down the figures of the next period to the remainder; the number thus formed is 356. Then write 2 below 2 on the left and add; the sum is 4; thus we get twice the part of the root already obtained. Dividing 35 by 4 we get 8 for the second figure of the root, but by trial we find it to be too large. We therefore take 7, the next lower one. Write 7 to the right of 2, the first figure of the root, and also to the right of 4; multiply 47 by 7; the product is 329; put it below 356 and subtract; the remainder is 27; bring down the figures of the next period, the number thus formed being 2725. Write 7 below 7, the unit digit of 47, and add the two numbers; the sum is 54; thus we get twice 27, the part of the root already obtained; dividing 272 by 54 we get 5, the third figure of the root; place it to the right of 7, the second figure of the root, and also to the right of 54; multiply 545 by 5; the product is 2725; write it below 2725 and subtract; the remainder is zero. Hence 275 is the square root required.

$$\begin{array}{r|l}
2 & \dot{7}5\dot{6}2\dot{5}\ (\ 275 \\
\underline{2} & \underline{4} \\
47 & 356 \\
\underline{7} & \underline{329} \\
545 & \ 2725 \\
 & \ \underline{2725} \\
 & \ \cdot\cdot\cdot\cdot
\end{array}$$

Ex. 3. Find the square root of 5745609.

$$\begin{array}{r|l}
2 & \dot{5}7\dot{4}5\dot{6}0\dot{9}\ (\ 2397 \\
\underline{2} & \underline{4} \\
43 & 174 \\
\underline{3} & \underline{129} \\
469 & \ 4556 \\
\underline{9} & \ \underline{4221} \\
4787 & \ \ 33509 \\
 & \ \ \underline{33509} \\
 & \ \ \cdot\cdot\cdot\cdot\cdot
\end{array}$$

∴ 2397 is the square root required.

Ex. 4. Find the square root of 502681.

$$\begin{array}{r|l} 7 & 5\dot{0}2\dot{6}8\dot{1}\ (\ 709 \\ \underline{7} & \underline{49} \\ 1409 & \quad 12681 \\ & \quad \underline{12681} \\ & \quad \cdots\cdots \end{array}$$

∴ 709 is the answer.

Here *in finding the second figure of the root, we divide* 12, the first two figures of 126 by 14 and find that 14 is not contained in 12 even once; therefore we write 0 as the second figure of the root, to the right of 7 the first figure and also to the right of 14. We then bring down 81, the figures of the next period, to 126, and divide 1268 of 12681 by 140 and get 9 as the third figure of the root; we then write 9 to the right of 0, the second figure of the root, and also to the right of 140; we next multiply 1409 by 9, place the product below 12681 and subtract, and we find that the remainder is zero. Hence 709 is the square root required.

Ex. 5. Find the square root of 1440·9616.

$$\begin{array}{r|l} 3 & 1\dot{4}4\dot{0}\cdot 9\dot{6}1\dot{6}\ (\ 37\cdot 96 \\ \underline{3} & \underline{9} \\ 67 & 540 \\ \underline{7} & \underline{469} \\ 749 & \quad 7196 \\ \underline{9} & \quad \underline{6741} \\ 7586 & \qquad 45516 \\ & \qquad \underline{45516} \\ & \qquad \cdots\cdots \end{array}$$

∴ 37·96 is the square root required.

Here the decimal point precedes the third period in the given number; therefore the decimal point is placed before the third figure in the root.

Ex. 6. Find the square root of ·000196.

$$\begin{array}{r|l} 1 & \cdot 0\dot{0}0\dot{1}9\dot{6}\ (\ \cdot 014 \\ \underline{1} & \quad \underline{1} \\ 24 & \quad \cdot 96 \\ & \quad \underline{96} \\ & \quad \cdot\cdot \end{array}$$

∴ ·014 is the square root required.

Here the work begins with the second period of the given number. The first period of the given number consists of zeros; therefore the first figure of the root immediately after the decimal point is zero.

Ex. 7. Find the square root of ·2 correct to seven decimal places.

4	$\cdot 2\dot{0}$ (·44721359
4	16
84	400
4	336
887	6400
7	6209
8942	19100
2	17884
89441	121600
1	89441
894423	3215900
3	2683269
8944265	53263100
5	44721325
89442709	854177500
	804984381
	49193119

∴ ·4472136 is the square root required.

130. The square root of a fraction may be found by reducing it to a decimal.

Ex. 8. Find the square root of $\frac{381}{508}$ correct to a hundredth.

We have here $\frac{381}{508} = \cdot 75$

8	$\cdot 7\dot{5}$ (·866
8	64
166	1100
6	996
1726	10400
	10356
	44

∴ ·87 *is the required square root.*

Since it is required to find the answer correct to a hundredth, that is correct to two decimal places only, the work is continued to three decimal places; and as the third figure is greater than 5, the second figure should be increased by 1 in stating the result. Hence ·87 is the answer.

131. The square root of the product of two or more squares is equal to the product of the square roots of the given squares.

Let it be required to find the square root of $4 \times 9 \times 25$. The product of these numbers is 900 and its square root is 30. Now the square roots of 4, 9 and 25 are respectively 2, 3 and 5, of which the product is also 30. The above may be expressed as follows:

$$\begin{aligned}\sqrt{4 \times 9 \times 25} &= \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5} \\ &= \sqrt{(2 \times 3 \times 5) \times (2 \times 3 \times 5)} \\ &= 2 \times 3 \times 5.\end{aligned}$$

Hence in the case of a number which is an exact square and which can be resolved into factors, the square root may be readily found by resolving the number into factors.

Ex. 9. Find the square root of 1764.

$$\begin{aligned}1764 &= 4 \times 441 \\ &= 4 \times 9 \times 49 \\ &= 2^2 \times 3^2 \times 7^2; \\ \therefore \sqrt{1764} &= \sqrt{2^2 \times 3^2 \times 7^2} \\ &= 2 \times 3 \times 7 \\ &= 42.\end{aligned}$$

Ex. 10. Find the square root of 39204.

$$\begin{aligned}39204 &= 4 \times 9801 \\ &= 4 \times 9 \times 1089 \\ &= 4 \times 9 \times 9 \times 121 \\ &= 2^2 \times 3^2 \times 3^2 \times 11^2; \\ \therefore \sqrt{39204} &= \sqrt{2^2 \times 3^2 \times 3^2 \times 11^2} \\ &= 2 \times 3 \times 3 \times 11 \\ &= 198.\end{aligned}$$

Ex. 11. Find the fourth root of 15752961.

Since the fourth root is the square root of the square root, we have

3	$1\dot{5}7\dot{5}2\dot{9}6\dot{1}$ (3969
3	9
69	675
9	621
786	5429
6	4716
7929	71361
	71361
	.....

6	$3\dot{9}6\dot{9}$ (63
6	36
123	369
	369
	...

∴ 63 is the fourth root of 15752961.

EXERCISE XXXVIII.

Find the square root of

1. 27225. **2.** 137641. **3.** 39601. **4.** 1522756.
5. 119025. **6.** 350464. **7.** 824464. **8.** 249001.
9. 97535376. **10.** 547600. **11.** 522729. **12.** 622521.
13. 38452401. **14.** 81812025. **15.** 5687573056.
16. 2540764836. **17.** 7890481. **18.** 4745757682576.
19. 64·6416. **20.** 2079·36 **21.** 45·9684.
22. 1·036324. **23.** 854700·25. **24.** 1069·29.
25. ·011449. **26.** ·03530641. **27.** 8·042896.

Express as a decimal correct to 3 decimal places, where necessary, the square root of

28. $\frac{507}{768}$; $1\frac{9}{16}$. **29.** $\frac{87}{145}$; $11\frac{1}{9}$. **30.** $\frac{13}{25}$; $2\frac{14}{15}$.
31. ·100 ; $\cdot\dot{4}$. **32.** $\frac{9\cdot16}{\cdot256}$. **33.** $2515\frac{294}{361}$.
34. $1\cdot7866\dot{7}$; $\cdot63\dot{9}$.

35. What number has its square equal to the difference of the squares of 992 and 1217 ?

Find by factors the square root of

36. 1156. **37.** 43264. **38.** 186624. **39.** 531441.

Find the square roots correct to 5 decimal places of

40. $\cdot 2564$. **41.** $\cdot 7\dot{0}\dot{3}$. **42.** $1\cdot 25$. **43.** $\cdot 4\dot{3}\dot{1}$.

44. $3\cdot\dot{5}1\dot{2}$. **45.** $\cdot 925$. **46.** $\cdot 7$.

Find the fourth root of

47. 84934656. **48.** 53974440976. **49.** 66074188401.

50. 81108054012001.

CUBE ROOT.

132. The cube root of a given number is that number, which, when multiplied by itself twice over, produces the given number. Thus 5 is the cube root of 125, for 125 is equal to 5 × 5 × 5.

133. In finding the cube root of a given number, the rule for which is given below, the cubes of the first nine numbers must be remembered for ready use. They are:

$1^3 = 1$.
$2^3 = 8$.
$3^3 = 27$.
$4^3 = 64$.
$5^3 = 125$.
$6^3 = 216$.
$7^3 = 343$.
$8^3 = 512$.
$9^3 = 729$.

134. The rule for extracting the cube root of a given number may be stated as follows:

Point every third figure by placing a dot over it, beginning with the units' place, proceeding to the left (and to the right if the number be a decimal); and thus divide the whole number into periods.

Find the greatest number whose cube is contained in the first period to the left. This is the first figure in the root. Subtract its cube from the first period and to the remainder bring down the second period. On the left hand of the given number arrange two columns, and call that which is to the extreme left the first column and the other the second column. Treble the first figure of the root and place it in the first column, and in the second column place three times the square of the first figure in the root; affix two ciphers to the quantity now placed in the second column, and using the quantity so formed as divisor, divide the number formed by bringing down the second period to the first remainder. The quotient is the next figure in the root. Affix this figure to the quantity in the first column, and multiply the quantity so formed by that figure, and add the product to the number in the second column. Multiply the quantity so formed in the second column by that figure and subtract the product from the number formed by bringing down the second period to the first remainder.

If there be more periods to be brought down the process should be continued as follows: To the quantity in the first column add twice the second figure in the root, *i.e.* the figure last obtained, so as to have in the first column thrice the part of the root already found. Place the square of the figure of the root last obtained below the last figure in the second column, and add together this square, the number next above it and the number next above that number, so as to have in the second column thrice the square of the part of the root already found. Affix two ciphers to the result thus obtained in the second column, and taking the number thus formed as divisor, divide the number formed by bringing down the third period to the second remainder. The quotient will be the next figure in the root; affix this figure to the number in the first column, multiply the number thus formed by that figure, and add the product to the number in the second column. Multiply the number so formed in the second column by that figure, and subtract the product from the number formed by bringing down the third period to the second remainder.

If there be more periods to be brought down the operation must be repeated.

If at any stage of the work the product to be subtracted is found to be too large, the figure of the root obtained by the trial divisor should be abandoned, and the next lower one should be tried; and so on till a product small enough is obtained.

Ex. 1. Extract the cube root of 262144.

1st col.	2nd col.	
184	10800	$26\dot{2}14\dot{4}$ (64
	736	216
	11536	46144
		46144
		.....

Ex. 2. Extract the cube root of 258474853.

183	10800	$25\dot{8}47\dot{4}85\dot{3}$ (637
6	549	216
1897	11349	42474
	9	34047
	1190700	8427853
	13279	8427853
	1203979	....

Ex. 3. Find the cube root of 89915392.

124	4800	$8\dot{9}91\dot{5}39\dot{2}$ (448
8	496	64
1328	5296	25915
	16	21184
	580800	4731392
	10624	4731392
	591424	....

Here dividing 25915, the number formed by bringing down the second period to the first remainder by the trial divisor 4800, the quotient is found to be 6; but with this 6 for the second figure in the root the product to be subtracted is found to be too large. So the next lower figure 5 is tried; but with the same result. Then 4 is tried, which is found to be the proper figure. Similarly, the trial divisor 580800 shows 9 as the quotient of 4731392, but on trial the product is found to be too large, and so the next lower figure 8 is tried and is found to suit.

Ex. 4. Required the cube root of ·153990656.

153	7500	$\cdot 15\dot{3}99\dot{0}65\dot{6}$ (·536
6	459	125
1596	7959	28990
	9	23877
	842700	5113656
	9576	5113656
	852276	....

The number being a decimal the periods are marked off from the left to the right.

Ex. 5. Find the cube root of 175701·626712833571.

156	7500	175̇701̇·626̇712̇833̇571̇ (56·0091
12	936	125
168009	8436	50701
18	36	50616
1680271	9408000000	85626712833
	1512081	84685608729
	9409512081	941104104571
	81	941104104571
	941102424300	....
	1680271	
	941104104571	

Here for marking periods the first dot is placed over the units' figure 1, and then we go on marking every third figure to the left and to the right.

135. To extract the cube root of a vulgar fraction :

If both the numerator and the denominator be perfect cubes the cube root of each should be extracted separately ; and the result will be the cube root of the given fraction. Thus $\frac{5}{7}$ is the cube root of $\frac{125}{343}$. If the numerator or the denominator be not a perfect cube the fraction should be reduced to a decimal and the cube root of the result should be extracted.

EXERCISE XXXIX.

Find the cube root of

1. 13824. 2. 175616. 3. 314432. 4. 32768.
5. 389017. 6. 592704. 7. 2985984. 8. 1953125.
9. 6539203. 10. 1771561. 11. 41063625. 12. 182284263.
13. 127263527. 14. 311665752. 15. 67917312.
16. 45844273539. 17. 280111385007. 18. 1371330631.
19. 665110004489. 20. 1134626507. 21. 470366406433.
22. 66459653874216. 23. 668430015037875. 24. ·079507.
25. ·753571. 26. 15·438249. 27. 629422·793.
28. 66·430125. 29. ·000002197. 30. 452·045367981.
31. 27054036·008. 32. $\frac{216}{343}$; $3\frac{3}{8}$. 33. $\frac{128}{250}$; $\frac{3993}{10125}$.

Find the cube root, to three places of decimals where necessary, of

34. $\cdot\dot{8}$; $\dot{0}3\dot{7}$. 35. 2; 3. 36. $\frac{3}{4}$; $\frac{23}{32}$; $\frac{47}{49}$.

Find the sixth root of

37. 148035889. 38. 34296347249.

39. 1·0161520150601. **40.** 10506·912570445824.

CHAPTER VII.

CONCRETE NUMBERS.

136. So far we have been dealing with abstract numbers, and we have shown how the several operations of Arithmetic may be performed on these numbers. We now proceed to consider questions which involve concrete numbers of different denominations. The tables given below show the relations which units of different denominations bear to one another.

TABLES OF ENGLISH MONEY, WEIGHTS AND MEASURES.

Money.

4 Farthings = 1 Penny (d.). 12 Pence = 1 Shilling (s.).
20 Shillings = 1 Pound (£).

A farthing was formerly denoted by q. Now, however, it is denoted only as a fraction of a penny. £., s., d., and q., are respectively the initial letters of the Latin words, libra, solidus, denarius, and quadrans—the names of certain Roman coins or sums of money.

There are at present 12 coins in circulation in England. Of these the following are of bronze or copper:

A Farthing. A Half-penny. A Penny.

The following are of silver:

A Three-penny piece. A Six-penny piece.
A Shilling. A Florin (fl.) = 2s.
A Half-crown = 2s. 6d. A Four-shilling piece.
A Crown = 5s.

The following are of gold :

A Half-sovereign = 10s. A Sovereign (or pound) = 20s.
A Sovereign weighs $123\frac{1}{4}$ grains.

The following coins were once in circulation :

A Groat = 4d. A Tester = 6d. A Noble = 6s. 8d.
An Angel = 10s. A Mark = 13s. 4d. A Guinea = 21s.
A Carolus = 23s. A Jacobus = 25s. A Moidore = 27s.

Of these, the first two, viz. groat and tester, were of silver and the rest of gold.

WEIGHTS.

Troy Weight.

For weighing gold, silver and jewellery :

24 Grains (grs.) = 1 Penny-weight (dwt.).
20 Penny-weights = 1 Ounce (oz.).
12 Ounces = 1 Pound (lb.).

A Carat of Diamond = $3\frac{1}{5}$ Grs.
A Carat of Gold or Silver = 240 Grs.

Avoirdupois Weight.

For all common goods :

16 Drams (drs.) = 1 Ounce (oz.).
16 Ounces = 1 Pound (lb.).
28 Pounds = 1 Quarter (qr.).
4 Quarters = 1 Hundredweight (cwt.).
20 Hundredweights = 1 Ton.
A Stone (st.) = 14 Lbs. A Sack (of flour) = 240 Lbs.

The pound Avoirdupois contains 7000 grs. Troy.

Apothecaries' Weight.

20 Grains (grs.) = 1 Scruple (sc.).
3 Scruples = 1 Dram or Drachm (dr.).
8 Drachms = 1 Ounce (oz.). 12 Ounces = 1 Pound (lb.).

The grain, the ounce and the pound in this weight are the same as in Troy weight. Apothecaries buy their drugs and medicines by the Avoirdupois weight, and sell them by the one given above.

The grain is the same in Troy, Avoirdupois and Apothecaries' weights.

7000 Grains = 1 Lb. (Avoirdupois).
5760 Grains = 1 Lb. (Troy).

This gives us the connection between a Troy pound and an Avoirdupois pound.

MEASURES.

Length.

12 Inches (in.) = 1 Foot (ft.).
3 Feet = 1 Yard (yd.).
5½ Yards = 1 Pole, Rod or Perch (pr.).
40 Poles = 1 Furlong (fur.).
8 Furlongs = 1 Mile (m.).
3 Miles = 1 League (lea.).
12 Lines = 1 Inch.
3 Inches = 1 Palm.
4 Inches = 1 Hand (in horse measure).
9 Inches = 1 Span.
18 Inches = 1 Cubit.
2½ Feet = 1 Military pace.
6 Feet = 1 Fathom.
100 Fathoms = 1 Cable's length.
22 Yards or 100 Links = 1 Chain.
69½ Miles or 60 Geographical or Nautical Miles. } = 1 Degree (1 deg. or 1°).

For cloth, the following measure is used :

2¼ Inches = 1 Nail.
9 Inches = 1 Quarter.
4 Quarters = 1 Yard.
5 Quarters = 1 English Ell.
6 Quarters = 1 French Ell.
3 Quarters = 1 Flemish Ell.

Surface of Area.

144 Square Inches = 1 Square Foot (1 sq. ft. or 1 ft.).
9 Square Feet = 1 Square Yard (1 sq. yd. or 1 yd.).
30¼ Square Yards = 1 Square Pole (1 sq. po. or 1 po.).
40 Square Poles = 1 Square Rood or Rood (1 rd.).
4 Roods = 1 Acre (1 ac.).
640 Acres = 1 Square Mile (sq. m.).
10000 Square Links = 1 Square Chain.
10 Square Chains = 1 Acre.
100 Acres = 1 Hide of Land.

8 Hides were formerly a Knight's Fee, and 40 Hides a Baron's Fee.

Solidity.

1728 Cubic Inches = 1 Cubic Foot (1 cub. ft.).
27 Cubic Feet = 1 Cubic Yard (1 cub. yd.).

Capacity.

For liquids and dry goods :

4 Gills = 1 Pint (pt.).
2 Pints = 1 Quart (qt.).
4 Quarts = 1 Gallon (gal.).

2 Gallons = 1 Peck (pk.).
4 Pecks = 1 Bushel (bus.).
8 Bushels = 1 Quarter (qr.).
5 Quarters = 1 Load.
For dry goods only.

36 Gallons (of beer) = 1 Barrel (bar.).
54 Gallons (of beer) = 1 Hogshead (hhd.).
63 Gallons (of wine) = 1 Hogshead.
2 Hhds. or 126 Gallons = 1 Pipe (of wine).

Apothecaries subdivide the pint in fluid measure as follows :

60 Minims or Drops = 1 Drachm.
8 Drachms = 1 Ounce.
20 Ounces = 1 Pint.

A Gallon contains 277·274 cubic inches.

A Gallon of pure or distilled water weighs 10 lbs. (Avoir.).

A Pint of water weighs a pound and a quarter.

A Cubic Foot of water weighs about 1000 oz. (Avoir.).

TIME.

60 Seconds (sec.) = 1 Minute (min.).
60 Minutes = 1 Hour (hr.).
24 Hours = 1 Day.
7 Days = 1 Week (wk.).
28, 29, 30, or 31 Days = 1 Calendar Month.
12 Calendar Months = 1 Year.
365 Days = 1 Common Year.
366 Days = 1 Leap Year.
100 Years = 1 Century.

The lengths of the Calendar months may be easily remembered by the following lines :

Thirty days hath September,
April, June, and November ;
February hath twenty-eight alone ;
And all the rest have thirty-one ;
But leap year coming once in four,
February then hath one day more.

Each day is considered to commence at midnight.

52 weeks are regarded as equal to one year.

A *Lunar month*, or the time in which the moon travels round the earth, is equal to 28 days nearly.

A *Solar year*, or the time in which the earth travels round the sun, is equal to 365 days, 5 hrs. 48 mins. 47½ secs. nearly.

Since the common year is 5 hrs. 48 mins. 47½ secs. (nearly) shorter than the solar year, there will be, in four years, a difference of 23 hrs. 15 mins. 10 secs. in the times measured by the two. In order to make good this difference, it is enacted that every fourth year—the year, of which the number is divisible by four—should be a leap year, *i.e.* should contain 366 days. The extra day is given to February, and thus in a leap year February has 29 days. It will, however, be seen that in this way, we add 24 hrs. where we ought to really add 23 hrs. 15 mins. 10 secs., *i.e.* the time in this way denoted by 4 years (3 common and 1 leap) exceeds the proper time—4 solar years—by about 44 mins. and 50 secs. and thus in 400 years, the excess will be a little over 3 days. To correct this error, there are allowed only 97 leap years in 400 years; the year completing a century, the number of which is not divisible by 4, not being considered a leap year. Thus 1600, 1760, 1888, 1892 are all leap years. But 1900 is not a leap year.

ANGULAR MEASURE.

60 Seconds (60″) = 1 Minute (1′).
60 Minutes = 1 Degree (1°).
90 Degrees = 1 Right Angle.

MISCELLANEOUS MEASURE.

24 Sheets of paper = 1 Quire.
20 Quires = 1 Ream.
10 Reams = 1 Bale.

12 Articles of any kind = 1 Dozen.
12 Dozen = 1 Gross.
20 Articles of any kind = 1 Score.

TABLES OF MONEY, WEIGHTS AND MEASURES IN BRITISH INDIA.

12 Pies = 1 Anna.
16 Annas = 1 Rupee.
15 Rupees = 1 Sovereign or pound.

There are at present 10 coins in circulation in British India. Of these the following are of copper:

A Pie or $\frac{1}{12}$ Anna piece.
A Half Paisa or $\frac{1}{8}$ Anna piece.
A Paisa or $\frac{1}{4}$ Anna piece.
A Half-anna piece.

And the following are of silver:

A Two-anna piece.
A Four-anna piece.
An Eight-anna or $\frac{1}{2}$ Rupee piece.
A Rupee.

The Sovereign and the Half-sovereign, which have been only recently introduced, are of gold. The Indian sovereign is the same as the English sovereign.

The Rupee weighs 180 grs. Troy; and all the other silver coins weigh proportionately to this. The Half-anna piece weighs 200 grs. Troy; and all the other copper coins weigh in proportion to this.

Besides these coins, a paper currency has been established by the Government of India; and notes of the value of *Rs*.10000, *Rs*.1000, *Rs*.500, *Rs*.100, *Rs*.50, *Rs*.20, *Rs*.10, and *Rs*.5 are issued.

WEIGHT.

Imperial.

(Weight of)
1 Rupee = 1 Tola.
5 Tolas = 1 Chatak.
4 Chataks = 1 Pawa.
4 Pawas = 1 Seer.
40 Seers = 1 Maund.

Bombay.

4½ Tâks = 1 Chatak.
4 Chataks = 1 Pawaseer.
4 Pawaseers = 1 Seer.
5 Seers = 1 Pasari.
2 Pasarees = 1 Dhadâ.
4 Dhadâs = 1 Maund.
20 Maunds = 1 Candy.
120 Seers = 1 Pallâ.

Bombay City Maund = 28 lbs. (Avoir.).

Bengal.

4 Sikis = 1 Tola.
5 Sikis = 1 Kaucha.
4 Kauchas = 1 Chatak.
4 Chataks = 1 Poa.
4 Poas = 1 Seer.
40 Seers = 1 Maund.

Madras.

(Weight of)
1 Rupee = 1 Tola.
3 Tolas = 1 Palam.
8 Palams = 1 Seer.
5 Seers = 1 Viss.
8 Viss = 1 Maund.
20 Maunds = 1 Candy.

Bombay.

For weighing gold and silver:

8 Gunjas = 1 Mâsâ.
12 Mâsâs = 1 Tóla.
24 Tólas = 1 Seer.
40 Vâls = 1 Tóla.

For jewels:

Bombay.

16 Tandúbas = 1 Rati.
24 Ratis = 1 Tâk.

Bengal.

4 Dhaus = 1 Rati.
6 Ratis = 1 Anna.
16 Annas = 1 Tola.

LINEAR MEASURES.

Bombay.

8 Yawas = 1 Angul.
2 Anguls = 1 Tasu.
12 Tasus = 1 Hath.
2 Haths = 1 Gunja.
4 Haths = 1 Danda.
2000 Dandas = 1 Kos.
4 Kos = 1 Yojan.

Bengal.

3 Jabs = 1 Anguli.
4 Angulis = 1 Mushti.
3 Mushtis = 1 Bighat.
2 Bighats = 1 Hath.
4 Haths = 1 Dhanu.
2000 Dhanus = 1 Kros.
4 Kroses = 1 Jojan.

Madras.

9 Inches = 1 Span.
2 Spans = 1 Cubit.
2 Cubits = 1 Yard.

SURFACE MEASURE.

Imperial.

16 Annas = 1 Gunthâ.
40 Gunthâs = 1 Acre.

Bombay.

4 Square Angulis = 1 Square Tasu.
144 Square Tasus = 1 Square Hath.
4 Square Haths = 1 Square Gaja.
4900 Square Tasus = 1 Square Rod.
20 Square Rods = 1 Paud.
20 Pauds = 1 Bigha.
120 Bighas = 1 Chahur.

Bengal.

1 Square Cubit = 1 Ganda.
20 Gandas = 1 Chatak.
16 Chataks = 1 Kottah.
20 Kottahs = 1 Bigha.

Madras.

2400 Square Feet = 1 Manie.
24 Manies = 1 Cawni.
484 Cawnies = 1 Square Mile.

CUBIC MEASURE.

Bombay.

8 Cubic Angulis = 1 Cubic Tasu.
1728 Cubic Tasus = 1 Cubic Hath.
8 Cubic Haths = 1 Cubic Gaja.

Bengal.

13824 Cubic Angulis = 1 Cubic Cubit or Hath.
8 Cubic Cubits = 1 Cubic Yard.
8 Cubic Yards or 64 Cubic Cubits = 1 Chouka.

CORN MEASURE.

Bombay.

2 Chiptis = 1 Mapat.
2 Mapatas = 1 Seer.
2 Seers = 1 Adholi.
2 Adholis = 1 Payali.
16 or 12 Payalis = 1 Maund.
20 Maunds = 1 Khandi.
30 Payalis = 1 Pallâ.

Bengal.

5 Chataks = 1 Kunika.
2 Kunikas = 1 Khunchi.
2 Khunchies = 1 Reck.
2 Recks = 1 Pali.
2 Palis = 1 Doan.
2 Doans = 1 Kati.
8 Doans = 1 Maund.
20 Doans = 1 Sali.
8 Katis = 1 Arhi.
20 Arhis = 1 Bish.
16 Bishes = 1 Kahnu.

Madras.

8 Ollocks = 1 Padi.
8 Padies = 1 Marakkal.
5 Marakkals = 1 Parah.
12 Marakkals = 1 Kalam.
400 Marakkals = 1 Garee.

LIQUID MEASURE.

4 Pawas = 1 Seer.
40 Seers = 1 Maund.

MEASURE OF TIME.

60 Vipals = 1 Pal.
60 Pals = 1 Ghatee.
2½ Ghatees = 1 Tas (or hour).
3 Tâsas = 1 Prahar.
8 Prahars = 1 Dina or Ahoratra.
7 Dinas = 1 Week.
2 Weeks = 1 Paksh.
2 Pakshas = 1 Masa.
30 Days = 1 Masa.
12 Masas = 1 Year.
360 Days = 1 Year.

We give below some of the coins that have been in use in some of the countries in Europe and America.

Franc = 100 Centimes = 9½d.
Mark = 100 Pfennige = 11¾d.
Rouble (gold) = 100 Copecks = 3s. 2d.
Dollar = 100 Cents = 4s. 1¼d.
1 Sovereign = 25·22 Francs = 20·43 Marks.
= 6·3 Roubles = 4·87 Dollars.

NOTE.—The values of the coins given above are approximate.

The following are the tables of weights and linear measures of the Metric System which is in general use in France. In this system the unit of any denomination is *10 times that of the next lower one.*

WEIGHT.

10 Milli-grammes = 1 Centi-gramme.
10 Centi-grammes = 1 Deci-gramme.
10 Deci-grammes = 1 *Gramme.*
10 Grammes = 1 Deca-gramme.
10 Deca-grammes = 1 Hecto-gramme.
10 Hecto-grammes = 1 Kilo-gramme.
10 Kilo-grammes = 1 Myria-gramme.

1 Gramme = 15·43 grains nearly.

LINEAR MEASURE.

10 Milli-metres = 1 Centi-metre.
10 Centi-metres = 1 Deci-metre.
10 Deci-metres = 1 *Metre.*
10 Metres = 1 Deca-metre.
10 Deca-metres = 1 Hecto-metre.
10 Hecto-metres = 1 Kilo-metre.
10 Kilo-metres = 1 Myria-metre.

1 Metre = 39·37 inches nearly.

REDUCTION.

137. A quantity is said to be *simple* when it is expressed in terms of one denomination only. Thus 3 rupees, 7 yards, 5 gallons are all *simple* quantities.

A quantity is said to be *compound*, when it is expressed in terms of two or more denominations. Thus 4 rupees 5 annas, 6 yards 2 feet 9 inches, 1 gallon 3 quarts are all *compound* quantities.

Reduction is the method by which simple or compound quantities of given denominations may be converted into others of lower or higher denominations.

138. To express a simple quantity in terms of a lower denomination :

Rule. Multiply the quantity by the number which represents how many units of the next lower denomination are contained in a unit of the given denomination. The product will express the given quantity in terms of the next lower denomination. Multiply this product, if necessary, by the number which represents how many units of the next lower denomination are contained in one unit of this denomination and repeat the process till the required denomination is reached.

Ex. 1. Reduce Rs.28 to pies.

Since R.1 = 16 as.,

∴ Rs.28 = 28 × 16 or 448 as.

Since 1 anna = 12 pies,

∴ 448 as. = 448 × 12 or 5376 pies ;

∴ Rs.28 = 5376 pies.

The work may be set down as in the margin.

$$\begin{array}{r} \text{Rs.} \\ 28 \\ 16 \\ \hline 448 \\ 12 \\ \hline 5376 \end{array}$$

∴ 5376 pies is the answer.

139. To express a compound quantity in terms of a lower denomination :

Rule. Multiply the number of the highest denomination in the given quantity by the number of units of the next lower denomination contained in one unit of the highest, and to the product add the number of that lower denomination, if there be any in the given quantity ; repeat this process for each succeeding denomination till the required denomination is reached.

Ex. 2. Reduce £15. 16s. 6d. to pence.

$$\begin{array}{rrr} £. & s. & d. \\ 15 & 16 & 6 \\ 20 & & \\ \hline 300 & & \\ 16 & & \\ \hline 316 & & \\ 12 & & \\ \hline 3792 & & \\ 6 & & \\ \hline 3798 & & \end{array}$$

Here we first multiply 15 by 20 ($\because$ £1 = 20s.) and add 16 to the product 300. The result is 316, which represents the number of shillings equal to £15. 16s. We next multiply 316 by 12 ($\because$ 1s. = 12d.) and add 6 to the product 3792. The result is 3798, which represents the number of pence equal to £15. 16s. 6d.

Ex. 3. Reduce 16 khandies 13 maunds 2 seers to seers.

$$\begin{array}{rrr} \text{Khandies} & \text{Maunds} & \text{Seers} \\ 16 & 13 & 2 \\ 20 & & \\ \hline 320 & & \\ 13 & & \\ \hline 333 & & \\ 40 & & \\ \hline 13320 & & \\ 2 & & \\ \hline 13322 & & \end{array}$$

We first multiply 16 by 20 ($\because$ 1 khandi = 20 maunds) and to the product add 13. We next multiply 333 by 40 ($\because$ 1 maund = 40 seers) and add to the product 2. The result 13322 is the number of seers equal to 16 khandies 13 maunds 2 seers.

Ex. 4. Reduce 3 tons 10 cwt. 4 lbs. to pounds.

$$\begin{array}{rrrr} \text{Tons} & \text{Cwt.} & \text{Qrs.} & \text{Lbs.} \\ 3 & 10 & 0 & 4 \\ 20 & & & \\ \hline 60 & & & \\ 10 & & & \\ \hline 70 & & & \\ 4 & & & \\ \hline 280 & & & \\ 28 & & & \\ \hline 7840 & & & \\ 4 & & & \\ \hline 7844 & & & \end{array}$$

$\therefore$ 3 tons 10 cwt. 4 lbs. = 7844 lbs.

EXERCISE XL.

Reduce to pies

1. Rs.18.
2. 14 as.
3. Rs.372.
4. Rs.4. 13 as.
5. Rs.9. 3 as.
6. Rs.65. 12 as.

Reduce to pence

7. £36.
8. 17s.
9. £8. 13s.
10. £12. 5s.
11. £45. 1s.
12. £21. 8s.
13. £8. 3 cr.
14. 5 g. 8s.
15. 28 cr. 9s.

Reduce to farthings

16. £2. 3s. 4d.
17. £18. 7s. 9d.
18. 18s. 3d.
19. £26. 7d.

20. Reduce Rs.5. 4 as. 9 pies to pies.

21. Reduce to farthings £7. 13s. 4d. 3 farthings.

Reduce to pounds avoirdupois

22. 7 cwt. 3 qrs.
23. 5 tons 2 cwt.
24. 9 tons 2 qrs.
25. 19 tons 18 cwts.
26. 5 cwt. 4 lbs.
27. 12 stones 9 lbs.

28. Reduce 13 tons 15 cwts. 2 qrs. 21 lbs. 10 oz. 9 drs. to drams.

29. Reduce to grains 9 lbs. (Troy) 7 oz. 19 gr.

Reduce to inches

30. 5 yds.
31. 3 yds. 2 ft.
32. 4 yds. 1 ft.
33. 7 fur. 2 yds.
34. 9 m. 5 fur.
35. 2 m. 4 fur.

36. Reduce to inches 3 m. 5 fur. 13 po. 4 yds. 1 ft. 9 in.

Reduce to pints

37. 3 qt. 1 pt.
38. 1 gal. 2 qt.
39. 5 gals. 1 qt.
40. 2 pks. 1 gal.
41. 4 qrs. 3 pks.
42. 7 bus. 1 gal.

43. Reduce to pints 9 gals. 3 qt. 1 pt.

44. Reduce to pints 4 qrs. 7 bus. 3 pks. 1 gal. 2 qt.

45. Reduce to seers 3 maunds 6 pásarees.

Reduce to seconds

46. 8 hrs. 32 mins.
47. 28 mins. 9 secs.
48. 2 dys. 8 hrs.
49. 2 wks. 6 dys.
50. 4 dys. 19 mins.
51. 18 hrs. 15 secs.

52. Reduce to seconds 3 dys. 21 hrs. 49 mins. 38 secs.

53. Reduce 5 yrs. 3 mo. 29 dys. to days.

54. Reduce 8 acres 3 roods to square yards.

55. Reduce 5 ac. 2 ro. 26 sq. po. 8 sq. yds. to square feet.

56. Reduce 25 cub. ft. 96 cub. in. to cubic inches.

57. Reduce 42 degrees 28 minutes to seconds.

58. Reduce 5 tolas 4 masâs to gunjas.

59. Reduce 2 pallâs 5 payalyas to seers.

60. Reduce 17 sovereigns to pies.

140. To convert a quantity of a given denomination into one of a higher denomination or denominations :

Rule. Divide the given quantity by the number of its units required to make a unit of the next higher denomination ; the remainder, if any, will represent in the result the units of this lower denomination ; the quotient standing for units of the next higher denomination. Divide this quotient as we divided the given quantity and repeat the process till the required conversion is completed.

Ex. 1. Express 829 pies in terms of rupees, annas and pies.

12) 829 pies
16) 69 annas + 1 pie
4 rupees + 5 annas.

∴ 829 pies = Rs.4. 5 as. 1 pie.

Here we first divide 829 by 12 (∵ 12 pies = 1 anna) ; the quotient 69 stands for annas and the remainder 1 will represent the number of pies in the result. Dividing 69 by 16 (∵ 16 as. = R.1) ; we get 4 as quotient and 5 as remainder. Thus 5 will represent in the result the number of annas and the quotient 4 will stand for rupees. Hence the given quantity is equal to Rs.4. 5 as. 1 p.

Ex. 2. Reduce 6936 pence to pounds, shillings and pence.

12) 6936d.
20) 578s.
£28 + 18s.

∴ 6936d. = £28. 18s.

We first divide 6936 by 12 (∵ 12d. = 1s.) ; the quotient is 578 and no remainder is left. Hence there will be no pence in the result. Dividing next 578 by 20 (∵ 20s. = £1), we get 28 as quotient and 18 as remainder. Therefore £28. 18s. is the answer.

Ex. 3. Reduce 684079 inches to miles, furlongs, poles, yards, feet and inches.

12) 684079 in.
3) 57006 ft. + 7 in.
19002 yds.
2
11) 38004 half-yards.
40) 3454 po. + 10 half-yards or 5 yards.
8) 86 fur. + 14 po.
10 m. + 6 fur.

∴ 684079 in. = 10 m. 6 fur. 14 po. 5 yds. 7 in.

We first divide 684079 by 12 (∵ 12 in. = 1 ft.); the quotient is 57006 and the remainder 7; 7 will denote inches in the result. We next divide 57006 by 3 (∵ 3 ft. = 1 yd.); the *quotient* is 19002 and there is no remainder; therefore there will be no feet in the result.

We have next to divide 19002 yards by $5\frac{1}{2}$ (∵ $5\frac{1}{2}$ yds. = 1 po.), to obtain poles. This is the same as dividing $19002 \times 2 = 38004$ half-yards by 11. The result is 3454 poles and ten half-yards, *i.e.* 5 yards. Dividing next 3454 by 40 (∵ 40 po. = 1 fur.), we obtain 86 furlongs and 14 poles. 86 furlongs again is equal to 10 miles and 6 furlongs (∵ 8 fur. = 1 mile). Therefore the result is 10 miles, 6 furlongs, 14 poles, 5 yards and 7 inches.

N.B.—In examples of linear measure, it is usual to omit the denominations of poles and furlongs unless we are expressly asked in the question to bring them in. Thus in the above example after reaching 19002 yards, we should proceed at once to miles by dividing 19002 by 1760 (∵ 1760 yards = 1 mile). The result is 10 miles, 1402 yards and 7 inches.

Since the processes of reduction given in Art. 134 or 135 and 136 are each the converse of the other the accuracy of the result obtained by one process may be tested by working back according to the other process.

Ex. 4. Reduce 6 lbs. 5 oz. 14 dwt. 21 gr. to grains and test the accuracy of the result.

$$\begin{array}{rlll}
\text{lbs.} & \text{oz.} & \text{dwt.} & \text{gr.} \\
6 & 5 & 14 & 21 \\
\underline{12} & & & \\
72 & & & \\
\underline{5} & & & \\
77 & \text{ounces.} & & \\
\underline{20} & & & \\
1540 & & & \\
\underline{14} & & & \\
1554 & \text{pennyweights.} & & \\
\underline{24} & & & \\
37296 & & & \\
\underline{21} & & & \\
37317 & \text{grains.} & &
\end{array}$$

∴ the given quantity = 37317 grains.

Now to reduce 37317 grains to pounds, ounces, etc., we proceed as follows :

24) 37317 grains.
20) 1554 dwt. + 21 gr.
12) 77 oz. + 14 dwt.
6 lbs. + 5 oz.

∴ 37317 grains = 6 lbs. 5 oz. 14 dwt. 21 gr., which is the given quantity.

Therefore the work is correct.

Ex. 5. Reduce £24 to guineas.

£
24
20
21) 480s.
22 g. + 18s.

As pounds are not contained in guineas an exact number of times, we must reduce the pounds to shillings by multiplying by 20. The result is 480 shillings. And as 21 shillings make one guinea, by dividing 480 by 21, we obtain guineas equal to £24. ∴ £24 = 22 guineas 18 shillings.

EXERCISE XLI.

1. Reduce to annas 96 pies, 168 pies, 252 pies and 756 pies.
2. Reduce to shillings 132d., 336d., 432d., 516d. and 948d.
3. Reduce to rupees 176 as., 224 as., 304 as., 768 as. and 4096 as.
4. Reduce to guineas 441s., 735s., 987s., 1239s. and 1764s.
5. Reduce to quarters 308 lbs., 784 lbs., 3136 lbs. and 4424 lbs.
6. Reduce to pennyweights 384 grs., 576 grs. and 2304 grs.
7. Reduce to yards 756 in., 1296 in. and 5184 in.
8. Reduce to gallons 512 pts., 1024 pts., 7744 pts. and 9604 pts.

Reduce to rupees, annas, etc.

9. 421 pies, 530 pies, 731 pies, 847 pies and 1000 pies.
10. 928 pies, 1324 pies, 1821 pies, 2000 pies and 54321 pies.
11. 500 as., 746 as., 8250 as., 12345 as., and 10203 as.

Reduce to pounds, shillings, etc.

12. 700d., 845d., 926d., 1020d. and 40301d.
13. 593s., 648s., 7124s. and 184692s.

Reduce to tons, cwts., etc.

14. 423 lbs., 7495 lbs., 43689 oz. and 236981 drs.
15. 198 qrs., 967 cwts., 10532 lbs. and 6451 cwts.

Reduce to miles, furlongs, etc.

16. 3517 in., 1392 ft., 18561 in. and 2736 yds.

17. 1095 ft., 361 fur., 34192 yds. and 498 po.

18. 765 in., 5214 ft., 520 yds., 238 fur. and 959 po.

Reduce to years, weeks, etc.

19. 1238 dys., 741 wks., 1000 dys. and 100000 hrs.

20. 175429 secs., 8415 min. and 7261 dys.

21. 9831 dys., 87654 wks. and 7654321 secs.

Reduce to pounds (Troy), ounces, etc.

22. 513 oz., 6921 dwts. and 81547 grs.

23. 123 oz., 456 dwts. and 78910 grs.

Reduce to acres, roods, etc.

24. 648 sq. po., 59621 sq. ft. and 5169 sq. yds.

25. 275 ros., 527 sq. po. and 7520 sq. yds.

26. 246810 sq. in., 4840 sq. yds. and 2030405 sq. ft.

Reduce to cubic yards, cubic feet, etc.

27. 438 cub. ft., 1054 cub. ft. and 79412 cub. in.

28. 1754 cub. ft. and 382395 cub. inches.

29. Reduce one thousand seers to maunds.

30. Reduce 28875 grains to Avoirdupois weight.

31. Reduce to Avoirdupois weight 525 lbs. (Troy).

32. Reduce 7161 sovereigns to guineas.

33. Reduce 324 nails to English ells, yards, etc.

34. Reduce one thousand gunjas to tolas, masâs, etc.

35. Reduce 7325 seers to khandis, maunds, etc.

36. Reduce 39 guineas to pounds and shillings.

37. Reduce 51 pounds to guineas and shillings.

38. Reduce 63360 pies to pounds.

39. Reduce 267 pounds to rupees.

40. Reduce 1680 pence to rupees.

COMPOUND ADDITION.

141. To add compound quantities of the same kind :

Rule. Place the given quantities under one another so that those of the same denomination may be under each other in the same column. Add the numbers of the lowest denomination together. Divide the sum by the number of units of this denomination which make a unit of the next higher denomination. Set down the remainder, if any, under the lowest denomination column and add the quotient to the numbers of the next higher denomination. Proceed in this way with the other columns in succession, the result is the sum required.

Ex. 1. Add together Rs.5. 7 as. 8 pies, Rs.9. 12 as. 5 pies, 9 as. 4 pies, and Rs.18. 3 as. 2 pies.

Rs.	as.	pies.
5 .	7 .	8
9 .	12 .	5
	9 .	4
18 .	3 .	2
34 .	0 .	7

Write the quantities as directed above. Add up the pies; they give 19 pies; divide 19 pies by 12 ($\because$ 12 pies = 1 anna); the quotient is 1 and the remainder 7. Write down 7 under the pies column and carry 1 to the annas column. This 1 and the annas in the annas column together give 32 annas, *i.e.* $32 \div 16$ or 2 rupees. Here there is no remainder to be placed under the annas column, therefore write zero under that column and carry 2 to the rupees column. This 2 and the rupees in the rupees column give 34 rupees. Therefore the answer is Rs.34. 0 as. 7 pies.

Ex. 2. Add together £4. 12s. $6\frac{1}{4}$d., £3. 10s. $3\frac{1}{3}$d., £7. 5s. $9\frac{3}{4}$d., 8s. $5\frac{1}{4}$d., £9. 4s. $2\frac{1}{2}$d.

£	s.	d.
4 .	12 .	$6\frac{1}{4}$
3 .	10 .	$3\frac{1}{3}$
7 .	5 .	$9\frac{3}{4}$
	8 .	$5\frac{1}{4}$
9 .	4 .	$2\frac{1}{2}$
25 .	1 .	$3\frac{1}{12}$

Here we take the sum of the pence by first adding together the fractional parts and then the integers. The sum of the former is $2\frac{1}{12}$. We therefore write $\frac{1}{12}$ as the fractional part of the sum of the pence; adding 2 to the integers we get 27, which is equal to 2s. 3d. Therefore we write 3 as the integral part of the pence in the result, *i.e.* under the column of the pence we set down $3\frac{1}{12}$. Carrying 2 to the column of shillings we obtain 41s. = £2. 1s. We therefore set down 1 under the shillings column. Carrying 2 to the pounds column we obtain £25. We therefore write 25 under the pounds column. The required sum therefore is £25. 1s. $3\frac{1}{12}$d.

EXERCISE XLII.

Add together

	Rs.	as.	pies.
1.	75	3	3
	14	2	2
	10	1	1
	11	4	4

	£	s.	d.
2.	28	9	9
	82	4	2
	23	0	5
	32	14	7

	£	Rs.	as.	pies.
3.	35	8	8	8
	38	12	6	3
	41	1	0	9
	40	8	0	5

	Guin.	s.	d.	far.
4.	48	20	10	0
	24	10	5	3
	36	15	8	2
	89	16	5	3

	£	s.	d.
5.	7	2	$10\frac{1}{4}$
	3	16	$4\frac{2}{3}$
	5	2	$11\frac{1}{6}$

	£	s.	d.
6.	10403	17	$5\frac{1}{4}$
	2487	9	$10\frac{5}{8}$
	7108	12	$8\frac{1}{8}$

	kds.	mds.	srs.
7.	78	18	23
	22	17	22
	56	14	14
	2	10	6

	lbs.	oz.	dwt.	gr.
8.	1	2	3	4
	5	6	7	8
	9	10	11	12
	13	0	14	15

	tolas	masâs	gunjâs.
9.	12	6	3
	8	4	2
	4	2	1
	23	4	2

	oz.	dr.	scr.	gr.
10.	5	3	2	1
	4	2	1	6
	8	4	0	12
	6	7	1	12

	yds.	ft.	in.
11.	63	2	11
	62	1	9
	1	0	1
	23	1	8

	miles	fur.	po.	yds.
12.	65	4	3	2
	54	3	2	1
	76	5	4	3
	4	3	30	4

	sq.yds.	sq.ft.	sq.in.
13.	23	6	86
	11	0	34
	12	8	99
	52	2	69

	acres	ro.	sq. po.
14.	9	1	16
	49	2	25
	64	3	36
	47	0	30

	ro.	sq.po.	sq.yds.
15.	3	29	17
	1	38	25
	1	12	30
	2	39	18

	cub.yds.	cub.ft.	cub.in.
16.	38	19	125
	71	9	250
	42	10	500
	33	26	1025

	hrs.	min.	sec.
17.	18	31	$49\frac{1}{2}$
	15	43	$24\frac{1}{6}$
	23	51	$36\frac{1}{3}$

	tons	cwt.	qrs.	lbs.	oz.	drs.
18.	2003	5	2	0	0	$7\frac{1}{9}$
	5228	19	3	27	1	$6\frac{2}{3}$
	7267	15	2	1	13	$15\frac{2}{9}$

	miles	fur.	po.	yds.	ft.	ins
19.	29	5	35	4	2	$11\frac{1}{8}$
	42	3	19	2	1	$9\frac{1}{2}$
	8	4	27	5	2	$8\frac{3}{8}$

	wks.	dys.	hrs.	min.	sec.
20.	2	5	9	47	21
	9	2	14	51	38
	5	6	23	59	25
	1	4	14	41	57

	qrs.	bus.	pks.	gals.	qts.	pts.	gills.
21.	9	7	2	1	3	1	3
	12	3	1	0	2	1	2
	8	4	3	1	3	1	2

COMPOUND SUBTRACTION:

142. To find the difference of two compound quantities.

Rule. Place the smaller of the two quantities under the other so that the numbers of the same denomination may be in the same column. Begin with the numbers of the lowest denomination, and subtract if possible, each number of the lower line from that which stands above it and set down the remainder underneath. If a number in the lower line be greater than the corresponding one above it, increase the latter by as many units as this denomination takes to make a unit of the next higher denomination and subtract, carrying 1 to the number in this higher denomination in the lower line. Proceed in this way till the subtraction is completed.

Ex. 1. Subtract Rs.4. 12 as. 8 pies from Rs.8. 15 as. 10 pies.

Rs.	as.	pies.
8	15	10
4	12	8
4	3	2

Setting down the smaller quantity under the greater and subtracting we get Rs.4. 3 as. 2 pies as the difference.

Ex. 2. Subtract Rs.12. 9 as. $5\frac{3}{4}$ pies from Rs.21. 6 as. $9\frac{1}{2}$ pies.

Rs.	as.	pies.
21	6	$9\frac{1}{2}$
12	9	$5\frac{3}{4}$
8	13	$3\frac{3}{4}$

Since $\frac{3}{4}$ cannot be subtracted from $\frac{1}{2}$ we increase $\frac{1}{2}$ by 1 and carry 1 to 5; and we get as the difference $3\frac{3}{4}$. We therefore set down $3\frac{3}{4}$ under the pies column. Next we find that 9 is greater than 6. We therefore increase 6 by 16 ($\because$ 16 as. $=1$ rupee) and setting down the difference between $6+16$, *i.e.* 22 and 9, which is 13, under the annas column, we carry 1 to 12 of the lower line under the rupees column. Subtracting 13 from 21, we set down 8 under the rupees column. Therefore the required difference is Rs.8 13 as. $3\frac{3}{4}$ p.

EXERCISE XLIII.

1.

Rs.	as.	p.
87	10	11
75	4	8

2.

£.	s.	d.	far.
123	16	11	3
35	8	3	0

3.

Rs.	as.	p.
12	6	$8\frac{1}{3}$
6	8	$9\frac{1}{2}$

	£.	s.	d.
4.	50	8	$4\frac{2}{5}$
	26	1	$5\frac{3}{4}$

	£.	Rs.	as.	p.
5.	103	8	13	5
		93	14	3 . 8

	Guin.	s.	d.	far.
6.	99	9	9	0
	95	6	6	3

	ton	cwt.	qrs.	lbs.
7.	1	0	0	0
		5	3	13

	cwt.	lbs.	oz.
8.	35	88	9
	22	87	12

	kds.	mds.	srs.
9.	45	0	0
	25	0	1

	lbs.	oz.	dwt.	gr.
10.	11	11	11	11
	1	10	13	13

	tolas	masâs	gunjâs
11.	20	7	5
	18	9	7

	oz.	dr.	scr.	gr.
12.	4	0	0	0
	3	0	2	17

	yds.	ft.	in.
13.	69	0	4
	59	2	7

	mi.	fur.	po.	yds.
14.	73	0	2	1
	64	7	38	0

	sq.yds.	sq.ft.	sq.in.
15.	19	0	0
	0	0	81

	sq. m.	ac.	ro.	po.
16.	4	0	0	0
		321	3	6

	cub.yds.	cub.ft.	cub.in.
17.	119	19	0
	19	8	728

	tons	cwt.	qr.	lbs.	oz.	drs.
18.	72	0	2	25	4	12
	35	3	0	27	13	8

	yrs.	wks.	dys.	hrs.	mins.	secs.
19.	832	47	2	20	37	45
	569	35	4	21	48	29

	mi.	fur.	po.	yds.	ft.	in.
20.	16	5	38	4	2	8
	15	6	17	5	1	11

	ac.	ro.	sq.po.	sq.yds.	sq.ft.	sq.in.
21.	45	1	32	18	4	93
	39	3	35	19	4	132

COMPOUND MULTIPLICATION.

143. Compound multiplication is the multiplication of compound quantities by abstract numbers. Since the multiplier denotes how many times the multiplicand is taken, it is obvious that the multiplier can never be a concrete number. The product in compound multiplication is a compound quantity.

144. To multiply a compound quantity by an abstract number :

Rule. Place the multiplier under the lowest denomination of the multiplicand ; multiply by it the number of the lowest denomination ; divide the product by the number of its units which make a unit of the next higher denomination ; place the remainder, if any, under the number of the lowest denomination and carry the quotient to the product of the number of the next denomination by the multiplier. Continue the process till the multiplication is completed.

Ex. 1. Multiply Rs.4. 7 as. 5 pies by 8.

$$\begin{array}{r}
\text{Rs.}\quad \text{as.}\ \text{pies} \\
4\ .\ \ 7\ .\ 5 \\
8 \\
\hline
35\ .\ 11\ .\ 4
\end{array}$$

The product of 5 and 8 is 40. Dividing 40 by 12, we get 3 as quotient and 4 as remainder. We therefore set down 4 under the pies column and carry 3 to the product of 8 and 7, *i.e.* to 56. We then divide 59 by 16; the quotient is 3 and the remainder 11; we set down 11 under the annas column and carry 3 to the product of 8 and 4, *i.e.* to 32. The result is 35, which we set down under the rupees column. Therefore the required product is Rs.35. 11 as. 4 p.

145. When the multiplier is resolvable into factors, it is sometimes convenient to so resolve it and multiply the given quantity by the several factors in succession.

Ex. 2. Multiply £3. 7s. 6¾d. by 21.

Since $21 = 7 \times 3$ we first multiply the given quantity by 7 and then the product by 3.

$$\begin{array}{r}
\text{£.}\quad \text{s.}\quad \text{d.} \\
3\ .\ \ 7\ .\ 6\tfrac{3}{4} \\
7 \\
\hline
23\ .\ 12\ .\ 11\tfrac{1}{4} \\
3 \\
\hline
70\ .\ 18\ .\ \ 9\tfrac{3}{4}
\end{array}$$

146. When the multiplier is not resolvable into convenient factors, we may represent it as the sum or difference of two numbers such that one of them is resolvable into convenient factors and the other is a small number in itself. The required product may thus be found by multiplying the given quantity by the two numbers and adding up the two products or deducting one from the other as the case may be.

Ex. 3. Multiply 3 yds. 2 ft. 6 inches by 43.

$$
\begin{array}{rcrcrl}
\text{yds.} & & \text{ft.} & & \text{in.} & \\
3 & . & 2 & . & 6 & \\
 & & & & 10 & \\
\hline
38 & . & 1 & & & =3\text{ yds. }2\text{ ft. }6\text{ in.}\times 10 \\
 & & 4 & & & \\
\hline
153 & . & 1 & & & =3\text{ yds. }2\text{ ft. }6\text{ in.}\times 40 \\
11 & . & 1 & . & 6 & =3\text{ yds. }2\text{ ft. }6\text{ in.}\times 3 \\
\hline
164 & . & 2 & . & 6 & =3\text{ yds. }2\text{ ft. }6\text{ in.}\times 43.
\end{array}
$$

43 is not resolvable into factors, but it may be written as 40 + 3. Now 40 may be resolved into 10 and 4. We therefore multiply the given quantity by 10 and the product by 4. And to this second product, we add the product of the given quantity and 3.

Ex. 4. Multiply Rs.6. 12 as. 3 pies by 47.

Since 47 = 48 − 1, we proceed as follows :

$$
\begin{array}{rcrcrl}
\textit{Rs.} & & \text{as.} & & \text{pies} & \\
6 & . & 12 & . & 3 & \\
 & & & & 8 & \\
\hline
54 & . & 2 & & & =\text{Rs.}6.\ 12\text{ as. }3\text{ p.}\times 8 \\
 & & 6 & & & \\
\hline
324 & . & 12 & & & =\text{Rs.}6.\ 12\text{ as. }3\text{ p.}\times 48 \\
6 & . & 12 & . & 3 & =\text{Rs.}6.\ 12\text{ as. }3\text{ p.}\times 1 \\
\hline
317 & . & 15 & . & 9 & =\text{Rs.}6.\ 12\text{ as. }3\text{ p.}\times 47.
\end{array}
$$

EXERCISE XLIV.

Multiply

1. Rs.35. 8 as. 7 p. by 5, 7 and 9.
2. Rs.78. 2 as. 8 p. by 12, 15 and 18.
3. Rs.54. 13 as. $6\frac{1}{4}$p. by 21, 24 and 28.
4. £26. 8s. 9d. by 8, 12 and 30.
5. £69. 12s. $8\frac{3}{4}$d. by 6, 25 and 48.
6. £85. 7s. 5d. by 17, 29 and 53.
7. 23 lbs. 7 oz. 13 drs. by 8, 14 and 26.
8. 185 tons 13 cwt. 3 qrs. 19 lbs. by 5, 16 and 24.
9. 38 lbs. 8 oz. 17 dwts. $23\frac{1}{3}$ grs. by 11 and 23.
10. 5 yds. 2 ft. $8\frac{3}{4}$ in. by 10, 35 and 84.
11. 7 fur. 19 po. 4 yds. by 32, 40 and 51.
12. 283 mi. 6 fur. 3 yds. 2 ft. by 7, 9 and 65.
13. 16 sq. yds. 5 sq. ft. 125 sq. in. by 6, 18 and 72.

Multiply

14. 92 ac. 3 r. 31 sq. po. 1½ sq. yds. by 8, 12 and 15.
15. 61 ac. 27 sq. po. 8 sq. yds. by 42, 86 and 95.
16. 47 cub. yds. 18 cub. ft. 821 cub. in. by 56 and 76.
17. 26 cub. ft. 1727 cub. in. by 1728.
18. 17 dys. 14 hrs. 12 mins. by 53.
19. 2 gals. 3 qts. 1¾ pts. by 24, 36 and 44.
20. 4 bus. 2 pks. 1 gal. by 128 and 196.
21. 6 qrs. 5 bus. 3 pks. 1 gal. by 37, 41 and 59.
22. 3 dys. 18 hrs. 32 mins. 49 secs. by 13, 19 and 31.
23. 125 yrs. 239 dys. 21 hrs. by 324 and 1681.
24. 26 khandis 15 maunds 9 seers by 1575 and 3087.
25. 23 tolas 8 masâs 6 gunjâs by 1000 and 2750.

COMPOUND DIVISION.

147. The term Compound Division is applied to the division of a compound quantity by an abstract number as also to the division of a compound quantity by another compound quantity. When a compound quantity is divided by an abstract number, the quotient is a compound quantity. When it is divided by another compound quantity, the result is an abstract number.

Division, as has already been explained, is the reverse operation of multiplication. The dividend in division corresponds to the product in multiplication. The divisor may be taken to correspond to either the multiplicand or the multiplier and the quotient to the third quantity. Now since in compound multiplication, the multiplier is an abstract number and the multiplicand a compound quantity, in compound division, either the divisor or the quotient must be an abstract number. If, in the first case, the quotient be a compound quantity, in the second, the divisor will be a compound quantity.

148. To divide a compound quantity by an abstract number :

Rule. Place the divisor on the left of the dividend, leaving the right side for the quotient as in simple division.

Begin with the quantity of the highest denomination in the dividend. Divide it, if possible, by the divisor, setting down the quotient in the quotient's place. If the number to be divided be smaller than the divisor, or if a remainder be left after dividing, reduce the number or the remainder, as the case may be, to the next lower denomination and add to it the number of that denomination in the dividend. Then divide the sum by the divisor; the quotient will represent in the result the quantity of this lower denomination. Proceed in this manner till the whole division is completed.

Ex. 1. Divide £118. 18s. 6½d. by 35.

£. s. d. £ s. d.
35) 118 . 18 . 6½ . (3 . 7 . 11½
105
13
20
260
18
278s.
245
33
12
396
6½
402½
35
52½
35
17½
17½
...

118 (pounds) divided by 35, gives 3 (pounds) as quotient and 13 (pounds) as remainder. Write £3 in the quotient's place as the first quantity. Multiply 13 by 20 (∵ £1 = 20s.) and to the product add 18 (shillings) from the dividend. We thus get 278 (shillings). Divide this number by 35. The quotient is 7 (shillings) and the remainder is 33 (shillings). Write 7s. as the second quantity in the quotient. Multiply 33 by 12 (∵ 1s. = 12d.) and to the product add 6½ (pence) from the dividend. We thus get 402½ (pence). Dividing this by 35 we get 11 as quotient and 17½ as remainder. But 17½ is exactly one-half of 35. We therefore get 11½d. as the last quantity in the quotient. Thus the required quotient is £3. 7s. 11½d.

Ex. 2. Divide 8 days 16 hours 25 minutes and 20 seconds by 36.

Here we cannot divide 8 (days) by 36. We therefore multiply 8 by 24 (∵ 1 day = 24 hours) and add to the product 16 (hours). We then proceed with the division as in Ex. 1 above. The first term in the quotient here is not days, but hours.

When the divisor is a sufficiently small number the quotient may be written below the dividend as in short division of abstract numbers.

Ex. 3. Divide Rs.116. 1 a. 11 pies by 7.

$$\begin{array}{rl}
 & \text{Rs.}\quad\ \text{as.}\quad \text{p.}\\
7\,) & \underline{116\ .\ \ 1\ .\ 11}\\
 & \ \ 16\ .\ \ 9\ .\ \ 5
\end{array}$$

Rs.116÷7 gives Rs.16 with Rs.4, *i.e.* 64 as. over. 64 as. + 1 a. or 65 as.÷7 gives 9 as., with 2 as. or 24 pies over. 24 pies + 11 pies or 35 pies÷7 gives 5 pies. Therefore the required quotient is Rs.16. 9 as. 5 pies.

When the divisor is resolvable into small factors, it is sometimes convenient *to divide by the several factors in succession*.

Ex. 4. Divide £361. 15s. 8d. by 32.

Since $32 = 4 \times 8$, we may first divide by 4 and the quotient by 8.

$$\begin{array}{rl}
 & \ \ \pounds\qquad \text{s.}\qquad \text{d.}\\
4\,) & \underline{361\ .\ 15\ .\ \ 8}\\
8\,) & \underline{\ \ 90\ .\ \ 8\ .\ 11} = \text{Dividend} \div 4\\
 & \ \ 11\ .\ \ 6\ .\ 1\tfrac{3}{8} = \text{Dividend} \div 32.
\end{array}$$

∴ £11. 6s. $1\frac{3}{8}$d. is the answer.

When the divisor is ten or some power of ten, the division may be performed by pointing off, from the right-hand of the dividend of each denomination, as many figures as the divisor contains ciphers, the portion on the left being the quotient and that on the right the remainder in each case.

Ex. 5. Divide £2568. 12s. 7d. by 100.

$$\begin{array}{rl}
 & \qquad \pounds\qquad\ \ \text{s.}\ \ \ \text{d.}\\
100\,) & 25{,}68\ .\ 12\ .\ 7\\
 & \underline{\quad\ 20}\\
 & 13{,}72\\
 & \underline{\quad\ 12}\\
 & \ \ 8{,}71
\end{array}$$

∴ the quotient is £25. 13s. $8\frac{71}{100}$d.

Here the first term in the quotient is 25 (pounds) and the first remainder 68 (pounds). The second term in the quotient is 13 (shillings) and the second remainder 72 (shillings). The last term in the quotient is 8 (pence) and the last remainder is 71 (pence). Therefore the last term itself may be written as $8\frac{71}{100}$d.

When the divisor is a compound quantity, it must be of the same kind as the dividend in order that the division should be possible. And in such a case, both the dividend and the divisor should be reduced to the same denomination and the division should be performed as in simple division. The quotient will be an abstract number showing the number of times the divisor is contained in the dividend.

Ex. 6. Divide 5 tons 9 cwts. 2 qrs. 21 lbs. by 2 cwts. 3 qrs. 7 lbs.

cwts. qrs. lbs.	tons cwts. qrs. lbs.	
2 . 3 . 7	5 . 9 . 2 . 21	
4	20	
8	100	
3	9	
11	109	
28	4	
308	436	
7	2	
315	438	315) 12285 (39
	28	945
	12264	2835
	21	2835
	12285	....

Reducing both to lbs., we get 2 cwts. 3 qrs. 7 lbs. = 315 lbs. and 3 tons 9 cwts. 2 qrs. 21 lbs. = 12285 lbs.

$$\therefore \frac{12285 \text{ lbs.}}{315 \text{ lbs.}} = 39,$$

since 39 (an abstract number) expresses how many times 315 lbs. is contained in 12285 lbs.

EXERCISE XLV.

Divide

1. Rs.26. 7 as. 6 p. by 6. 2. Rs.525. 13 as. 11 p. by 12.
3. £191. 10s. 9d. by 11. 4. £516. 12s. 6d. by 12.
5. £25932. 13s. $3\frac{1}{2}$d. by 19. 6. £86051. 8s. 9d. by 63.
7. 14 tons 17 cwt. 3 qrs. 9 lbs. 8 oz. 4 drs. by 9.
8. 55 tons 19 cwt. 3 qrs. 27 lbs. by 96.
9. 2 tons 4 cwt. 15 lbs. 12 drs. by 18.
10. 51 lbs. 2 oz. 9 dwt. 2 grs. by 14.
11. 63 mi. 5 fur. 24 po. by 13. 12. 271 yds. 1 ft. 7 in. by 17.
13. 8 mi. 3 fur. 25 po. 3 yds. 1 ft. 6 in. by 6.
14. 16 sq. yds. 4 sq. ft. 105 sq. in. by 3.
15. 2728 ac. 3 ro. 6 po. $8\frac{3}{4}$ yds. by 13.
16. 64 cub. yds. 18 cub. ft. 936 cub. in. by 18.
17. 36 yrs. 29 wks. 4 dys. 8 hrs. by 9.
18. 100 gals. 1 qt. 1 pt. by 73.
19. 53 qrs. 6 bus. 2 pks. 3 qts. 1 pt. by 217.
20. 487 khandis 18 maunds 5 seers by 392.
21. 814 tolâs 10 masâs 4 gunjâs by 480.

Divide

22. £357668. 3s. $5\frac{1}{4}$d. by £3764. 18s. $6\frac{3}{4}$d.
23. £2. 12s. 3d. by 1s. $4\frac{1}{2}$d.
24. 30 tons 6 cwt. 1 qr. 24 lbs. 8 oz. by 3 cwt. 2 qrs. 12 lbs. 5 oz.
25. 497 khandis 16 maunds 4 seers by 2 khandis 3 maunds and 4 seers.
26. Divide £5718. 18s. 6d. by 100.
27. Divide Rs.6842. 13 as. 9 p. by 100.
28. Divide £487. 13s. 8d. by 10.
29. Divide £43631. 1s. 2d. by 1000.
30. Divide 2728 ac. 3 ro. 6 po. $8\frac{3}{4}$ sq. yds. by 209 ac. 3 ro. 25 po. 3 sq. yds.

REDUCTION OF FRACTIONS.

149. To multiply a compound quantity by a fraction or (what is the same thing) to take a fractional part of a compound quantity :

Rule. Multiply the given quantity by the numerator of the fraction and divide the product by the denominator.

Mixed numbers should be reduced to improper fractions, and compound and complex fractions to simple ones before applying the above rule.

Ex. 1. Find the value of $\frac{5}{24}$ of Rs.7.

$$\tfrac{5}{24} \text{ of Rs.}7 = \frac{\text{Rs.}7 \times 5}{24} = \frac{\text{Rs.}35}{24} = \text{R.1. 7 as. 4 pies.}$$

Ex. 2. Find the value of $3\frac{4}{5}$ of £9.

$$\begin{aligned} 3\tfrac{4}{5} \text{ of } £9 &= \tfrac{19}{5} \text{ of } £9 \\ &= £\tfrac{171}{5} = £34.\ 4\text{s.} \end{aligned}$$

Or thus :

$$\begin{aligned} 3\tfrac{4}{5} \text{ of } £9 &= 3 \times £9 + \tfrac{4}{5} \times £9 \\ &= £27 + £\tfrac{36}{5} \\ &= £27 + £7.\ 4\text{s.} \\ &= £34.\ 4\text{s.} \end{aligned}$$

Ex. 3. Find the value of $\frac{4}{9}$ of $1\frac{7}{8}$ of Rs.5. 3 as.

$$\tfrac{4}{9} \text{ of } 1\tfrac{7}{8} = \tfrac{4}{9} \times \tfrac{15}{8} = \tfrac{5}{6}\,;$$

$$\begin{aligned} \therefore\ \tfrac{4}{9} \text{ of } 1\tfrac{7}{8} \text{ of Rs.5. 3 as.} &= \tfrac{5}{6} \text{ of Rs.5. 3 as.} \\ &= \frac{(\text{Rs.5. 3 as.}) \times 5}{6} \\ &= (\text{Rs.25. 15 as.}) \div 6 \\ &= \text{Rs.4. 5 as. 2 pies.} \end{aligned}$$

Or thus: $\frac{4}{9}$ of $1\frac{7}{8} = \frac{5}{6}$ and Rs.5 3 as. = 83 as.

∴ $\frac{4}{9}$ of $1\frac{7}{8}$ of Rs.5. 3 as. = $\frac{5}{6} \times 83$ as.

= $\frac{415}{6}$ as.

= 69 as. 2 pies

= Rs.4. 5 as. 2 pies.

Ex. 4. Find the value of $\frac{75}{88}$ pole + $\frac{3}{8}$ yd. + $\frac{3}{16}$ foot.

$\frac{75}{88}$ pole = $\frac{75}{88} \times 5\frac{1}{2}$ yards = $\frac{75}{16}$ yds.

= $4\frac{11}{16}$ yds.

∴ $\frac{75}{88}$ pole + $\frac{3}{8}$ yard = $4\frac{11}{16}$ yds. + $\frac{3}{8}$ yd. = $5\frac{1}{16}$ yds.

= 5 yds. $\frac{3}{16}$ ft.

∴ $\frac{75}{88}$ po. + $\frac{3}{8}$ yd. + $\frac{3}{16}$ ft. = 5 yds. $\frac{3}{16}$ ft. + $\frac{3}{16}$ ft.

= 5 yds. $\frac{3}{8}$ ft.

= 5 yds. 0 ft. $4\frac{1}{2}$ in.

Ex. 5. Find the value of

$\frac{3}{8}$ of £1. 10s. + $\frac{2}{3}$ of $1\frac{1}{2}$ of $\frac{4}{5}$ of £1. 5s. − $\frac{45}{80}$ of £1.

$\frac{3}{8}$ of £1. 10s. = $\frac{(\text{£1. 10s.}) \times 3}{8} = \frac{\text{£4. 10s.}}{8}$

= $\frac{90\text{s.}}{8}$ = $11\frac{1}{4}$s. = 11s. 3d.

$\frac{2}{3}$ of $1\frac{1}{2}$ of $\frac{4}{5}$ of £1. 5s. = $\frac{2}{3} \times \frac{3}{2} \times \frac{4}{5}$ of £1. 5s.

= $\frac{4}{5}$ of £1. 5s.

= $\frac{(\text{£1. 5s.}) \times 4}{5}$ = £1.

$\frac{45}{80}$ of £1 = $\frac{45}{80} \times \frac{20}{1}$s. = $\frac{45}{4}$s. = 11s. 3d.

∴ the given expression = 11s. 3d. + £1. − 11s. 3d.

= £1.

150. To express a simple or compound quantity as the fraction of another simple or compound quantity :

Rule. Reduce both the quantities to the same denomination and write as numerator the first, *i.e.* the one to be expressed as a fraction, and write the other as denominator. Reduce the fraction, where necessary, to its lowest terms.

Ex. 6. Express 6 as. as the fraction of a rupee.

Since R. = 16 as.

The required fraction = $\frac{6\text{ as.}}{16\text{ as.}} = \frac{3}{8}$.

Ex. 7. What part of 3 rupees is 5 as. 4 pies ?

Since 5 as. 4 pies $=5\frac{1}{3}$ as. $=\frac{16}{3}\times\frac{1}{16}$ or $\frac{1}{3}$ of a rupee.

The required fraction $=\frac{1}{3}\div 3=\frac{1}{9}$.

Ex. 8. Express 7 cwts. 3 qrs. 14 lbs. as the fraction of 1 ton 19 cwts. 1 qr. 14 lbs.

```
cwts. qrs. lbs.          ton  cwts. qr.  lbs.
 7  .  3  .  14           1  .  19  . 1  . 14
 4                       20
---                      --
 28                      20
  3                      19
---                      --
 31                      39
 38                       4                          882 ) 4410 ( 5
---                      ---                               4410
868                      156                               ----
 14                        1                               ....
---                      ---
882 lbs.                 157
                          28
                        ----
                        4396
                          14
                        ----
                        4410 lbs.
```

Reducing both quantities to pounds we have

7 cwts. 3 qrs. 14 lbs. $=$ 882 lbs.

1 ton 19 cwts. 1 qr. 14 lbs. $=$ 4410 lbs.

$\therefore$ the required fraction $=\dfrac{882\text{ lbs.}}{4410\text{ lbs.}}=\frac{1}{5}$.

EXERCISE XLVI.

Find the value of

1. $\frac{5}{6}$ of R.1.
2. $\frac{3}{5}$ of £1.
3. $\frac{7}{8}$ of 3 as.
4. $\frac{3}{7}$ of a guinea.
5. $\frac{9}{32}$ of Rs.7.
6. $\frac{3}{4}$ of a crown.
7. $\frac{11}{24}$ of 5 pies.
8. $\frac{23}{48}$ of 1s. 1d.
9. $\frac{9}{16}$ of 2 as. 8 pies.
10. $\frac{2}{5}$ of 1 ton.
11. $\frac{7}{8}$ of a cwt.
12. $\frac{2}{7}$ of 1 lb.
13. $\frac{3}{8}$ of 1 dwt.
14. $\frac{1}{12}$ of 1 khandi.
15. $\frac{1}{6}$ of a tola.
16. $\frac{13}{36}$ of 1 mile.
17. $\frac{3}{4}$ of 1 fur.
18. $\frac{17}{22}$ of 1 po
19. $\frac{7}{9}$ of 1 day.
20. $\frac{13}{15}$ of 1 min.
21. $\frac{2}{7}$ of 3 wks.
22. $\frac{3}{8}$ of Rs.5 $+\frac{5}{6}$ of 3 as. $+\frac{2}{3}$ of $1\frac{1}{2}$ pies.
23. $\frac{4}{5}$ of £3 $+\frac{7}{9}$ of 6s. $+1\frac{2}{3}$ of $2\frac{1}{4}$d.

Find the value of

24. $1\frac{1}{5}$ of $\frac{2}{3}$ ton $+ 2\frac{1}{4}$ of $\frac{5}{6}$ cwt. $+ 3\frac{2}{5}$ of $1\frac{3}{7}$ lb.

25. $\frac{3}{10}$ of $2\frac{1}{2}$ khandis $+ 1\frac{3}{8}$ of 5 maunds $+ 9\frac{1}{2}$ of $1\frac{1}{3}$ seers.

26. $3\frac{7}{9}$ of $4\frac{1}{2}$ days $- 1\frac{1}{6}$ of 51 mins. $+ 12\frac{1}{3}$ of 9 secs.

27. $\frac{2}{5}$ of $1\frac{4}{11}$ of $\frac{7}{12}$ acre $+ 3\frac{5}{6}$ of 2 ro. $- 25\frac{1}{2}$ of $2\frac{1}{3}$ sq. yds.

28. $\frac{2}{7}$ of 3 miles $- \frac{8}{21}$ of 1 fur. $+ \frac{24}{25}$ of 1 po. $+ 2\frac{2}{15}$ yds.

29. Reduce $9\frac{3}{5}$ pies to the fraction of a rupee.

30. Reduce 2s. 11d. to the fraction of a guinea.

31. Reduce 1s. 6d. to the fraction of £1. 10s.

32. Reduce 2 as. 6 p. to the fraction of R.1. 2 as. 4 p.

33. Reduce 7d. 2f. to the fraction of a crown.

34. Reduce 2 ft. 3 in. to the fraction of $4\frac{1}{2}$ yds.

35. Reduce 3 ac. 1 ro. 20 po. to the fraction of 21 acres.

36. Reduce 1 fur. 20 po. to the fraction of 30 miles.

37. Reduce 16 ft. 6 in. to the fraction of a furlong.

38. Reduce 7 cwt. 3 qrs. 12 lbs. to the fraction of a ton.

39. Reduce 3 dwt. 8 grs. to the fraction of a pound (Troy).

40. Reduce 4 hrs. 9 mins. 36 secs. to the fraction of a day.

41. Reduce 2 maunds 20 seers to the fraction of a khandi.

42. Reduce 1 mâsâ 4 gunjas to the fraction of a tola.

43. What fraction of Rs.10. is Rs.6. 10 as. 8 pies ?

44. What part of $\frac{5}{8}$ of 5 cwt. is $\frac{1}{100}$ of a ton ?

45. Express $2\frac{1}{2}$ guineas as the fraction of £$1\frac{3}{4}$.

46. Find the sum of £$3\frac{43}{96}$ and $\left(9\frac{1}{12} - 2\frac{7}{8}\right)$ of 5s.

Find the value of

47. $\frac{1}{4}$ of £13. 1s. 1d. $+ \frac{2}{5}$ of £4. 3s. 4d. $+ \frac{1}{32}$ of £2. 4s. 8d.

48. $\frac{3}{4}$ of £5. 7s. 11d. $+ \frac{4}{3}$ of £20. 5s. $9\frac{3}{4}$d. $- \frac{16}{9}$ of £11. 11s. 6d.

49. $\frac{4}{15}$ of £1 $+ \frac{5}{8}$ of 1s. $+ \frac{5}{28}$ of a guinea $+ \frac{3}{5}$ of £1. 6s. 8d.

50. Simplify

$\frac{1664}{1408}$ of $\left\{\frac{7}{10}\right.$ of £$3\frac{3}{14} + 6\frac{2}{3}$ of £3. 0s. 9d. $- 4\frac{13}{31}$ of $\left.\text{£3. 2s.}\right\}$

REDUCTION OF DECIMALS.

151. The value of a decimal quantity of any denomination may be found, as in ordinary Reduction.

A recurring decimal should be reduced to a vulgar fraction and then treated as such.

Ex. 1. Reduce ·675 of a rupee to annas and pies.

$$\begin{array}{rl} \text{Rs. } \cdot675 & \\ 16 & \\ \hline 10{\cdot}8\cancel{0}\cancel{0} & \text{as.} \\ 12 & \\ \hline 9{\cdot}6 & \text{pies} \end{array}$$

∴ ·675 of a rupee = 10 as. 9·6 pies.

·675 of R.1 = 10·8 as.

And ·8 of an anna = 9·6 pies ;

∴ ·675 of R.1 = 10 as. 9·6 pies.

Ex. 2. Find the value of £·728.

$$\begin{array}{rl} \pounds\cdot728 & \\ 20 & \\ \hline 14{\cdot}56\cancel{0} & \text{shillings} \\ 12 & \\ \hline 6{\cdot}72 & \text{pence} \end{array}$$

∴ £·728 = 14s. 6·72d.

Ex. 3. Reduce ·0375 of 1 oz. 5 dwt. 12 grs. to grains.

1 oz. 5 dwt. 12 grs. = (480 + 120 + 12) grs. = 612 grs.

$$\begin{array}{rl} 612 & \text{grs.} \\ \cdot0375 & \\ \hline 3060 & \\ 4284 & \\ 1836 & \\ \hline 22{\cdot}95\cancel{0}\cancel{0} & \text{grs.} \end{array}$$

∴ the required number of grains = 22·95.

Here we first reduce 1 oz. 5 dwt. 12 grs. to grains and then multiply the result by ·0375.

Ex. 4. Find the value of

·45 of 3 cwts. + ·81 of 2 qrs. + ·54 of 18 lbs.

$$\begin{array}{lll} \text{cwts.} & \text{qrs.} & \text{lbs.} \\ 3 & 2 & 18 \\ \cdot45 & \cdot81 & \cdot54 \\ \hline 1{\cdot}35 \text{ cwts.} & 1{\cdot}62 \text{ qrs.} & 9{\cdot}72 \text{ lbs.} \\ 4 & 28 & \\ \hline 1{\cdot}40 \text{ qrs.} & 17{\cdot}36 \text{ lbs.} & \\ 28 & & \\ \hline 11{\cdot}20 \text{ lbs.} & & \end{array}$$

$\therefore$ the given expression = (1 cwt. 1 qr. 11·20 lbs.) +
(1 qr. 17·36 lbs.) + (9·72 lbs.)
= 1 cwt. 3 qrs. 10·28 lbs.
= 1 cwt. 3 qrs. 10 lbs. ·28 × 16 oz.
= 1 cwt. 3 qrs. 10 lbs. 4·48 oz.

Ex. 5. Find the difference between

$3{\cdot}\dot{0}69\dot{3}$ of 8 as. 5 pies and $\cdot 641\dot{6}$ of Rs.1·25.

$$3{\cdot}\dot{0}69\dot{3} = \frac{30693-3}{9999} = \frac{30690}{9999} = \frac{310}{101}.$$

8 as. 5 pies = 101 pies.

$$\therefore\ 3{\cdot}\dot{0}69\dot{3} \text{ of 8 as. 5 pies} = \frac{310}{101} \times \frac{101}{1} \text{ pies} = 310 \text{ pies.}$$

Again, $\cdot 641\dot{6} = \frac{6416-641}{9000} = \frac{5775}{9000} = \frac{77}{120}.$

And Rs.1·25 = 1·25 × 16 as. = 20 as. = 240 pies.

$$\therefore\ \cdot 641\dot{6} \text{ of Rs. } 1{\cdot}25 = \frac{77}{120} \times \frac{240}{1} = 154 \text{ pies.}$$

$\therefore$ the required difference = 310 pies − 154 pies
= 156 pies
= 13 as.

152. To express one compound quantity as the decimal of another quantity :

Rule. Reduce the quantities to the same denomination and then divide the first by the second.

Ex. 6. Express R.1. 3 as. 6 pies as the decimal of Rs.8.

Rs.1. 3 as. 6 pies = (192 + 36 + 6) pies = 234 pies.

And Rs.8 = 8 × 16 × 12 = 1536 pies.

$\therefore$ The required decimal $= \frac{234}{1536} = \frac{78}{512} = \frac{39}{256}$
= ·15234375.

The operation may be performed as follows :

12) 6 pies
16) 3·5 as.
8) 1·21875 Rs.
·15234375

Here we first set down 6 pies and divide by 12 to reduce them to annas. We add 3 to the quotient which is ·5, and divide 3·5 annas by 16 to reduce them to rupees. To the quotient we add one rupee. The result is Rs.1·21875. This result being divided by 8 we obtain the required decimal

Ex. 7. Express 6 hrs. 31 mins. 48 secs. as the decimal of 8 hrs. 9 min. 45 secs.

hrs. mins. secs.	hrs. mins. secs.
6 . 31 . 48	8 . 9 . 45
60	60
391 mins.	489 mins.
60	60
23508 secs.	29385 secs.

∴ the required decimal $=\frac{23508}{29385}=\frac{4}{5}=\cdot 8$.

The example may also be worked as follows :

60) 48 secs.	60) 45 secs.
60) 31·8 mins.	60) 9·75 mins.
6·53 hrs	8·1625 hrs.

∴ the required decimal $=\frac{6\cdot 53}{8\cdot 1625}=\cdot 8$.

Here we express each of the given quantities as the decimal of an hour and then divide the first by the second.

EXERCISE XLVII.

Find the value of

1. ·625 of R.1.
2. ·125 of £1.
3. ·45 of 1 guinea.
4. ·5 of 1 anna.
5. ·35 of 1s.
6. ·75 of 1d.
7. ·6 of 1 ton.
8. ·85 of 1 cwt.
9. ·9 of 1 qr.
10. ·412 of 1 dwt.
11. ·8 of 1 tola.
12. ·375 of 1 khandi.
13. ·36 of 1 maund.
14. ·27 of 1 mâsâ.
15. ·05 of 1 lb. (Avoir.).
16. $\cdot\dot{4}$ of 1 mile.
17. $\cdot 1\dot{6}$ of 1 fur.
18. $\cdot 8\dot{3}$ of 1 yd.
19. $\cdot 2\dot{7}$ of 1 gal.
20. $\cdot 2\dot{8}$ of 1 bus.
21. $\cdot 7\dot{9}$ of 1 qt.
22. $\cdot 1\dot{3}\dot{6}$ of 1 acre.
23. ·07 of 1 cub. ft.
24. ·275 of 1 hr.
25. ·0875 of Rs.7. 8 as. 9 pies.
26. ·3125 of £3. 4s. 8d.
27. ·324 of 2 cwt. 1 qr. 9 lbs.
28. ·068 of 5 oz. 13 dwt. 8 grs.
29. $\cdot\dot{5}\dot{4}$ of $\cdot 30\dot{7}\dot{2}$ of 1 mile 5 fur. 30 poles.
30. $3\cdot\dot{6}$ of $\cdot 9\dot{5}\dot{4}$ of $\cdot\dot{4}2857\dot{1}$ of 7 sq. ft. 102 sq. in.
31. Express 3 as. 6 pies as the decimal of a rupee.
32. Express 11s. 9¼d. as the decimal of a pound.
33. Express 2 qrs. 21 lbs. as the decimal of a ton.
34. Express 14·7 maunds as the decimal of a khandi.
35. Express 12 ft. 4½ in. as the decimal of a mile.
36. Express 9 sq. in. as the decimal of a square yard.
37. Express 5·5 sq. yds. as the decimal of an acre.

38. Express 13 mins. 18 secs. as the decimal of an hour.

39. Express Rs.6. 7 as. 7½ pies as the decimal of a rupee.

40. Express ·005 of a pound as the decimal of a penny.

Find the value of

41. 11·1375 of Rs.6. 8 as. – ·56 of Rs.7. 8 as.

42. *·0125 of £1 + ·0625 of 1s. + ·5 of 1d.*

43. ·03125 of Rs.2 + $\cdot\dot{7}2\dot{9}$ of Rs.3. 1 a. 4 pies + $\cdot72\dot{9}$ of Rs.6. 4 as.

44. $\cdot3\dot{7}\dot{8}$ of 2 as. 9 pies – $\cdot\dot{3}7\dot{8}$ of 1 a. 6·5 pies.

45. ·175 of a ton + $\cdot10\dot{7}1428\dot{5}$ of a cwt. + ·15 of 1 *lb*.

46. ·029 of 1 day 3 hrs. + ·45 of 11 hrs. 10 mins. + ·89 of 1 hour.

47. ·1751 of 3 bus. 2½ gals. – ·00107 of 1 qt.

48. 7·042 of 1 rood – ·657 of 2 acres.

49. Reduce Rs.483. 12 as. 6 pies to the decimal of Rs.1290. 1 a. 4 pies.

50. What decimal of £4. 3s. 4d. is $7\cdot\dot{6}9230\dot{7}$ of £5. 8s. 4d. ?

MISCELLANEOUS EXAMPLES IN CONCRETE QUANTITIES.

Ex. 1. Rama has ten thousand annas and Krishna has one million pies ; which of them has more money and how much ?

Reducing the money which each has to rupees, annas and pies, we have

16) 10000 as.
Rs.625

∴ 10000 as. = Rs.625.

And 12) 1000000 pies
16) 83333 as. + 4 pies
5208 + 5 as.

∴ 1000000 pies = Rs.5208. 5 as. 4 pies.

∴ the required difference = Rs.5208. 5 as. 4 pies – Rs.625
= Rs.4583. 5 as. 4 pies.

Hence Krishna has Rs.4583. 5 as. 4 pies more than Rama.

Ex. 2. A man bought from a merchant, out of a banknote for Rs.1000, 2 horses at Rs.78. 13 as. each, 8 bullocks at Rs.52. 6 as. 3 pies each, and 3 cows at Rs.48. 6 pies each. What change will he receive ?

Rs.	as.	Rs.	as.	pies.	Rs.	as.	pies.
78 .	13	52 .	6 .	3	48 .	0 .	6
	2			8			3
157 .	10	419 .	2 .	0	144 .	1 .	6

∴ The cost of 2 horses = Rs.157. 10 as.
„ 8 bullocks = Rs.419. 2 as.
„ 3 cows = Rs.144. 1 a. 6 pies.
∴ The total cost = Rs.720. 13 as. 6 pies.

∴ The required change = Rs.1000 − (Rs.720. 13 as. 6 pies)
= Rs.279. 2 as. 6 pies.

Ex. 3. Three persons have respectively Rs.220. 8 as., Rs.283 8 as., and Rs.409. 8 as., with which they agree to purchase cows at the highest price per head that will allow each man to invest all his money. Find the price of each cow and also the number of cows which each can buy.

Since each of the three persons can buy an exact number of cows by investing the whole of his money, the price of each cow will be contained in each sum an exact number of times. But since the price is required to be the highest possible, it must be the G.C.M. of the three sums.

Now Rs.220. 8 as. = 3528 as.
Rs.283. 8 as. = 4536 as.
Rs.409. 8 as. = 6552 as.

Again, since $3528 = 7 \times 63 \times 8$,
$4536 = 9 \times 63 \times 8$,
$6552 = 13 \times 63 \times 8$,

∴ the G.C.M. of 3528, 4536, 6552 is 63×8 or 504.
∴ 504 as. or 31 rupees 8 as. is the price of each cow.

And 3528 as. ÷ 504 as. = 7,
The number which the first man can buy.
4536 as. ÷ 504 as. = 9,
The number which the second man can buy.
6552 as. ÷ 504 as. = 13,
The number which the third man can buy.

Ex. 4. Two men walk at the same rate starting with the same foot at the same time. But one takes a pace of 2 ft. 4 in. and the other of 2 ft. 8 in. In what distance will they be in step again for the first time and how often will they do so in the distance of one mile?

2 ft. 4 in. = 28 in.
2 ft. 8 in. = 32 in.

They will be in step again when the distance travelled by each is an exact multiple of 28 and 32. Therefore to find when they will be in step for the first time after starting, we must take the L.C.M. of 28 inches and 32 inches.

$28 = 7 \times 4$; $32 = 8 \times 4$.

∴ the G.C.M. of 28 and 32 is 4.
∴ the L.C.M. of 28 and 32 is $32 \times 28 \div 4 = 224$.
∴ the required distance is 224 inches.

Again : since 1 mile $= 1760 \times 3 \times 12$ or 63360 inches and since 224 is contained in 63360 more than 282 (but less than 283 times), the number of times they will be in step in one mile is 282.

Ex. 5. Find the value of

$$\frac{\cdot 75 \text{ of Rs.3. 2 as.}}{3\frac{3}{4} \text{ of R.1. 14 as.}} + \frac{\cdot 18\dot{9}1\dot{8} \text{ of 2 qrs. 18 lbs.}}{\cdot\dot{1}4285\dot{7} \text{ of 1 cwt. 1 qr. 7 lbs.}}.$$

$$\cdot 75 \text{ of Rs.3. 2 as.} = \tfrac{75}{100} \times 50 \text{ as.} = \tfrac{75}{2} \text{ as.},$$

$$3\tfrac{3}{4} \text{ of R.1 14 as.} = \tfrac{15}{4} \times 30 \text{ as.} = \tfrac{225}{2} \text{ as.},$$

$$\cdot 18\dot{9}1\dot{8} \text{ of 2 qrs. 18 lbs.} = \tfrac{18918-18}{99900} \times 74 \text{ lbs.}$$

$$= \tfrac{18900}{99900} \times 74 \text{ lbs.} = \tfrac{7}{37} \times 74 = 14 \text{ lbs.}$$

$$\cdot\dot{1}4285\dot{7} \text{ of 1 cwt. 1 qr. 7 lbs.} = \tfrac{142857}{999999} \times 147 \text{ lbs.}$$

$$= \tfrac{1}{7} \times 147 \text{ lbs.} = 21 \text{ lbs.};$$

$$\therefore \textit{ the given expression} = \frac{\frac{75}{2} \text{ as.}}{\frac{225}{2} \text{ as.}} + \frac{14 \text{ lbs.}}{21 \text{ lbs.}}$$

$$= \tfrac{75}{225} + \tfrac{14}{21} = \tfrac{1}{3} + \tfrac{2}{3} = 1.$$

Ex. 6. Find the nearest sum of money to Rs.364. 6 as. 4 p. that can be divided by 37 without a remainder.

```
        Rs.  as.  p.
   37 ) 364 . 6 . 4 ( 9
        333
        ---
         31
         16
        ---
        496
          6
        -----
   37 ) 502 ( 13
        37
        ---
        132
        111
        ---
         21
         12
        ---
        252
          4
        -----
   37 ) 256 (6
        222
        ---
         34
```

It will thus be seen that if the given sum be diminished by 34 pies, there will be no remainder, or if it be increased by $37 - 34$ or 3 pies there will be no remainder. Hence the nearest sum required is Rs.364. 6 as. 4 p. + 3 p. = Rs.364. 6 as. 7 p.

EXERCISE XLVIII.

1. How many pounds, shillings, pence and farthings are there in 19 guineas, 13 crowns, 21 florins, 16 shillings and 11 farthings, added together ?

2. How many more inches than yards are there in a mile ?

3. *Multiply* the *difference* between 12 cwt. 3 qrs. 17 lbs. 10 oz. and 5 cwt. 2 qrs. 23 lbs. 11 oz. by 528. Divide the sum of the same two quantities by 324.

4. What length of time is the sun up if he rises at a quarter to 7 A.M. and sets at 46 minutes past 5 P.M. ?

5. A person bought an equal number of half-anna stamps and quarter-anna stamps for Rs.6. How many of each did he buy ?

6. Divide the sum of one hundred two-anna pieces, as many four-anna pieces and half as many eight-anna pieces by Rs.2. 1 a. 4 pies.

7. Find the least sum that must be added to Rs.69. 13 as. 6 pies to make it divisible by 14.

8. A bicycle has two unequal wheels, the larger one having a circumference of 8 ft. and the smaller of 5 ft. How many more turns will the smaller make than the larger in a distance of 100 miles ?

9. Find the least sum that can be paid either in guineas or in sovereigns.

10. A grocer buys one Khandi of sugar at Rs.9. 12 as. a maund and sells it at 4 as. 9 pies a seer ; find his gain in the transaction.

11. A dozen mangoes and one cocoanut cost 10 as. ; the mangoes cost 9 pies each ; find the cost of the cocoanut.

12. What is the largest sum of money which will divide Rs.5. 5 as. 4 pies and Rs.13. 8 as. exactly ?

13. A manufacturer employs 59 men at 5 as. each a day and 23 women at 3 as. 6 pies each a day. How much will he have to pay as wages in a year ?

14. The cost of 5 cwt. 2 qrs. 11 lbs. of coffee is £56. 13s. 9d. If the whole is sold at 1s. 11½d. per lb., what is the total gain or loss ?

15. A man earns Rs.91 in a year of 52 weeks. If he wishes to lay by 1 a. 6 pies a day, how much per week should he spend ?

16. How many payments of 14s. 11¾d. amount to £89. 2s. 6¼d. ? And how many plots each containing 2 ro. 16 po. 8 sq. yds. are there in 10 ac. 36 po. 15 sq. yards ?

17. A gentleman left to his daughter the contents of a writing-table, in which were found 13 drawers, in each drawer 7 divisions, and in each division 3 purses, each purse containing £7, 14 crowns, 3 half-crowns and 217 pennies. Find the value of the legacy.

18. A man earns Rs.592. 8 as. a year; he gives 2 as. 9 pies a day in charity and lays by Rs.3. 12 as. every month. How much per day should he spend so as to keep free from debt ?

19. Find the greatest unit of time with which 1 hr. 16 mins. 18 secs., and 1 hr. 35 mins. 12 seconds can both be represented as integers.

20. If a person has 6 twenty-rupee notes, 7 eight-anna pieces, 19 two-anna pieces, 3 sovereigns, 92 pice and 79 pies, how much money has he altogether ?

21. I change 19 sovereigns into rupees, four-anna pieces and two-anna pieces. If the number of four-anna pieces be 124 and that of two-anna pieces be twelve times as large, how many rupees do I get ?

22. Certain goods were bought for Rs.700. 15 as. 6 p.; R.1. 15 as. was paid for packing; for carriage Rs.15. 6 as. 8½ p.; and for import duty Rs.4. 4 as. 6¾ p. The goods were sold for Rs.819 4 as. 5 p. What was the profit on the transaction ?

23. Eighty-four men are employed on a piece of work and each of them receives 3s. 5d. per day; what will be the total amount of the daily wages of each man if two of them fall ill and their daily work and wages are divided among the others ?

24. A lady bought equal quantities of sugar at 3½d. per lb., tea at 3s. 9d. per lb., butter at 1s. 11½d. per lb., coffee at 1s. 4d. per lb. The whole cost was £1. 16s. 8d. How many pounds of each did she buy ?

25. A grocer bought two maunds of sugar for Rs.25. At what rate per seer should he retail it so as to gain Rs.7. 8 as. on the whole ?

26. Two horses and a carriage together cost £250; one of the horses and the carriage cost £181. 13s. 7d.; the other horse and the carriage cost £155. 6s. 5d. What was the separate cost of each of the three ?

27. A had sovereigns and pice while B had ten-rupee notes and pies. A gave B 3 sovereigns, but received from B 3 ten-rupee notes and six thousand pies. How much more than his due did A receive ?

28. Three farmers had respectively Rs.221, Rs.255, and Rs.323, which they separately invested in buying an exact number of oxen at the highest price per head. Find the price.

29. Divide Rs.1489. 9 as. 4 p. equally among 6 men and 7 women.

30. If the first day of January in a common year be a Sunday, on what date will the first Sunday in April be ? and on what date if the year be a leap year ?

Find the value of

31. $\cdot 41\dot{6}$ of $\dfrac{\cdot 015+2\cdot 1}{\cdot 035}$ of $\cdot 0\dot{2}8571\dot{4}$ of 11s. 8d.

Find the value of

32. $\dfrac{1\frac{1}{8} \div 1\frac{1}{5}}{\frac{2}{3} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{6\frac{15}{16} \text{ of } 5\frac{1}{3}}$ of £1.

33. $\cdot 3\dot{4}\dot{8} \times \cdot\dot{1}5384\dot{6}$ of $\dfrac{3\frac{5}{16} - 1\frac{1}{4}}{2\frac{5}{11} \text{ of } \dfrac{1\frac{1}{2}}{9}}$ of £1. 9s. 3d.

34. $\left[\dfrac{\frac{5}{6} + \frac{1}{2}\left(\frac{2}{3} - \frac{1}{2}\right) - \frac{6}{7}\left(\frac{4}{5} + \frac{1}{9}\right)}{\frac{3}{4}\left(\frac{7}{8} - \frac{1}{2}\right) - \frac{1}{3}\left(\frac{1}{7} - \frac{1}{10}\right)}\right]^{\text{th}}$ part of £44. 17s.

35. Reduce $8\frac{1}{2}$ inches to the decimal of a mile.

36. Find the sum of 2·35 of 2s. 1d. and $0\cdot\dot{0}\dot{3}$ of £6. 3s. 9d., and express it as a vulgar fraction of £2. 1s. 6d.

37. Find the value of

$$\frac{\text{£3. 12s. 6d.}}{\text{£8. 16s. 9d.}} \times 29 \text{ dys. } 11 \text{ hrs.}$$

38. What fraction of 12s. 6d. must be added to $\frac{5}{7}$ of a guinea to make a pound sterling ?

39. Reduce $11\frac{3}{4}$d. to the decimal of a pound, correct to five places of decimals.

40. Add together $\frac{1}{6}$ of a guinea, $\frac{9}{32}$ of a pound, $\frac{1}{20}$ of a crown, and $\frac{15}{16}$ of a shilling ; and reduce the result to the decimal of a pound.

41. Find the decimal of an acre, which differs from a square foot by less than one-thousand-millionth part of an acre.

42. What fraction is 1 week, 7 hours, 12 minutes of the time from January 1st 1800, to February 26th 1864, both days inclusive ?

43. What must be subtracted from 84 tons 13 cwts. 0 qr. 27 lbs. in order that the sum may be divided by 23 without a remainder ?

44. Find the value of ·01625 of £204. 3s. 4d. ; and reduce 8 lbs. 5 oz. 14 drs. to the decimal of a quarter.

45. Reduce nine inches and nine-tenths to the decimal of a mile ; and find the value of ·0625 of 1 ton 2 cwts. 3 qrs. 12 lbs.

46. Express £4. 6s. $4\frac{3}{4}$d. + $\frac{1}{5}$ of a farthing as the decimal of £5.

47. From $\frac{5}{24}$ of a guinea take $\frac{7}{12}$ of a crown ; and add $\frac{9}{16}$ of a shilling to $\frac{7}{32}$ of a sovereign.

48. Add together $\frac{1}{14}$ of a guinea, $\frac{3}{8}$ of £4, $\frac{13}{15}$ of a half-crown and $\frac{1}{3}$ of a shilling. Reduce the result to the decimal of 8s. 4d.

49. Express 17 lbs. 10 oz. 6 dwts. 15 grs. as the decimal of 1 lb. troy, and also as the decimal of 1 lb. avoirdupois.

50. Find the value of $\frac{7}{8}$ of £5. 10s. 6d. − $\frac{4}{7}$ of 2 guineas + $1\frac{2}{3}$ of $1\frac{1}{2}$ guineas.

51. What must be added to 96 wks. 4 dys. 6 hrs. so that the sum may be exactly divisible by 17 ?

52. If the unit of time be denoted by 5 hrs. 2 mins. 24 secs., what number will denote a week ?

53. A man pays a tenth of his income in rates and taxes, and a twelfth in insurances : he has left £492. 13s. 1d. What is his income ?

54. An imperial gallon is 277·274 cubic inches ; a Winchester bushel is 2150·42 cubic inches. How many Winchester bushels are equal to 100 imperial bushels ?

55. In a manufactory there are 273 men, of whom 100 are paid each 12s. 6d. a week, 100 are paid 15s. 4d. a week, and the remainder 16s. 8d. Find the sum expended weekly in wages.

56. Find the difference between

$$\frac{\text{£2. 17s. 6d.}}{\text{£12. 18s. 9d.}} \text{ and } \frac{\text{2 cwts. 2 qrs. 10 lbs.}}{\text{12 cwts. 0 qrs. 19 lbs.}}.$$

57. Find the nearest sum of money to £84. 16s. 4d. that can be divided by 13 without a remainder.

58. How many dollars will contain an exact number of sovereigns ?

59. A sum of money is divided among A, B and C. If the shares of A and B together amount to Rs.9. 1 a., of B and C Rs.8. 5 as. 3 p. and of A and C Rs.10. 6 as. 9 p., find the share of each.

60. A person paid 1088 rupees for cows and sheep ; of this sum he paid Rs.768 for 24 cows. If a cow cost 12 times as much as a sheep, how many sheep did he buy with the rest of the money ?

61. The fore and hind wheels of a carriage are 9 ft. 7 in. and 15 ft. 4 in. in circumference. Find the least number of revolutions of each, which will give the same length.

CHAPTER VIII.

PRACTICE.

SIMPLE PRACTICE.

153. When any number, taken a certain number of times, exactly makes up another, it is called an '*Aliquot* part' of the number made up. Thus 2 annas is an aliquot part of 1 rupee ; 3d. is an aliquot part of 1 shilling.

154. *Practice* is the method of calculating the value of a number of articles or of a quantity, by means of aliquot parts. It is called '*Simple*,' when we have to find the value of a number of simple quantities, the value of one of these quantities being given. It is called '*Compound*,' when the value of a compound quantity is to be found, the *value of a simple unit of one denomination being given.*

No general rules can be laid down as to what aliquot parts should be selected for use in working examples. Practice alone will guide the student in his selection of them.

155. The following aliquot parts may be remembered with advantage :

Of £1.

10s. 0d. = $\frac{1}{2}$. 4s. 0d. = $\frac{1}{5}$. 2s. 0d. = $\frac{1}{10}$. 1s. 3d. = $\frac{1}{16}$.
6s. 8d. = $\frac{1}{3}$. 3s. 4d. = $\frac{1}{6}$. 1s. 8d. = $\frac{1}{12}$. 1s. 0d = $\frac{1}{20}$.
5s. 0d. = $\frac{1}{4}$. 2s. 6d. = $\frac{1}{8}$. 1s. 4d. = $\frac{1}{15}$.

Of R.1.

8 as. 0 p. = $\frac{1}{2}$. 4 as. 0 p. = $\frac{1}{4}$. 2 as. 0 p. = $\frac{1}{8}$.
5 as. 4 p. = $\frac{1}{3}$. 2 as. 8 p. = $\frac{1}{6}$. 1 as. 0 p. = $\frac{1}{16}$.

Of 1s.

6d. = $\frac{1}{2}$. 2d. = $\frac{1}{6}$.
4d. = $\frac{1}{3}$. $1\frac{1}{2}$d. = $\frac{1}{8}$.
3d. = $\frac{1}{4}$. 1d. = $\frac{1}{12}$.

Of 1 anna.

6 p. = $\frac{1}{2}$. 2 p. = $\frac{1}{6}$.
4 p. = $\frac{1}{3}$. $1\frac{1}{2}$ p. = $\frac{1}{8}$.
3 p. = $\frac{1}{4}$. 1 p. = $\frac{1}{12}$.

Of 1 ton.

10 cwts. = $\frac{1}{2}$. $2\frac{1}{2}$ cwts. = $\frac{1}{8}$.
5 cwts. = $\frac{1}{4}$. 2 cwts. = $\frac{1}{10}$.
4 cwts. = $\frac{1}{5}$. 1 cwt. = $\frac{1}{20}$.

Of 1 khandi.

10 mds. = $\frac{1}{2}$. $2\frac{1}{2}$ mds. = $\frac{1}{8}$.
5 mds. = $\frac{1}{4}$. 2 mds. = $\frac{1}{10}$.
4 mds. = $\frac{1}{5}$. 1 md. = $\frac{1}{20}$.

Of 1 cwt.

2 qrs. = $\frac{1}{2}$. 16 lbs. = $\frac{1}{7}$. 8 lbs. = $\frac{1}{14}$.
1 qr. = $\frac{1}{4}$. 14 lbs. = $\frac{1}{8}$. 7 lbs. = $\frac{1}{16}$.

Ex. 1. Find by practice the value of 29 maunds of sugar at Rs.10. 9 as. 8 p. per maund.

		Rs.	as.	p.				
Cost at Rs.10	10 }							
,, 8 as.	$\frac{1}{2}$ }	29	0	0	is the cost at R.1 each.			
			10					
		290	0	0 =	,,	,,	Rs.10 each.
,, 1 a.	$\frac{1}{8}$	14	8	0 =	,,	,,	8 as. each.
,, 6 p.	$\frac{1}{2}$	1	13	0 =	,,	,,	1 anna each.
,, 2 p.	$\frac{1}{3}$	0	14	6 =	,,	,,	6 p. each.
		0	4	10 =	,,	,,	2 p. each.
∴ by adding,		307	8	4 =	,,	,,	Rs.10. 9 as. 8 p. each.

Sometimes it facilitates work to introduce aliquot parts which are not directly required but which are useful for deriving other parts from them. A mark should be put against those which are thus introduced, and they should be omitted in taking the sum.

Ex. 2. A bankrupt pays 11s. $3\frac{3}{10}$d. *in the pound; find the* dividend on a debt of £40.

		£	s.	d.	
Amount at 10s.	$\frac{1}{2}$	40	0	0	= amount of debt.
,, ,, 1s.	$\frac{1}{10}$	20	0	0	= amount at 10s. in the £.
,, ,, 3d.	$\frac{1}{4}$	2	0	0	= ,, ,, 1s. ,, ,,
,, ,, $1\frac{1}{2}$d.	$\frac{1}{2}$	0	10	0	= ,, ,, 3d. ,, ,,
,, ,, $\frac{3}{10}$d.	$\frac{1}{5}$	*0	5	0	= ,, ,, $1\frac{1}{2}$d. ,, ,,
		0	1	0	= ,, ,, $\frac{3}{10}$d. ,, ,,
∴ by adding,		22	11	0	= ,, ,, 11s. $3\frac{3}{10}$d. ,,

Sometimes it is convenient to find the price of a number of articles at a rate higher than the one given in the sum and then to subtract the price at the rate of the difference.

Ex. 3. Find by practice the value of 100 articles at £1. 19s. each.

£1. 19s. = £2 − 1s. ($\frac{1}{20}$ of a pound).

Hence we proceed as follows:

		£	s.	d.	
Cost at £2	2	100	0	0	cost at £1 each.
,, ,, 1s.	$\frac{1}{20}$			2	
		200	0	0	,, £2 ,,
		5	0	0	,, 1s. ,,
∴ by subtracting,		195	0	0	,, £1. 19s. each.

The required price, therefore, is £195.

EXERCISE XLIX.

Find by practice the value of the following articles:

1. 124 at 9 as. 6 p.
2. 87 at 5s. 4d.
3. 90 at 13 as. 9 p.
4. 144 at 13s. 8d.
5. 238 at 4s. 10d.
6. 75 at 6 as. 4 p.
7. 37 at 7s. 10d.
8. 326 at 15s. 7d.
9. 106 at 7 as. 3 p.
10. 364 at 11 as. 4 p.
11. 432 at 15 as. 9 p.
12. 628 at 19s. 11d.
13. 49 at 3 as. 8 p.
14. 520 at 4s. $7\frac{1}{2}$d.
15. 57 at 9s. 10d.
16. 240 at 11s. $10\frac{1}{2}$d.
17. 832 at 12s. 6d.
18. 99 at 16s. 10d.

---

* Here the amount at $1\frac{1}{2}$d. in the pound not being required is omitted in the sum.

Find by practice the value of the following articles :

19. 187 at Rs.5. 2 as. 6 p.
20. 400 at R.1. 7 as. 3 p.
21. 720 at £9. 1s. 6d.
22. 2345 at £2. 13s. 2$\frac{1}{2}$d.
23. 548 at Rs.36. 11 as. 2 p.
24. 320 at £28. 10s. 9d.
25. 1264 at £19. 7s. 1$\frac{1}{2}$d.
26. 2460 at £13. 9s. 4$\frac{1}{2}$d.
27. 168 at £51. 17s. 6d.
28. 2784 at £128. 3s. 2d.
29. 872 at £105. 15s. 7d.
30. 654 at £37. 5s. 7$\frac{1}{2}$d.

COMPOUND PRACTICE.

Ex. 1. Find the cost of a farm 35 acres 3 roods 26 poles at the rate of Rs.9. 12 as. an acre.

		Rs.	as.	p.	
35 acres = 35 × 1 acre	35	9	12	0	= Rent of 1 acre
2 roods = $\frac{1}{2}$ of 1 acre	$\frac{1}{2}$	7			
		68	4	0	= Rent of 7 acres
		5			
		341	4	0	= Rent of 35 acres
1 rood = $\frac{1}{2}$ of 2 roods	$\frac{1}{2}$	4	14	0	= Rent of 2 roods
20 po. = $\frac{1}{2}$ of 1 rood	$\frac{1}{2}$	2	7	0	= Rent of 1 rood
5 po. = $\frac{1}{4}$ of 20 po.	$\frac{1}{4}$	1	3	6	= Rent of 20 po.
1 po. = $\frac{1}{5}$ of 5 po.	$\frac{1}{5}$		4	10·5	= Rent of 5 po.
				11·7	= Rent of 1 po.
∴ by adding,		350	2	4·2	

= Rent of 35 acres 3 roods 26 po.

Ex. 2. Find by practice the value of 6 cwt. 3 qrs. 21 lbs. 14 ozs. at £12 per cwt.

		£	s.	d.	
6 cwt. = 6 × 1 cwt.	6	12	0	0	is the cost of 1 cwt.
2 qrs. = $\frac{1}{2}$ of 1 cwt.	$\frac{1}{2}$			6	
		72	0	0	= the cost of 6 cwt.
1 qr. = $\frac{1}{2}$ of 2 qrs.	$\frac{1}{2}$	6	0	0	= ,, ,, 2 qrs.
14 lbs. = $\frac{1}{2}$ of 1 qr.	$\frac{1}{2}$	3	0	0	= ,, ,, 1 qr.
7 lbs. = $\frac{1}{2}$ of 14 lbs.	$\frac{1}{2}$	1	10	0	= ,, ,, 14 lbs.
1 lb. = $\frac{1}{7}$ of 7 lbs.	$\frac{1}{7}$	0	15	0	= ,, ,, 7 lbs.
8 ozs. = $\frac{1}{2}$ of 1 lb.	$\frac{1}{2}$	*0	2	1$\frac{5}{7}$	= ,, ,, 1 lb.
4 ozs. = $\frac{1}{2}$ of 8 ozs.	$\frac{1}{2}$	0	1	0$\frac{6}{7}$	= ,, ,, 8 ozs.
2 ozs. = $\frac{1}{2}$ of 4 ozs.	$\frac{1}{2}$	0	0	6$\frac{3}{7}$	= ,, ,, 4 ozs.
		0	0	3$\frac{3}{14}$	= ,, ,, 2 ozs.
		83	6	10$\frac{1}{2}$	

is the cost of 6 tons 3 cwt. 21 lbs. 14 ozs.

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* Here the price of 1 lb. is omitted in adding up as it is not required.

Ex. 3. *Find by practice the value of* 2 *khandis* 12 *maunds* 27½ *seers at* Rs.65. 6 *as.* 8 *pies per khandi.*

			Rs.	as.	p.	
2 khandis	= 2 × 1 khandi	2	65	6	8	= cost of 1 khandi
10 maunds	= ½ of 1 khandi	½			2	
			130	13	4	= cost of 2 khandis
2 mds. 20 srs.	= ¼ of 10 mds.	¼	32	11	4	= „ 10 maunds
20 seers	= ⅕ of 2 mds. 20 srs.	⅕	8	2	10	= „ 2 mds. 20 srs.
10 seers	= ½ of 20 seers	½	*1	10	2	= „ 20 seers
5 seers	= ½ of 10 seers	½	*0	13	1	= „ 10 seers
2½ seers	= ½ of 5 seers	½	0	6	6½	= „ 5 seers
			0	3	3¼	= „ 2½ seers
			172	5	3¾	

= cost of 2 khandis 12 maunds 27½ seers.

EXERCISE L.

1. Find the cost of 22 cwt. 3 qrs. 21 lbs. at £2. 9s. 6d. per cwt.

2. Find the price of 3 khandis, 7 maunds, 9 seers at Rs.95. 7 as. 3 p. per khandi.

3. Find the value of 7 lbs. 8 oz. 15 dwt. 18 grs. at Rs.236. 13 as. 4 p. per lb. (Troy).

4. Find the cost of 7 Tolas 9 masas 5 gunjas at Rs.24. 6 as. 4 p. a Tola.

5. Find the value of 6 lbs. 13 oz. 11 drs. at £1. 1s. 4d. per lb.

6. Find the cost of 369 miles 3 furlongs 22 yards of telegraph wire at £14. 13s. 4d. per mile.

7. Find the produce of 529 acres 3 ro. at 5 qrs., 1 bush. 3 pks. per acre.

8. Calculate a person's wages for 5 months 3 weeks 6 days at £1. 7s. 5d. per month.

9. Find the value of 5 st. 3 lbs. 2 ozs. at £1. 4s. 10d. per stone of 14 lbs.

10. Find the value of 17 cub. yds. 9 cub. ft. 1296 cub. in. at £3. 3s. per cub. yard.

11. Find by practice the price of 5 ac. 3 ro. 27 po. at Rs.7. 9 as. 3 p. an acre.

12. Find the cost of 31 yards 2 ft. 9 in. at Rs.15. 7 as. 9 p. per yard.

13. Find the price of 3 qrs. 14 lbs. at £12. 13s. 4d. a ton.

14. Find the cost of a fence 3 fur. 11 po. 3¼ yds. long at £183. 6s. 8d. per furlong.

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* Omitted in adding up.

15. Find the cost of warming a building for 11 days 17 hrs. 28 min. at £4. 10s. per day.

16. Find the dividend on a debt of £263. 9s. 6d. at the rate of 6s. 10½d. per £.

17. Find the cost of 27 cwt. 3 qrs. 6 lbs. of metal at £22. 8s. per cwt.

18. Find the cost of 14 yds. 3 qrs. 1 nail of Genoa velvet at 27s. 4d. per yard.

19. Find by practice the rent of a farm containing 279 acres 3 rds. 36 po. at £2. 11s. 8d. per acre.

20. Find the value of 157 miles 3 fur. 24 po. of telegraph wire at £11. 10s. per mile.

INVOICES AND BILLS.

156. Merchants make all their calculations by practice in making out invoices, bills and accounts of sales.

157. An *invoice* is a statement of goods with prices always sent along with the goods to the buyer by the seller.

158. Each separate amount in an invoice is called an *item*.

159. A tradesman's bill is a list of goods with their prices, dates of purchase, rates, etc., which is presented or sent for payment to a customer who has an account with the tradesman.

Ex. 1. Make out an invoice of the following sale by Mrs. Radhabai Atmaram Sagoon to Messrs. Shiralkar & Co. :—10 copies of Homer's *Iliad* at R.1. 2 as. each ; 5 copies of Goldsmith's Works at Rs.2. 13 as. each ; 5 vols. from Cassell's National Library at 3 as. each ; and 2 doz. Exercise Books at 6 as. the dozen.

Bombay, 27*th Jan.*, 1901.

Messrs. Shiralkar & Co.,
Bought of Mrs. Radhabai Atmaram Sagoon,
Kalbadevi Road.

	Rs.	as.	p.
10 copies of Homer's *Iliad* at R.1. 2 as.	11	4	0
5 copies of Goldsmith's Works at Rs.2. 13 as.	14	1	0
5 vols. from Cassell's National Library at 3 as.	0	15	0
2 doz. Exercise Books at 6 as. per doz.	0	12	0
	27	0	0

Ex. 2. Make out a bill for the following transactions:

Hajee Kasam bought of Laddha Ibrahim, 34 Hornby Road, Bombay, 39½ yds. of *Brussel's carpet at 5s. 4d.*, and 62¾ yds. *of Kidderminster carpet at 3s. 6d., on the* 1st of September 1900; 27 yds. of Cocoa-nut matting at 1s. 2d., 34½ yds. of Drugget at 2s. 3d., and 43½ yds. of India matting at 1s. 1d., on the 15th October 1900.

Bombay, 31*st December,* 1900.

Hajee Kasam, Esq.

To Mr. Laddha Ibrahim,
34 Hornby Road.

		£	s.	d.
1st Sept.	39½ yds. Brussel's carpet at 5s. 4d.	10	10	8
„	62¾ yds. Kidderminster carpet at 3s. 6d.	10	19	7½
15th Oct.	27 yds. Cocoa-nut matting at 1s. 2d.	1	11	6
„	34½ yds. Drugget at 2s. 3d.	3	17	7½
„	43½ yds. India matting at 1s. 1d.	2	7	1½
		29	6	6½

EXERCISE LI.

Make out a bill for the following articles:

1. 56 lbs. of sugar at 7d. for 1 lb.; 22 oz. of cloves at 1s. 6d. per lb.; 2¾ lbs. of arrowroot at 8¾d. per lb.; 3 stones of flour at 8¼d. per lb.; ¾ cwt. of soap at 2¼d. per lb.

2. 17 yds. of calico at 7¾d. per yd.; 9 yds. of trimming at 2s. 7½d. per yd.; 1 silk umbrella at half a guinea; 19 yds. of flannel at 1s. 10½d. per yd.

3. 17½ tons at 55s. per ton; 147 links at 2¼d. per link; 37 yds. at 7s. 6¾d. per foot; 2½ cwt. at 3½d. per lb.

4. 12 bars of soap each weighing 3½ lbs. at 5½d. per lb.; 164 lbs. of tea at 30s. per doz. lbs.; 12 packets of candles each containing 6½ lbs. at 11½d. per lb.

5. 5 yds. of muslin at 1s. 2½d. per yd.; 39 yds. of merino at 4s. 3¼d. per yd.; 3 doz. buttons at 4¾d. per doz.; 198 yds. of braid at 1s. 3½d. per doz. yds.

6. 10 lbs. of sugar at 5d. per lb.; 6 lbs. of tea at 3s. 8d. per lb.; 8 lbs. of coffee at 1s. 4d. per lb.; 12 lbs. of currants at 5¾d. per lb.; 10 lbs. of rice at 4½d. per lb.; 9 lbs. of cheese at 11d. per lb.

7. 12½ tons of hay at £3. 15s. a ton; 8¼ qrs. of beans at 2 guineas a quarter; 12¾ qrs. of oats at £1. 6s. a quarter; 5 sacks of flour at 27s. 6d. a sack.

Make out a bill for the following articles :

8. 27 yds. of flannel at 3s. 4d. per yd. ; 32 yds. of calico at $5\frac{1}{2}$d. per yd. ; $3\frac{1}{2}$ doz. of stockings at 8s. 4d. per doz. ; 6 prs. of gloves at 3s. 6d. per pair ; 4 collars at 1s. $5\frac{1}{2}$d. each.

9. 17 lbs. of candles at $4\frac{3}{4}$d. per lb. ; $7\frac{3}{4}$ lbs. of tea at 2s. 3d. per lb. ; 27 lbs. of rice at $3\frac{3}{4}$d. per lb. ; 42 lbs. of sugar at 21s. per cwt. ; $11\frac{1}{2}$ lbs. of currants at 2d. per lb. ; $\frac{1}{4}$ cwt. of soap at $3\frac{1}{4}$d. per lb. ; 220 oranges at $7\frac{1}{2}$d. per doz.

10. 1 cwt. of indigo at 14s. 6d. per lb. ; 1 ton of cloves at 1s. 2d. per lb. ; 5 cwt. 3 qrs. 18 lbs. spelter at $4\frac{1}{2}$d. per lb. ; 7 cwt. 1 qr. 14 lbs. block-tin at £64 per ton. Subtract 10 p.c. discount for cash.

CHAPTER IX.

UNITARY METHOD.

160. The unitary method is the method of solving problems by ascertaining by division the value of a unit from the value of a given quantity and then finding from it by multiplication the value of another given quantity. Thus if it be given that 7 horses are worth Rs.630 and if it be required to find the value of 13 horses we proceed as follows :

7 horses cost Rs.630 ;
∴ 1 horse costs Rs.$\frac{630}{7}$ = Rs.90 ;
∴ 13 horses cost Rs.13×90 = Rs.1170.

Ex. 1. If 5 yards of cloth cost Rs.6. 4 as. what will 9 yards cost at the same rate ?

5 yards cost Rs.6. 4 as. ;
∴ 1 yard costs $\frac{1}{5}$ of Rs.6. 4 as. = R.1. 4 as. ;
∴ 9 yards cost $9 \times$ R.1. 4 as. = Rs.11. 4 as. ;
∴ Rs.11. 4 as. is the answer.

Ex. 2. If 15 men earn Rs.26. 4 as. in one week, what will 24 men earn in the same time ?

15 men earn Rs.26. 4 as. ;
∴ 1 man earns $\frac{1}{15}$ of Rs.26. 4 as. = R.1. 12 as. ;
∴ 24 men earn $24 \times$ R.1. 12 as. = Rs.42. ;
∴ Rs.42 is the answer.

Ex. 3. If $\frac{3}{5}$ of a maund of sugar cost Rs.6. 12 as. what will $\frac{7}{12}$ of a maund cost ?

$\frac{3}{5}$ of a maund costs Rs.6. 12 as. :

$\therefore$ $\frac{1}{5}$ of a maund costs $\frac{1}{3}$ of Rs.6. 12 as. = Rs.2. 4 as. ;

$\therefore$ 1 maund costs $5 \times$ Rs.2. 4 as. = Rs.11. 4 as. ;

$\therefore$ $\frac{7}{12}$ of a maund costs $\frac{7 \times \text{Rs.11. 4 as.}}{12}$ = Rs.6. 9 as. ;

$\therefore$ Rs.6. 9 as. is the answer.

Ex. 4. A bankrupt pays 9 as. in the rupee and his assets are worth Rs.423 ; find the amount of his debts.

Debt on which 9 as. or R.$\frac{9}{16}$ is paid is R.1 ;

$\therefore$ " " " R.$\frac{1}{16}$ " " " R.$\frac{1}{9}$;

$\therefore$ " " " R.1. " " " Rs.$\frac{16}{9}$;

$\therefore$ " " " Rs.423 " " " Rs.$\frac{16}{9} \times 423$ = Rs.752 ;

$\therefore$ Rs.752 is the answer.

Ex. 5. If 15 men do a piece of work in 12 days, how long will 18 men take to do it ?

15 men do the work in 12 days ;

$\therefore$ 1 man will do the work in 15×12 or 180 days ;

$\therefore$ 18 men " " $\frac{180}{18}$ or 10 days ;

$\therefore$ 10 days is the answer.

Ex. 6. If 32 men can do a piece of work in 21 days, how many men will do it in 28 days ?

In 21 days the work is done by 32 men ;

$\therefore$ In 1 day " " " 32×21 men ;

$\therefore$ In 28 days " " " $\frac{32 \times 21}{28}$ or 24 men :

$\therefore$ 24 men is the answer.

Ex. 7. If 4 men earn Rs.30 in 5 days, what will 7 men earn in 6 days ?

4 men in 5 days earn Rs.30 ;

$\therefore$ 1 man in 5 days earns Rs.$\frac{30}{4}$;

$\therefore$ 1 man in 1 day earns Rs.$\frac{30}{4 \times 5}$;

$\therefore$ 7 men in 1 day earn Rs.$\frac{30 \times 7}{4 \times 5}$;

$\therefore$ 7 men in 6 days earn Rs.$\frac{30 \times 7 \times 6}{4 \times 5}$.

But Rs.$\frac{30 \times 7 \times 6}{4 \times 5}$ = Rs.63 ;

$\therefore$ Rs.63 is the answer.

Ex. 8. If 8 men earn Rs.48 in 12 days, how many men will earn Rs.75 in 10 days ?

Rs.48 can be earned in 12 days by 8 men ;
∴ Rs.48 „ „ „ 1 day „ 8×12 men ;
∴ R.1 „ „ „ 1 day „ $\frac{8\times12}{48}$ men ;
∴ Rs.75 „ „ „ 1 day „ $\frac{8\times12\times75}{48}$ men ;
∴ Rs.75 „ „ „ 10 days „ $\frac{8\times12\times75}{48\times10}$ men.
But $\frac{8\times12\times75}{48\times10} = 15$;
∴ 15 men is the answer.

The student will notice that, in these examples, the quantity required to be found is always written to the right.

EXERCISE LII.

1. If 18 seers of sugar cost R.1. 6 as. 6 p., what will one maund of sugar cost ?

2. If 16 mangoes cost 12 annas, find the price of 124 mangoes.

3. If 9 men earn Rs.31 8 as., what will 53 men earn ?

4. If 12 boys earn Rs.33 in 8 days, how many boys will earn Rs.110 in the same time ?

5. If 21 oxen eat 19 stone of grass in a certain time, what weight of grass will 32 oxen consume in the same time ?

6. If $\frac{5}{6}$ palla of rice cost $12\cdot\dot{4}$ rupees, find the price of ·375 of a palla of rice.

7. If a man walk 45 miles in $3\frac{3}{4}$ days, how far will he walk in $6\frac{1}{4}$ days ?

8. A bankrupt owes Rs.1000 and his assets are worth Rs.625 ; what dividend does he pay in the rupee ?

9. If the income-tax on Rs.495 be Rs.10. 5 as., what is it on R.1 ?

10. If it require Rs.22. 8 as. to maintain 12 horses for 2 weeks, find the cost of keeping 21 horses for 24 days.

11. A man has $\frac{2}{3}$ of an estate, he sells $\frac{3}{4}$ of it for Rs.481. 9 as. 6 p. ; what is the whole estate worth ?

12. If 3 tons of copper cost £210, what will be the cost of $1\frac{1}{2}$ lbs. ?

13. How much will 3630 sq. yds. of land cost when an estate of 144 acres is worth Rs.46275 ?

14. If 32 men earn Rs.84 in one week, what will 24 men earn in the same time ?

15. If 14 men earn Rs.$14\frac{2}{3}$ in 11 days, what will 6 men earn in 2 weeks ?

16. If 25 men do a piece of work in 18 days, how long will 15 men take to do it ?

17. If 4 oxen plough a piece of land in 6 days, how many oxen will plough it in 2 days ?

18. If 8 oxen plough 24 acres of land in 12 days, how many acres will 6 oxen plough in 14 days ?

19. A man has an income of Rs.678 a year ; find his net income after paying an income-tax at the rate of $7\frac{1}{2}$ *pies for every 5 rupees* of his income.

20. A person after paying an income-tax of 2 pies in the rupee, has Rs.855 a year ; what is his gross income ?

21. If 28 men can do a piece of work in 40 days of 9 hours each, how many men will it take to do three times the amount if they work 36 days of 8 hours each ?

22. If a family of 6 persons can live on Rs.25 for 15 days, how long can a family of 9 persons live on Rs.20 ?

23. If the wages of 37 men for 27 days are £233. 2s., what will be the wages of 42 men for 45 days at the same rate ?

24. If 1323 oranges can be bought for Rs.4. 9 as. 6 pies, what would be the cost of 1656 oranges ?

25. If 7 oxen can eat 3 tons of hay in a month, how long will it take 49 sheep to eat 35 tons, an ox consuming 3 times as much as a sheep ?

26. If $1002\frac{7}{13}$ cub. yds. of earth are carted $3\frac{5}{7}$ miles for £70, how far should 181 cub. yds. 648 cub. in. be carted for the same sum ?

27. A man's hotel bill for 4 weeks 5 days amounted to £28. 17s. 6d. How much would he have had to pay at the same rate for 6 weeks 4 days ?

28. If $3\frac{1}{5}$ lbs. cost $8\frac{4}{7}$s., what will 97·3 lbs. cost ?

29. If $6\frac{2}{3}$ lbs. cost 3s. 4d., what will be the cost of $11\frac{1}{2}$ lbs. ?

30. A grocer bought 11 cwt. 3 qrs. of cheese for £18. 16s. ; he sold one quarter of his stock at £3 per cwt. and the remainder at 30s. per cwt. ; what was the gain upon the whole ?

31. A coach travels $4\frac{1}{2}$ miles an hour ; how far will it have gone between 8.35 A.M. and 4.25 P.M. ?

32. If 15 yds. of silk cost £1. 13s. 9d., how much will 20 yds. 1 ft. cost ?

33. If 4 men reap 5 acres 3 ro. 39 po. in a week, how many men would reap 35 acres 3 ro. 34 po. in the same time ?

34. If the price of 1250 copies of a book be Rs.3125, what sum will the sale of 1531 copies produce ?

35. If a railway journey of 177 miles 3 fur. takes 3 hrs. 56 min. 30 secs., what distance will a man accomplish in 4 hrs. 30 min. 45 secs. ?

36. Seven men agree to mow a field in 5 days working 8 hours a day; but at the end of 3 days 2 of the men leave the work; how many hours a day must the remainder work to complete the task in the time agreed upon?

37. If $16\frac{3}{4}$ tons of coal bought for £7. 5s. 2d. are carried 324 miles at a further charge of $\frac{1}{8}$d. per ton per mile, what is the total cost of coal to the purchaser?

38. If a man walk 252 miles at the rate of $3\frac{1}{2}$ miles per hour 8 hours a day, find how many hours a day another man must walk at the rate of $4\frac{1}{2}$ miles an hour in order to walk 243 miles in the same time.

39. If the rent of a farm containing 35 ac. 2 ro. 4 po. be £78. 9s. 2d., what would be the rent of another farm containing 26 ac. 2 ro. 23 po. supposing that 6 acres of the former be worth 7 acres of the latter?

40. If 15 horses and 148 sheep can be kept for 9 days for £75. 15s., what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses eat as much as 84 sheep?

CHAPTER X.

PROPORTION.

RATIO AND PROPORTION.

161. Ratio is the relation which one quantity bears to another of the same kind with respect to magnitude, the comparison being made by considering what multiple, part or parts the first is of the second. Thus if we compare Rs.5 with Rs.15, we find that the first is $\frac{1}{3}$ of the second. This relation between two quantities is called their ratio.

A ratio is expressed by placing two dots (:) between the quantities, or sometimes also in the form of a fraction of which one quantity is the numerator and the other the denominator. Thus the ratio of 3 to 5 may be expressed as 3 : 5 (read as three to five) or as $\frac{3}{5}$.

162. The two numbers between which a ratio is expressed are called the *terms* of that ratio; the first being called the *antecedent* and the second the *consequent*. Thus in the ratio

3 : 5, the numbers 3 and 5 are called the terms of the ratio ; 3 is called the antecedent and 5 the consequent.

Since we cannot compare quantities of different kinds in point of magnitude, it is obvious that the two terms of a ratio must either be both abstract numbers or both concrete numbers of the same kind ; it is also obvious that a ratio itself is always an abstract number since it is the quotient resulting from the division of one abstract number by another abstract number, or of one concrete quantity by another of the same kind. Thus the ratio between 4 and 8 is $\frac{1}{2}$, an abstract number ; also the ratio between 5 books and 15 books, which is $\frac{1}{3}$, is an abstract number.

When the two terms of a ratio are prime to each other the ratio is said to be in its *simplest form.* When the terms of a ratio have a common factor it is convenient to divide them by that factor so as to reduce the ratio to its simplest form.

Since the value of a fraction is not altered by multiplying both the numerator and the denominator by the same number, and since the ratio is a fraction with the antecedent as the numerator and the consequent as the denominator, it is obvious that the value of a ratio is not altered by multiplying both its terms by the same number.

Ex. 1. Express the ratio between 207 and 483 in its simplest form.

Since the G.C.M. of the given numbers is 69, dividing both the numbers by 69 we get 3 and 7 respectively. The required ratio, therefore, is 3 : 7 or $\frac{3}{7}$.

Ex. 2. Find the ratio between Rs.4 and 5 as. 4 pies.

Here we first reduce both quantities to the same denomination as in the division of concrete numbers. Reducing both, for instance, to annas we have Rs.4 = 64 as. ; 5 as. 4 p. = $\frac{16}{3}$ as. Therefore the required ratio is 64 as. : $\frac{16}{3}$ as., *i.e.* 192 as : 16 as., *i.e.* 12 : 1.

163. When two ratios are equal the equality is denoted by writing the sign :: or = between them. Thus 12 : 6 :: 4 : 2 (read 12 is to 6 as 4 is to 2) means that the ratio of 12 to 6 is equal to the ratio of 4 to 2. When this is the case, *i.e.* when two ratios are equal, the four terms of the ratios are said to be in proportion and are called proportionals. Proportion thus expresses the equality between two ratios.

164. In a proportion, the antecedent of the first ratio and the consequent of the second are called *extremes*, and the consequent of the first ratio and the antecedent of the second are called the *means* of the proportion. Thus in the proportion 6 : 18 :: 14 : 42, 6 and 42 are called the extremes and 18 and 14 are called the means of the proportion.

165. In a proportion the product of the extremes is equal to the product of the means. Thus in the proportion 6 : 11 :: 18 : 33, it will be seen that $6 \times 33 = 11 \times 18$.

From this it follows that if the product of the extremes be divided by one mean the quotient will be the other mean; and if the product of the means be divided by one extreme, the quotient will be the other extreme.

Ex. 1. In a proportion the product of the means is 48 and one extreme is 3; find the other extreme.

As explained above the required extreme $= \frac{48}{3} = 16$.

Ex. 2. The first three *terms of a* proportion are 6, 9 and 10; find the fourth.

We set down the proportion as follows:

$$6 : 9 :: 10 : \text{fourth}.$$

$$6 \times \text{fourth term} = 9 \times 10;$$

$$\therefore \text{ fourth term} = \frac{9 \times 10}{6} = 15.$$

EXERCISE LIII.

Express in its simplest form the ratio of:

1. 24 to 16. **2.** 25 to 15. **3.** 576 : 288. **4.** $3\frac{1}{2}$ to $5\frac{1}{2}$.

5. $2\frac{1}{3}$ to $3\frac{1}{4}$. **6.** $3\frac{3}{4}$ to $8\frac{1}{3}$. **7.** 8 as. to 10 as. **8.** 4s. to 6s.

9. 3 cwt. to 16 lbs. **10.** 15 seers to 1 maund.

11. R.1 to 5 as. 4 p. **12.** 12s. 6d. to £1.

Express as a fraction in its lowest terms the ratio of

13. 3·2 to ·96. **14.** 209 to 253.

15. 2 wks. to 9 dys. 8 hrs. **16.** 3 yds. 2 ft. to 1 ft. 10 in.

17. Find a fourth proportional to 3, 5 and 9.

Find the term omitted in the following proportions:

18. 2 : 5 :: : 10. **19.** 7 : :: 35 : 15. **20.** : 8 :: 65 : 104.

21. Which of the ratios is greater, 98 : 103 or 97 : 102 ?

22. The consequent of a ratio is Rs.3. 6 as. ; if the ratio be equal to $\frac{4}{3}$ find its antecedent.

23. The antecedent of a ratio which is equal to $\frac{5}{8}$ is 145 ; find the consequent.

24. Find a number which is the same fraction of 63 as $2\frac{1}{3}$ is of 49.

25. Find the quantity which is to Rs.13. 14 as. as 287 is to 861.

SIMPLE PROPORTION, OR THE RULE OF THREE.

166. *Simple proportion* or the *Rule of three*, as it is called, is the method of finding the fourth term of a proportion, the other three being given. It is obvious that of these three quantities, two must be of the same kind, and the third, of the same kind as the required term.

167. When two quantities are so connected with one another that one of them increases or decreases, in proportion as the other increases or decreases, they are said to ***vary directly*** with one another, and a proportion is said to be *direct* when it involves such quantities. On the other hand, two quantities are said to *vary inversely* when they are so connected that one of them increases or decreases in proportion as the other decreases or increases; and a proportion which involves such quantities is said to be *inverse*. Thus, for instance, a labourer's wages will vary *directly* with the number of days that he works. If he work for more days, the amount will be more, and if for less days, the amount will be less. On the other hand, the number of articles which a man can buy for a given sum of money will vary *inversely* with the price of each article. If the price be greater, the number of articles bought will be smaller ; and *vice versa*.

168. To find the fourth term of a proportion, the other three being given :

Rule. Set down as the third term of the proportion that quantity which is of the same kind as the fourth or required term. Then consider whether the answer must be greater or less than the third term; if greater, write the greater of the other two quantities as the second term of the proportion; if less, the less. Lastly set down the remaining quantity as the first term of the proportion.

The proportion being thus stated reduce the first and the second terms to the same denomination if they be not already so expressed. Multiply the numbers denoting the second and the third terms and divide the product by the number representing the first. The quotient will be the answer expressed in the denomination of the third term.

Ex. 1. If 18 books cost Rs.63, what will 26 books cost?

18 books : 26 books :: Rs.63 : cost of 26 books or the answer;

$$\therefore \text{ the answer} = \frac{\overset{7}{\cancel{63}} \times \overset{13}{\cancel{26}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{18}}} \text{ rupees}$$

$$= 91 \text{ rupees.}$$

Here the required term, being the cost of 26 books, will be in rupees; therefore Rs.63 must be set down as the third term of the proportion. Now, since the cost of 26 books will be greater than the cost of 18 books, we must write the greater of 18 and 26 books, *i.e.* 26 books, as the second term of the proportion, and the remaining quantity, *i.e.* 18 books, as the first term. Then since one *extreme* is equal to the product of the means divided by the other extreme, the required cost $= \frac{63 \times 26}{18} = 91$ rupees.

The proportion here is *direct.*

It always saves work to strike out any common factors which the first and second terms or the first and third terms of a proportion may have. Thus in the above example, 18 and 63 have 9 in common, and 18 and 26 have 2 in common. We therefore strike out 9 from the former and 2 from the latter.

Ex. 2. If 2 qrs. 6 lbs. of coffee cost £5. 3s. 4d., what is the value of 6 cwts. 12 lbs.?

$$2 \text{ qrs. } 6 \text{ lbs.} = 62 \text{ lbs.,}$$

$$6 \text{ cwts. } 12 \text{ lbs.} = (6 \times 4 \times 28) \text{ lbs.} + 12 \text{ lbs.}$$

$$= 684 \text{ lbs.,}$$

$$£5.\ 3s.\ 4d. = £5\tfrac{1}{6} = £\tfrac{31}{6},$$

2 qrs. 6 lbs. : 6 cwts. 12 lbs. :: £5. 3s. 4d. : the required value;

$$\therefore \text{ the required value} = \frac{\overset{\overset{57}{\cancel{114}}}{\cancel{684}}}{\underset{\cancel{2}}{\cancel{62}}} \times \frac{\cancel{31}}{\cancel{6}} \text{ pounds} = £57.$$

Here the proportion is direct.

Ex. 3. If 5 yds. of cloth cost 14 as. 2 pies, what must be given for 9 pieces each measuring 13 yds. 2 ft. ?

Each piece contains 13 yds. 2 ft. ;

$\therefore$ 9 pieces contain 13 yds. 2 ft. × 9 = 123 yds.

To find the value of 123 yds. we have

$$\begin{array}{rcl}
\cancel{5}\ \text{yds.} : 123\ \text{yds.} & :: & 14\ \text{as.}\ 2\ \text{pies : the required price.}\\
1 \qquad \underline{34} & & \underline{12}\\
492 & & 168\\
\underline{369} & & \underline{2}\\
12\,)\,\underline{4182}\ \text{pies} & & \cancel{170}\ \text{pies}\\
16\,)\ \underline{348}\ \text{as.} + 6\ \text{pies} & & 34\\
\text{Rs.}21 + 12\ \text{as.} & &
\end{array}$$

$\therefore$ Rs.21. 12 as. 6 pies is the price required.

Or thus, 13 yds. 2 ft. $= 13\frac{2}{3}$ yds.

$= \frac{41}{3}$ yds.

The length of 9 pieces $= \frac{41}{3} \times 9$ yds. $= 123$ yds.

And 14 as. 2 pies $= 14\frac{2}{12}$ or $14\frac{1}{6}$ as. $= \frac{85}{6}$ as. $=$ Rs.$\frac{85}{96}$.

5 yds. : 123 yds. :: Rs.$\frac{85}{96}$: the required price ;

$\therefore$ the required price $=$ Rs.$\frac{123}{1} \times \frac{85}{96} \times \frac{1}{5}$

$=$ Rs.$\frac{123 \times 17}{96}$

$= \frac{2091}{96}$

$=$ Rs.21. 12 as. 6 pies.

The proportion here is direct.

Ex. 4. A man buys 48 sheep at Rs.2. 8 as. per head ; if the price had been Rs.3 each sheep, how many should he have bought for the same sum ?

At Rs.3 per head he would obviously have bought a smaller number of sheep for his money ; therefore we have the following proportion :

Rs.3 : Rs.$2\frac{1}{2}$ :: 48 sheep : the required number of sheep ;

$\therefore$ the required number of sheep $= \frac{48 \times 5}{3 \times 2}$

$= 40.$

The proportion here is *inverse.*

Ex. 5. If 24 men can do a piece of work in 36 days, in what time will 32 men do it ?

The number of men being now greater than 24 the number of days will obviously be smaller than 36.

Hence 36 days being the third term of the proportion, 24 men must be made the second term and 32 men the first term. We thus have

32 men : 24 men :: 36 days : the required number of days ;

$$\therefore \text{ the required number} = \frac{\overset{9}{\cancel{36}} \times \overset{3}{\cancel{24}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{32}}} = 27.$$

Hence 27 men is the answer.

The proportion here is *inverse.*

Ex. 6. How many yards of calico at 4 as. 6 pies a yard should be exchanged for 72 yds. of muslin at 3 as. 9 pies a yard ?

Since the price of calico is greater than that of muslin, the number of yards of calico will be smaller than the number of yards of muslin. Hence we have the following proportion :

4 as. 6 p. : 3 as. 9 p. :: 72 yards : the required number of yards.

Now 4 as. 6 p. $= \frac{9}{2}$ as.

and 3 as. 9 p. $= \frac{15}{4}$ as. ;

$$\therefore \text{ the required number of yds.} = \overset{\overset{4}{\cancel{36}}}{\cancel{72}} \times \frac{15}{\underset{\underset{1}{\cancel{2}}}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{9}}} = 60 \;;$$

$\therefore$ **60 yards of calico is the answer.**

The proportion here is *inverse.*

Ex. 7. If 35 maunds of sugar are carried 43 miles for Rs.3. 8 as., what will be the cost of carrying 2 khandies of sugar the same distance ?

Rs.3. 8 as. $=$ Rs.$3\frac{8}{16}$ $=$ Rs.$3\frac{1}{2}$ $=$ Rs.$\frac{7}{2}$.

2 khandis $= 2 \times 20$ maunds $= 40$ maunds.

Therefore we have

35 maunds : 40 maunds :: Rs.$\frac{7}{2}$: the required number of Rs. ;

$$\therefore \text{ the required number of Rs.} = \frac{\overset{\overset{4}{\cancel{8}}}{\cancel{40}}}{1} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}} \times \frac{1}{\underset{\underset{1}{\cancel{5}}}{\cancel{35}}}$$
$$= 4.$$

The student will note that in this example the term 43 miles does not enter into calculation at all, since the distance in both cases is the same. The proportion here is direct.

Ex. 8. A garrison of 465 men has provisions for 64 days ; if after 8 days 186 men are sent to their assistance, for how many more days will the provisions last ?

The provisions would have lasted for 64 − 8 or 56 days more if no additional men had come. But as 186 more men have arrived, there will now be $465+186=651$ men to consume the food in place of 465 men.

Therefore we have the following proportion :

651 men : 465 men : : 56 days : the required number of days;

$$\therefore \text{ the required number of days} = \frac{465\times\cancel{56}^{8}}{\cancel{651}_{93}} = \frac{465\times 8}{93}$$

$$= \frac{5\times 93\times 8}{93} = 5\times 8 = 40\ ;$$

∴ 40 days is the answer.

The proportion here is *inverse.*

Ex. 9. A bankrupt's debts are Rs.3760 and his assets are Rs.1410 ; how much in the rupee can he pay his creditors ?

Since 1410 rupees are the assets, *i.e.* the property which will just realize by sale Rs.1410, it is clear that the bankrupt can pay his creditors Rs.1410 only ; but his debts amount to Rs.3760.

To find therefore how much dividend per rupee he can pay, we have

Rs.3760 : R.1 :: Rs.1410 : the required dividend in Rs. ;

$$\therefore \text{ the required dividend} = \text{Rs.}\frac{1410}{3760} = \frac{3\times 47\times 10}{8\times 47\times 10} = \frac{3}{8}.$$

But $\text{R.}\frac{3}{8}\times 16$ as. $=6$ as. ;

∴ the bankrupt can pay 6 as. in the rupee.

Ex. 10. A man has an income of Rs.570 a year. What will be his net income after paying an income tax of 2 pies in the rupee ? And what should be his gross income to leave a net income of Rs.570 ?

In the present instance the man's net income will be what remains after paying the income-tax. Since this tax is 2 pies in the rupee, we have

R.1 : Rs.570 : : 2 pies : total amount of tax ;

∴ the total amount of tax $=1140$ pies $=$ Rs.5. 15 as. ;

∴ the man's net income $=$ Rs.570 $-$ Rs.5. 15 as.

$=$ Rs.564. 1 a. ;

∴ the net income $=$ Rs.564. 1 a.

Again, since the man's net income for every rupee of gross income is $(192-2)$ pies $=190$ pies $=\frac{190}{192}$ or $\frac{95}{96}$ rupee, to find the gross income which will produce a net income of Rs.570, we have

$\text{R.}\frac{95}{96}$: Rs.570 : : R.1 : the required number of Rs. ;

$$\therefore \text{ the required number of Rs.} = \frac{570\times 96}{95} = 6\times 96 = 576\ ;$$

∴ Rs.576 is the required gross income.

Ex. 11. A merchant invested Rs.245 in buying rice at 8 measures a rupee and sold it at seven measures a rupee. What was his gain on the transaction ? If he had to gain Rs.55 on his outlay, at what rate should he have sold it ?

To find the number of measures bought we have

Re.1 : Rs.245 :: 8 measures : the number of measures bought ;

∴ *the number of measures* $=245 \times 8 = 1960$.

To find the money realized by the sale of 1960 measures we have

7 measures : 1960 measures :: Re.1 : the number of Rs. realized ;

∴ the number of Rs. realized $= \frac{1960}{7} = 280$;

∴ the gain on the transaction = Rs.280 − Rs.245 = Rs.35.

Again, since his outlay is Rs.245, if he had to gain Rs.55, the sum to be realized by the sale of 1960 measures should be

Rs.245 + 55 = Rs.300.

To find the rate, therefore, we have

Rs.300 : Re.1 :: 1960 measures : the required number of measures ;

∴ the required number $= \frac{1960}{300} = \frac{196}{30} = 6\frac{8}{15}$;

∴ the required rate is $6\frac{8}{15}$ measures a rupee.

Ex. 12. A clerk was employed on the understanding that he was to receive Rs.475 and a shawl for a year's service ; he served only for 8 months, and received Rs.251 and the shawl. Find the cost of the shawl.

The clerk served for 4 months less than the stipulated time and received (475 − 251) rupees, *i.e.* Rs.224 less. Rs.224 is equivalent to 4 months' pay. Therefore to find the remuneration for a year's service we have 4 mo : 12 mo. :: Rs.224 : a year's pay.

∴ a year's pay $= \frac{224 \times 12}{4} = 672$ rupees ;

∴ the cost of the shawl = Rs.672 − Rs.475 = Rs.197 ;

∴ Rs.197 is the answer.

EXERCISE LIV.

1. If 216 mangoes can be bought for Rs.16, how many can be had for Rs.270 ?

2. If 6 seers of sugar cost Re.1. 14 as., how much will a maund and 8 seers cost ?

3. If 7 cwt. of sugar cost £12. 5s., how many pounds can be bought for 17s. 6d. ?

4. If 5 maunds of rice cost Rs.62 8 as., what will 2 khandis 7 maunds cost ?

5. If 257 lbs. of tea cost £34. 16s. 0½d., what is the price of a pound ?

6. If 25 tolas of gold cost Rs.637. 8 as., find the cost of a gunja.

7. Find the value of 17 qrs. 1 bus. of corn at the rate of 4s. $8\frac{1}{2}$d. per bushel.

8. If 100 quills cost 3 as. 9 pies, how many can be bought for Rs.129. 1 a. 6 p.?

9. If 5 acres 2 roods 2 poles of land are let for Rs.1470, what will be the rent of 9 ac. 1 r. 25 po. of the same land?

10. A box of soap containing 1 cwt. 2 qrs. 3 lbs. costs £3. 5s. 9d.; what will be the cost of 6 boxes, each containing 2 cwt. 4 lbs.?

11. A bankrupt's debts are Rs.1326. 1 a. 6 p., and his assets are Rs.795. 10 as. 6 p.; what will a creditor receive on a debt of Rs.150?

12. What will a creditor lose on a debt of Rs.2580 if he receive a dividend of 9 as. in the rupee?

13. What is the amount of tax on £945. 17s. 6d. at the rate of 9d. per pound?

14. If an ounce of gold cost 5 pounds, find the value of a grain.

15. A man spends 9 guineas a week; what must his income be to enable him to lay by £100 a year?

16. If by walking 8 hours a day a man can perform a journey in 54 days, in how many days will he do it if he travelled 6 hours a day at the same rate?

17. If 56 men can do a piece work in 36 days, how many men can do it in 24 days?

18. 91 men can do a piece of work in 19 days; in what time can 39 men do it?

19. If 55 men can do a piece of work in 30 days, how many more men should be engaged to complete it in 22 days?

20. A garrison of 1200 men had provisions for five months. They were reinforced with 800 men; how long would the provisions last?

21. If 96 yards of paper 2 ft. 9 in. wide is required to cover the surface of a wall, what length of paper 1 ft. 10 in. wide will cover the same surface?

22. How much tea at 3s. 6d. per lb. can be exchanged for 6 cwt. 1 qr. 5·6 lbs. of coffee at £2. 1s. 8d. per cwt.?

23. If a twopenny loaf weigh 1 lb. and 5 oz. when wheat sells at 48s. a quarter, what should it weigh when wheat sells at 7s. a bushel?

24. What will be the charge of keeping 4 horses for a year at the rate of 4 as. 9 p. a day for each horse?

25. Find the expense of lodgings for 90 days at the rate of a guinea a week.

26. If the carriage of 1 khandi and 15 maunds for 86 miles be Rs.3. 2 as., how far ought 2 khandis 3 maunds be carried for the same sum ?

27. A man working 8 hours a day can complete a piece of work in 18 days ; in how many days should he do it working 9 hours a day ?

28. If 30 men can mow a field in 20 days, how long will it take 25 men to mow it ?

29. If a man walking at the rate of 4 miles an hour can travel a certain distance in 3 hours 25 minutes, in what time could he run the distance at the rate of 7 miles an hour ?

30. If a staff 5 ft. high casts a shadow 7 ft. 3 in. long, what is the height of a steeple which casts a shadow 232 ft. long ?

31. A pole 8 ft. high casts a shadow 6 ft. 9 in. long ; what will be the length of the shadow cast by a house 9 yds. 1 ft. high ?

32. A person paid an income-tax Rs.5. 3 as. 4 p. when the tax was 2 pies in the rupee ; what was his income ?

33. If 24 yards of railway iron weigh 12 cwt. 3 qrs. 7 lbs., what weight of iron will be required for a railway 32 miles long ?

34. After paying an income-tax at the rate of 4d. in the pound a man has £491. 13s. 4d. remaining. What is his income ?

35. If 78 tons 10 cwt. 3 qrs. 10 lbs. 1 oz. of iron cost £722. 4s. 4$\frac{1}{4}$d., find the cost of 123 tons 8 cwt. 1 qr. 23 lbs. 13 oz.

36. A merchant invested Rs.965 in buying rice at 7 seers a rupee and sold it at 6 seers a rupee ; what did he gain by the transaction ?

37. A merchant bought a certain quantity of wheat at 9 seers a rupee for Rs.369 and immediately sold it for Rs.492 ; at what rate did he sell it ?

38. A merchant having bought a certain quantity of Bajri for Rs.960 sold it at the rate of 12 measures a rupee ; if his gain in the transaction be Rs.240, at what rate did he buy it ?

39. A lends B Rs.328 for 9 months ; how long should B lend Rs.432 to A so as to requite his favour ?

40. A gentleman engaged a servant on the understanding that he was to receive a turban and Rs.69 in cash after serving for 6 months ; but the servant left his service at the end of the 4th month receiving the turban and Rs.41 in cash. Find the cost of the turban.

41. If a man walking at the rate of 3$\frac{1}{2}$ miles an hour walks to a place in 4 hrs. 20 min., how long will it take another man walking at the rate of 3$\frac{1}{4}$ miles an hour to walk there and back ?

42. If it is high water at noon on a certain day, find after how many days it will again be high water at noon supposing the time of high water to be three quarters of an hour later every day.

43. How much muslin at R.1. 5 as. 8 p. per yard is equal in value to 143 yards of cambric at Rs.3. 13 as. 8 p. per yard ?

44. A man can count at the rate of 100 a minute ; how long will it take him to count 500 lacs ?

45. If an estate be worth £2374. 16s. per annum and the land-tax be assessed at 1s. $11\frac{1}{2}$d. in the pound, what will be the net annual income ?

46. A shopkeeper purchased $250\frac{3}{4}$ yards of cloth for Rs.900 and paid expenses amounting to Rs.103 ; what must he charge per yard in order to make a profit of Rs.501. 8 as. ?

47. A person owes the sums of Rs.31500 and Rs.8500 and his property only amounts to Rs.14125. How much is he able to pay in the rupee and what is the loss upon the second debt ?

48. If the carriage of $9\frac{3}{4}$ maunds for a distance of 80 miles be Rs.3, how many miles should 130 maunds be carried for the same sum ?

49. If an ounce of gold be worth £$4\cdot\dot{0}09\dot{9}$, what is the value of a bar of gold weighing $1\cdot68\dot{3}$ lbs. ?

50. A person travelled 120 miles by railway at 15 miles an hour, 120 by coach at 8 miles an hour, and 80 by bullock-cart at 2 miles an hour ; how long did he take ?

51. If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in $\frac{3}{5}$ of the time ?

52. A man hired a labourer for 8 months on condition that he was to be given a silver bracelet and Rs.35 in cash for his service. But the servant being allowed to continue his service till the end of the year received the bracelet and Rs.59 in cash. Find the cost of the bracelet.

53. A creditor receives on a debt of Rs.3270 a dividend of 9 as. p. in the rupee and afterwards receives a further dividend upon the deficiency of 3 as. 4 p. in the rupee ; how much does he receive on the whole ?

54. Supposing that the ratio which the diameter of a circle bears to its circumference is as 1 is to 3, find the diameter of a wheel which makes 126 revolutions in 1 fur. 2 yds.

55. If the diameter of the fore wheel of a carriage is 2 ft. 3 in. and that of the hind wheel be 3 ft. 6 in. ; find how many complete revolutions the one will make more than the other in a distance of 10 miles, it being given that the circumference of a circle is to the diameter as 22 to 7.

56. A person after paying an income-tax of 6 pies in the rupee and other expenses amounting to one-fourth of the remainder has Rs.465 left. What is his gross income ?

57. If 4000 men have provisions for 190 days and if after 30 days 800 men go away, find how long the remaining provisions will serve the number left.

58. A man went to the market with 57 rupees to buy coffee and sugar, of which he bought 128 seers at 4½ as. a seer. If coffee be sold at 14 as. a seer, what weight of coffee did he buy ?

59. If it cost twice as much to feed a horse for 4 days as to feed a cow for 5 days, and if the cost of feeding 2 horses for 15 days be Rs.9. 6 as., what would it cost to feed 3 cows for 40 days ?

60. A person bought 12 cows and a number of oxen for Rs.806. Of this sum he paid Rs.372 for all the cows he bought. If an ox cost twice as much as a cow, find the number of oxen he bought.

COMPOUND PROPORTION.

169. When the antecedents of two or more ratios are multiplied together and also their consequents the ratio thus produced is said to be *compounded* of the others. Thus the ratio of $2 \times 4 \times 5 : 3 \times 7 \times 9$, *i.e.* of $40 : 189$, may be said to be compounded of the three ratios $2 : 3$, $4 : 7$, and $5 : 9$.

170. *Compound Proportion* (or The Double Rule of Three as it is called) is the method of finding a quantity to which a given quantity bears a ratio which is compounded of two or more ratios.

171. In questions involving compound proportion one of the given quantities must be of the same kind as the quantity required, viz., the answer, and the others taken two and two must be of the same kind.

172. As a compound proportion contains two or more simple proportions, examples in compound proportion can be solved by reducing it to two or more simple proportions. The following process however shortens the work, and should therefore be preferred.

Place that quantity, the third, which is of the same kind as the answer. Then take each pair of the quantities of the same kind separately, and consider in reference to that pair alone, whether the answer should be greater or less than the third term. If greater, place the greater of that pair as the second term ; if less, the less ; and place the remaining quantity as the first term. The whole proportion being thus written, reduce, where necessary, the first and second terms of a pair to the same denomination. Then multiply all the second terms and the third term together, and divide their product by the product of all the first terms. The quotient will give the answer, which will be of the same denomination as the third term.

Ex. 1. If 27 men earn Rs.153 in 34 days, how much will 45 men earn in 16 days ?

Here the quantity required is the number of rupees ; put therefore Rs.153 as the third term. Then, as 45 men will earn more rupees than 27 men, put 45 men (the greater of the two quantities, 45 men +27 men) as the second term, and the remaining quantity, 27 men, as the first term. Again, as in 16 days less will be earned than in 34 days, put 16 days as the second term, and the remaining quantity, 34 days, as the first term. Thus the compound proportion should be stated as follows :

$$\left.\begin{array}{l}27 \text{ men} : 45 \text{ men} \\ 34 \text{ days} : 16 \text{ days}\end{array}\right\} :: \text{Rs.}153 : \textit{required number of rupees.}$$

Now, by multiplying the second terms and the third term together, and dividing the product by the product of the first terms, we obtain

$$\text{Required number of Rs.} = \frac{\overset{5}{\cancel{45}} \times \overset{8}{\cancel{16}} \times \overset{\overset{3}{\cancel{9}}}{\cancel{153}}}{\underset{\cancel{3}}{\cancel{27}} \times \underset{\cancel{2}}{\cancel{34}}} = \text{Rs. } 120.$$

Ex. 2. If 12 men working 8 hours a day can do a piece of work in 15 days, in what time will 20 men working 6 hours a day do the same ?

Here the quantity required is ‘ days ’ ; put therefore 15 days as the third term. Then, since 20 men will require a smaller number of days than 12 men, put 12 men as the second term, and the remaining quantity, 20 men, as the first term. Again, since 6, the number of working hours, is less than 8, the number of days required to complete the given piece of work must be more than 15 days ; therefore put 8 hours as the second term, and the remaining quantity, 6 hours, as the first term.

Thus we have

$$\left.\begin{array}{l}20 \text{ men} \;\; : 12 \text{ men} \\ \;\,6 \text{ hours} : \;\,8 \text{ hours}\end{array}\right\} :: 15 \text{ days} : \text{required number of days ;}$$

$$\therefore \text{ required number of days} = \frac{\overset{2}{\cancel{12}} \times \overset{2}{\cancel{8}} \times \overset{3}{\cancel{15}}}{\underset{4}{\cancel{20}} \times \cancel{6}} = 12 \text{ days.}$$

Ex. 3. If 16 maunds of goods be carried 90 miles for Rs.3, how far can 12 maunds be carried for Rs.5 ?

Here the quantity required is the distance. Now, the less the weight to be carried the greater will be the distance to which it will be carried for the same sum. Therefore the proportion of maunds to miles is inverse.

Again, the greater the cost of carriage the greater must be the distance to which a given weight is carried. Therefore the proportion of rupees to miles is direct.

Therefore, to find the required distance, we have

$$\left.\begin{array}{r}\text{12 maunds : 16 maunds}\\ \text{Rs.3 : Rs. 5}\end{array}\right\} : : \text{90 miles : the required number of miles;}$$

$$\therefore \text{ the required number} = \frac{\overset{8}{\cancel{16}} \times 5 \times \overset{\overset{5}{\cancel{15}}}{\cancel{90}}}{\underset{\cancel{2}}{\cancel{12}} \times \cancel{3}} = 200 \text{ miles.}$$

Ex. 4. If a half-anna loaf weighs 8 oz. when wheat is 3 as. a seer, what weight of bread will be purchased for 1 anna when wheat is 2 as. a seer ?

$\frac{1}{2}$ anna = 6 pies, and 1 a. = 12 pies.

The greater the price charged for a loaf of bread, the greater must be its weight. Therefore the proportion of the cost to the weight is direct.

The less the cost of wheat the greater the weight of bread which can be made out of a fixed sum. Therefore the proportion of the price of wheat per seer to the weight of bread is inverse.

Therefore to find the required weight of bread we have

$$\left.\begin{array}{l}\text{6 pies : 12 pies}\\ \text{2 as. : 3 as.}\end{array}\right\} : : \text{8 oz. : the required weight;}$$

$$\therefore \text{ the required weight} = \frac{\overset{\cancel{2}}{\cancel{12}} \times 3 \times 8}{\cancel{6} \times \cancel{2}} = 24 \text{ oz.;}$$

$\therefore$ the required weight is 24 oz. = 1 lb. 8 oz.

Ex. 5. If the gas for 5 burners lighted 5 hours every night for 10 days cost Rs.3. 12 as., what will be the cost of 75 burners which are lighted 4 hours every night for 15 days ?

It will be seen that the proportion in each case is direct :

$$\left.\begin{array}{ll}\text{5 burners} & \text{: 75 burners}\\ \text{5 hours} & \text{: 4 hours}\\ \text{10 days} & \text{: 15 days}\end{array}\right\} : : \text{Rs.}\frac{15}{4} \text{ : the required number of Rs.;}$$

$$\therefore \text{ the required number} = \frac{\overset{3}{\cancel{15}} \times \overset{15}{\cancel{75}} \times \overset{1}{\cancel{4}} \times \overset{3}{\cancel{15}}}{\cancel{4} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}} \times \underset{2}{\cancel{10}}} = \frac{3 \times 15 \times 3}{2} = \frac{135}{2} = 67\tfrac{1}{2};$$

$\therefore$ the required cost = Rs.$67\frac{1}{2}$ = Rs.67. 8 as.

Ex. 6. If 72 men dig a canal 187 yards long, 8 ft. broad, 5 ft. deep in 55 days, how long will it take 78 men to dig a canal 221 yds. long, 3 yds. broad, and 4 ft. deep ?

The greater the number of men the less the number of days required to dig a canal. Therefore the proportion of men to days is inverse.

The greater the length of the canal the greater the number of days required for digging it. Therefore the proportion of the length of the canal to days is direct ; for the same reason the proportions of the width and depth of the canal to days are also direct.

Therefore to find the required number of days we have

$$\left.\begin{array}{rcl} 78 \text{ men} & : & 72 \text{ men} \\ 187 \text{ yds.} & : & 221 \text{ yds.} \\ 8 \text{ ft.} & : & 9 \text{ ft.}^{*} \\ 5 \text{ ft.} & : & 4 \text{ ft.} \end{array}\right\} :: 55 \text{ days} : \text{the required number of days} ;$$

$$\therefore \text{ the required number} = \frac{\overset{\cancel{11}}{\cancel{55}} \times \overset{\overset{6}{\cancel{12}}}{\cancel{72}} \times \overset{\cancel{13}}{\cancel{221}} \times 9 \times \cancel{4}}{\cancel{7} \times \underset{\cancel{13}}{\cancel{18}}\underset{\cancel{17}}{\cancel{78}} \times \underset{\cancel{2}}{\cancel{8}} \times \cancel{5}} = 54 ;$$

$\therefore$ the required number of days is 54.

EXERCISE LV.

1. If 36 men earn Rs.54 in 21 days, how many rupees will 42 men earn in 24 days ?

2. If a man spend Rs.15 in 10 days, in what time will 25 men spend Rs.450 at the same rate ?

3. If 6 oxen can plough 7½ acres in 5 days, how many acres will 10 oxen plough in 9 days ?

4. If it costs Rs.112. 8 as. to keep 6 horses for 5 weeks, what will it cost to keep 3 horses for two weeks ?

5. If a family of 9 people require Rs.20 in 8 months for their maintenance, how many rupees will serve a family of 24 people for 16 months at the same rate ?

6. If 35 men, working 9 hours a day, can do a piece of work in 32 days, how long will it take 48 men, working 12 hours a day, to complete it ?

7. If the carriage of 3 cwt. 2 qrs. 12 lbs. for 95 miles cost Rs.9. 8 as., what weight should be carried for 82 miles for Rs.10. 4 as. ?

8. If I have 15 maunds of goods carried 80 miles for Rs.1. 12 as., how far can I have one khandi carried for Rs.2. 6 as. 6 pies ?

9. If 98 men earn Rs.245 in 3 weeks, how long will it take 35 men to earn Rs.375 ?

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* 3 yds. = 9 ft.

10. If a man can perform a journey of 170 miles in $4\frac{1}{4}$ days of 11 hours each, in how many days of $8\frac{3}{4}$ hours will he perform a journey of 470 miles ?

11. Two gangs of 6 men and 9 men are set to reap two fields of 35 and 45 acres respectively. The first gang complete their work in 12 days, in how many days will the second gang complete theirs ?

12. If 10 compositors, who can set 3 letters in 5 seconds, finish 27 pages in an hour and a half, how many compositors, who can set 5 letters in 6 seconds, will complete 50 pages in an hour ?

13. If 210 coolies in 7 days of 10 hours each dig a channel 1 mile long, 6 feet broad, and 2 feet deep, in how many days of 7 hours each will 35 coolies dig a channel 660 feet long, 75 feet broad, and $2\frac{1}{4}$ feet deep ?

14. If 27 men take 15 days to mow 225 acres of grass, how long will 33 men take to mow 165 acres ?

15. If a family of 9 persons spend Rs.4,800 in 8 months, how much will serve a family (living upon the same scale) of 24 persons for 16 months ?

16. If 12 men, in 30 days of 4 hours each, can raise an embankment 30 yards long, 2 ft. broad, and 4 ft. high ; how many men will be required to raise an embankment 50 yards long, 6 feet high, and $1\frac{1}{2}$ yards broad, in 9 days of 5 hours each ?

17. If the cost of printing a book of 320 leaves with 21 lines on each page, and on an average 11 words in each line, be £19, find that of printing a book with 297 leaves, 28 lines on each page, and 10 words in each line.

18. If the wages for 54 men for 36 days amount to £60. 15s., what will be the wages of 30 men and 50 boys for 32 days, supposing 2 men to do as much work as 5 boys ?

19. If 12 furnaces consume 12 tons 9 cwt. 21 lbs. of coal in 10 hours, how long will 7 furnaces be consuming 15 tons ?

20. If 16 fires consume 6 tons 6 cwt. of coal in 21 working days of 12 hours each, what weight of coal can be consumed by 24 fires, in 7 working days of 15 hours each ?

21. If £100 will pay for the Railway travelling of 20 travellers for 5 days of 12 hours each, how long would it last 16 travellers, travelling 6 hours a day, the rate of travelling being the same ?

22. A 20 horse-power engine pumps 40 gallons of water from a pit 100 yards deep in 3 hours. What quantity of water would an engine of 35 horse-power pump from a depth of 235 feet in 141 days working night and day ?

23. If 180 men can make a road in 15 days, in what time would 270 men make a road twice as long as the first ?

24. If 3000 copies of a book of 11 sheets require 66 reams of paper, how many quires of paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets ?

25. When wheat is 9s. a bushel, a six-penny loaf weighs 8 oz.; what must be the price of wheat per bushel when a four-penny loaf weighs 12 oz. ?

26. If a four-penny loaf weigh 8 oz. when wheat is 5s. 3d. a bushel, what should be the weight of a six-penny loaf when wheat is 6s. 9d. a bushel ?

27. If a four-penny loaf weigh 8 oz. when wheat is 5s. 3d. a bushel, what should be the price of a loaf weighing 12 oz. when wheat is 6s. 9d. a bushel ?

28. A family of 18 men can be maintained for 10 days at a certain cost when *Bajri* is at Rs.12 a *palla* ; for how many days can a family of 12 men be maintained for the same cost and at the same rate when *Bajri* is at Rs.9 a *palla* ?

29. If 20 men can be fed for 15 days at a certain cost when *Jwari* is at Rs.9 a *palla*, what must be the price of *Jwari* per *palla*, in order that 25 men can be fed for 18 days at the same cost and at the same rate ?

30. If 8 men can be fed for 12 days at a certain cost when wheat is at Rs.15. a *palla*, how many men can be fed for 20 days at the same cost and at the same rate when wheat is at Rs.12 a *palla* ?

31. If 5 men can reap a field 700 yards long and 200 feet broad in 3 days working 7 hours a day, in how many days will 7 men working 12 hours a day reap a field 800 yards long and 350 feet broad ?

32. *If 48 pioneers, in 5 days of* $12\frac{1}{2}$ *hours long, can dig a trench* 139·75 yds. long, $4\frac{1}{2}$ yds. wide and $2\frac{1}{2}$ yds. deep, how many hours per day must 360 pioneers work during 42 days in order to dig a trench $4910\frac{1}{16}$ yds. long, $4\frac{7}{8}$ yds. wide and $3\frac{1}{5}$ yds. deep ?

MISCELLANEOUS EXAMPLES IN UNITARY METHOD AND PROPORTION.

Ex. 1. If 8 cows or 56 sheep eat a certain quantity of grass in 25 days, in how many days will 5 cows and 5 sheep eat it ?

8 cows eat as much as 56 sheep.
$\therefore$ 1 cow eats as much as 7 sheep ;
$\therefore$ 5 cows eat as much as 35 sheep ;
$\therefore$ 5 cows and 5 sheep eat as much as
(35 + 5) sheep, *i.e.* 40 sheep.
Again $\because$ 56 sheep eat the grass in 25 days ;
$\therefore$ 1 sheep will eat the grass in 25×56 days ;
$\therefore$ 40 sheep will eat the grass in $\frac{25 \times 56}{40}$ days ;
but $\frac{25 \times 56}{40} = 35$;
$\therefore$ 35 days is the answer.

Ex. 2. If a man can travel 336 miles in 24 days walking 6 hours **a day, how many miles will he travel in 15 days walking 9 hours a day ?**

The man, walking 6 hrs. a day, travels in 24 days, 336 miles.

∴ the man, ,, 1 hr. ,, 24 days, $\frac{336}{6}$ miles;

∴ the man, ,, 1 hr. ,, 1 day, $\frac{336}{6\times24}$,,

∴ the man, ,, 9 hrs. ,, 1 day, $\frac{336\times9}{6\times24}$,,

∴ the man, ,, 9 hrs. ,, 15 days, $\frac{336\times9\times15}{6\times24}$,,

But $\frac{336\times9\times15}{6\times24}=315$;

∴ 315 miles is the answer.

Ex. 3. If 5 men, 9 women or 12 boys earn Rs.20 in a week, what will 4 men, 3 women and 6 boys earn in 6 weeks ?

5 men earn in 1 week Rs.20 ;

∴ 1 man earns in 1 week Rs.$\frac{20}{5}$ = Rs.4 ;

∴ 4 men earn in 1 week Rs.4 × 4 or Rs. 16.

Again 9 women earn in 1 week Rs.20 ;

∴ 1 woman earns in 1 week Rs.$\frac{20}{9}$;

∴ 3 women earn in 1 week Rs.$\frac{20}{9}\times3$ or Rs.$6\frac{2}{3}$.

Again 12 boys earn in 1 week Rs.20 ;

∴ 1 boy earns in 1 week Rs.$\frac{20}{12}$;

∴ 6 boys earn in 1 week Rs.$\frac{20}{12}\times6$ or Rs.10 ;

∴ 4 men 3 women and 6 boys will earn in 1 week Rs.$(16+6\frac{2}{3}+10)$
= Rs.$32\frac{2}{3}$;

∴ 4 men 3 women and 6 boys will earn in 9 weeks Rs.$32\frac{2}{3}\times9$
= Rs.294.

Ex. 4. If a tradesman with a capital of Rs.1800 gain Rs.75 in 8 months, in what time will he gain Rs.125 with a capital of Rs.1200 ?

By Proportion :

$$\left.\begin{matrix}\text{Rs.}1200 : \text{Rs.}1800 \\ \text{Rs.}75 : \text{Rs.}125\end{matrix}\right\} :: 8 \text{ months} : \text{required number of months};$$

$$\therefore \text{ required number} = \frac{\overset{4}{\cancel{8}}\times\overset{\cancel{3}}{\cancel{1800}}\times\overset{5}{\cancel{125}}}{\underset{2}{\cancel{1200}}\times\underset{\cancel{3}}{\cancel{75}}} = 4\times5 = 20;$$

∴ 20 months is the answer.

By the Unitary Method :

Rs.1800 produce Rs.75 as gain in 8 months.

∴ Rs.1 produces R.75 as gain in 8 × 1800 months;

∴ R.1 ,, R.1 ,, $\frac{8\times1800}{75}$,,

∴ Rs.1200 produce R.1 as gain in $\frac{8\times 1800}{75\times 1200}$ months;

∴ Rs.1200 „ Rs.125 „ „ $\frac{8\times 1800\times 125}{75\times 1200}$ „

But $$\frac{\overset{4}{\cancel{8}}\times\overset{\cancel{3}}{\cancel{1800}}\times\overset{5}{\cancel{125}}}{\underset{\cancel{3}}{\cancel{75}}\times\underset{\cancel{2}}{\cancel{1200}}}=20;$$

∴ 20 months is the answer.

Ex. 5. If 30 men require 15 rupees worth of rice for 18 days when rice is sold at 8 seers a rupee, how many men will 24 rupees maintain for 27 days when 6 seers of rice can be had for one rupee?

By Proportion:

The larger the sum spent in buying rice, the more will be the men maintained on it. Therefore the proportion of the sum spent in buying rice to the men maintained thereon is direct.

The greater the number of days for which a given quantity of rice is to last, the smaller the number of men that will be maintained. Therefore the proportion of days to men is inverse.

Again, the less the number of seers that can be bought for one rupee, the less will be the number of men that can be maintained out of a given sum. Therefore the proportion of seers bought for a rupee to men is direct.

To find therefore the required number of men, we have

$$\left.\begin{array}{l}\text{Rs.15 : Rs.24}\\ \text{27 days : 18 days}\\ \text{8 seers : 6 seers}\end{array}\right\}::\text{30 men : required number of men.}$$

∴ the required number of men $=\dfrac{\overset{2}{\cancel{30}}\times\overset{\overset{1}{\cancel{3}}}{\cancel{24}}\times\overset{2}{\cancel{18}}\times 6}{\underset{1}{\cancel{15}}\times\underset{\underset{1}{\cancel{3}}}{\cancel{27}}\times\underset{1}{\cancel{8}}}=24$;

∴ 24 men is the answer.

By the Unitary Method:

Rs.15 maintain for 18 days, when rice is 8 seers a rupee, 30 men.

∴ R.1 maintains for 18 dys., when rice is 8 srs. a rupee, $\frac{30}{15}$ men.

∴ R.1 „ 1 dy., „ 8 „ $\frac{30\times 18}{15}$ „

∴ R.1 „ 1 dy., „ 1 sr. „ $\frac{30\times 18}{15\times 8}$ „

∴ Rs.24 maintain for 1 dy., „ 1 „ $\frac{30\times 18\times 24}{15\times 8}$ „

∴ Rs.24 „ 27 dys., „ 1 „ $\frac{30\times 18\times 24}{15\times 8\times 27}$ „

∴ Rs.24 „ 27 dys., „ 6 srs. „ $\frac{30\times 18\times 24\times 6}{15\times 8\times 27}$ „

but $$\frac{\overset{2}{\cancel{30}}\times\overset{2}{\cancel{18}}\times\overset{\overset{1}{\cancel{3}}}{\cancel{24}}\times 6}{\underset{1}{\cancel{15}}\times\underset{1}{\cancel{8}}\times\underset{\underset{1}{\cancel{3}}}{\cancel{27}}}=24;$$

∴ 24 men is the answer.

Ex. 6. A garrison of 1500 men had provisions for 30 days, but after 9 days they were reinforced by 250 men; how long will the provisions last?

We have to find the time in which 1500 + 250 or 1750 men will consume as much as would suffice to maintain 1500 men for 30 − 9 or 21 days.

The garrison now has 21 days' provisions for 1500 men, *i.e.* 21 × 1500 days' provisions for 1 man.

∴ The provisions will be consumed by 1750 men in

$$\frac{21 \times 1500}{1750} \text{ days} = 18 \text{ days}.$$

Ex. 7. If 15 men eat 28 shillings worth of bread in 14 days when wheat is 52 shillings per quarter, what must be the price of wheat per quarter that 18 shillings worth of bread may suffice for 13 men for 5 days?

The less the number of men to be fed, the greater will be the rate of wheat per quarter at which we can buy wheat out of a fixed sum. Therefore the proportion of men to be fed to the rate of wheat is inverse.

The less the total sum to be spent on buying wheat, the less must be the rate at which wheat is bought. Therefore the proportion of the total cost of wheat to the rate at which it is bought is direct.

The less the number of days for which the men are to be fed out of a given sum, the greater will be the rate of wheat at which we can buy. Therefore the proportion of days to the rate of wheat is inverse.

Hence we have:

$$\left.\begin{array}{ll} 13 \text{ men} & : 15 \text{ men} \\ 28 \text{ shillings} & : 18 \text{ shillings} \\ 5 \text{ days} & : 14 \text{ days} \end{array}\right\} :: 52 \text{ shillings} : \text{required number of shillings.}$$

$$\therefore \text{ The required number of shillings} = \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{52}} \times \overset{3}{\cancel{15}} \times 18 \times \overset{1}{\cancel{14}}}{\underset{1}{\cancel{13}} \times \underset{\cancel{2}}{\cancel{28}} \times \underset{1}{\cancel{5}}} = 108.$$

∴ 108 shillings per quarter is the rate.

Ex. 8. If 25 masons working 10 hours a day build a wall 118 ft. long, 8 ft. high and 3 ft. thick in 48 days, what must be the thickness of a wall 177 ft. long, 9 ft. high which 45 masons working 9 hours a day build in 25 days?

The greater the number of masons employed, the greater will be the thickness of the wall (other things remaining the same); the less the number of working hours per day, the less will be the thickness; the less the number of days for which work is done, the less will be the thickness. Therefore these three proportions are direct.

The greater the length of the wall, the less will be the thickness if the quantity of work done remains the same ; similarly the greater the height, the less will be the thickness ; therefore these two proportions are inverse.

Thus to find the required thickness we have :

$$\left.\begin{array}{rcl} 25 \text{ masons} & : & 45 \text{ masons} \\ 10 \text{ hours} & : & 9 \text{ hours} \\ 177 \text{ ft.} & : & 118 \text{ ft.} \\ 9 \text{ ft.} & : & 8 \text{ ft.} \\ 48 \text{ days} & : & 25 \text{ days} \end{array}\right\} :: 3 \text{ ft.} : \text{required number of feet.}$$

$$\therefore \text{ The required number} = \frac{\overset{1}{\cancel{3}} \times \overset{\overset{1}{\cancel{9}}}{\cancel{45}} \times \overset{3}{\cancel{9}} \times \overset{\overset{1}{\cancel{2}}}{\cancel{118}} \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{25}}}{\underset{1}{\cancel{25}} \times \underset{\underset{1}{\cancel{2}}}{\cancel{10}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{177}} \times \underset{1}{\cancel{9}} \times \underset{\underset{2}{\cancel{6}}}{\cancel{48}}} = \frac{3}{2} = 1\tfrac{1}{2} \text{ ;}$$

$\therefore$ the required thickness is 1 ft. 6 in.

EXERCISE LVI.

1. A railway train travels 1 fur. 44 yds. in 15 seconds; find the rate of the train in miles.

2. 8 horses or 13 oxen or 19 cows eat 7 acres of pasture in 29 days ; how many acres will 26 oxen and 24 horses consume in the same time ?

3. If I can travel 198 miles by train for Rs.3. 1 a. 6 p., how far ought I to be carried at the same rate for Rs.10. 0 a. 10½ p. ?

4. A man runs 2¼ miles in 36 minutes; what time will he take to run 1595 yds. ?

5. A garrison of 800 men had provisions for 30 days ; how long will the provisions last if 200 more men are introduced, but the daily allowance per man diminished by a fourth ?

6. If 3 maunds of goods be carried 150 miles for Rs.3. 2 as., what should be paid for 7 maunds to be carried to a distance of 90 miles ?

7. If a number of men in a certain time earn £86. 5s., how much will four times that number of men with double wages earn in one-third of that time ?

8. If 12 oxen are fed for 17 days at the cost of Rs.13. 13 as., for how many days can 4 oxen be fed for Rs.14. 10 as. ?

9. If 40 men, 60 women or 80 children, can do a work in 6 months, in what time will 10 men, 10 women and 10 children do ⅓ of the work ?

10. A ship with 1200 men on board had sufficient provisions to last 17 days. The survivors of a wreck having been taken aboard the provisions were consumed in 15 days. How many men were taken aboard ?

11. The wine in a pipe when full is worth £19. 8s. 6d. ; how much has leaked away if what is left is worth £19. 2s. 4d. ?

12. A certain number of men, twice as many women, and four times as many boys, earn Rs.525 a week. If a man gets 4 as., a woman 2 as., and a boy 1 a. a day, how many men, women, and boys were there ?

13. An equal number of men and women earned £54. 12s. in 4 weeks, the men receiving 15s. and the women 7s. 9d. each per week; how many men and women were there ?

14. If 90 men can dig a ditch 50 yards long in 4½ days, how many men can dig a ditch 360 ft. long in 27 days ?

15. If 4 men earn as much in a day as 7 women, and one woman as much as two boys, and if 6 men, 10 women, and 14 boys working together for 8 days earn £22, what will be the earnings of 8 men and 6 women working together for 10 days ?

16. A party of work-people consisting of 17 men and 16 boys is paid £116. 17s. 6d., a man's wages being 5s. 6d. per day, and a boy's half that sum. For how many days have they worked ?

17. A fort is provisioned for 30 days ; after 5 days a reinforcement of 200 men arrives and the food will now hold out only 17 days. How many men were there in the fort ?

18. A party of men, women and boys, 32 of each, earns £235. 4s. Find what each man, woman, and boy earns ; a man earning twice as much as a woman, and a woman twice as much as a boy.

19. Two steamers leave a place at the same time for the same port, distant 1260 miles ; the faster steamer goes at the rate of 11¼ miles an hour and arrives at the port 28 hours before the other. Find the rate of the other.

20. If the cost of provisioning a gun-boat carrying 84 men be £598. 10s. when the ship is at sea for 95 days, what will it cost to provision for 33 days a ship carrying a crew of 110 men ?

21. How many hours a day must 24 men work to accomplish as much in 5 days as 25 men could do in 4 days if they worked 6 hours a day ?

22. A barters some sugar with B for flour which is worth 2s. 3*d* per stone, but uses a false stone weight of 13½ lbs. ; what value should B set upon his flour that the exchange may be fair ?

23. A landlord agreed to receive from his tenant a certain number of rupees in cash, together with a certain number of maunds of paddy. When paddy is at Rs.20 a maund, the value of the whole rent amounted to Rs.605, and when paddy is at Rs.28 the value of the whole rent is Rs.685. What will be the value of the whole rent when paddy is at Rs.24 a maund ?

24. The rent of a farm valued at Rs.756 cash was converted into corn rent of paddy, wheat, and barley in equal proportions at the respective rates of Rs.28, Rs.32, and Rs.24 a maund. What will be the value of the corn rent when paddy is at Rs.25 a maund, wheat at Rs.36 a maund, and barley at Rs.28 a maund ?

25. If 5 boys do as much work in 4 days as 2 men take 3 days to do, compare the strength of a man to that of a boy.

26. When the income-tax of 6 pies in the rupee is reduced to 4 pies in the rupee, a person has to pay Rs.2. 9 as. 8 p. less; what is his income ?

27. A person distributed Rs.14. 13 as. 6 p. among a number of beggars. If he had given 9 pies more to each, he would have required Rs.16. 0 as. 3 p.; how many beggars were there ?

28. Find how much rice a family requires monthly when a reduction in the price of rice from 7 seers to 10 seers a rupee reduces the total monthly expenses from Rs.31. 8 as. to Rs.30.

29. A man spends as much in 6 months as he earns in 4; if his yearly income be Rs.275, how much will he lay by at the end of the year ?

30. A mass of pure silver weighs as much as Rs.25; the mass together with Rs.15 weighs $1\frac{1}{2}$ seers; what is the weight of the mass of silver ?

31. One man or two women can do a piece of work in 24 days; one man and three boys can do it in 12 days; in what time can 4 women and 2 boys do it ?

32. If a man, whose annual income is Rs.480, spend as much in 3 months as he earns in 4, what debt will he incur at the end of the year ?

33. The expenses of a family when rice is 10 seers a rupee are Rs.42 a month; when rice is 12 seers a rupee the expenses are Rs.40 a month; what will the expenses be when rice is 8 seers a rupee ?

34. A vessel, whose speed was $9\frac{1}{4}$ miles per hour, started at 8 o'clock to go a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 : 5, starting from the same place, arrived 5 minutes before the first; when did the second vessel start ?

35. If the work done by a man, a woman, and a child be in the ratio of 3 : 2 : 1; and if there be in a factory 16 men, 24 women, and 30 children, whose daily wages amount to Rs.10. 5 as., what will be the wages of 20 men, 30 women, and 40 children for 16 weeks ?

36. If 14 horses require 69 rupees worth of gram in a fortnight when gram is at Rs.36 a maund, what must be the price of gram per maund when Rs.66. 2 as. suffice to feed 23 horses for a week ?

37. A family of 12 men requires Rs.6 worth of sugar for 21 weeks at the rate of $1\frac{1}{4}$ tolas per man per day; what quantity of sugar per day should each man eat when the same quantity is required to last for 28 weeks though the number of men in the family is increased to 15 ?

38. The produce of a field of wheat yielding 4 quarters per acre is worth £148 when wheat is at £3 per quarter. What will the produce of the same field be worth when the yield is 5 qrs. per acre and wheat is 50s. per quarter ?

39. If a man performs a journey of 200 miles in 8 days when he travels 8 hrs. a day, how many hours a day must he travel to perform a journey of 1125 miles in 60 days ?

40. A besieged garrison of 1200 men had provisions sufficient to maintain them for 30 days at the rate of 12 oz. per day per man. But they received a reinforcement of 300 men. How many ounces a day should each man receive so that they may be able to hold out for 10 days more ?

41. A garrison of 2400 men was supplied with provisions for 9 weeks at the rate of 10 ounces per day per man. How many men should leave so that in case of a siege the same provisions may enable the remaining men to hold out for 3 weeks more at the rate of 8 oz. per day per man ?

42. If 15 men, 12 women and 9 boys can complete a piece of work in 51 days, what time would 9 men, 15 women and 15 boys take to do 4 times as much, supposing that a woman does twice as much as a boy, and a man half as much again as a woman per day ?

43. If 5 men can perform a piece of work in 12 days of 10 hours each, how many men will perform a piece of work 4 times as large in a fifth part of the time if they work the same number of hours in the day supposing that three of the second set can do as much work in an hour as 2 of the first set ?

44. If 15 men, 20 women and 45 boys, each working 9 hours a day, build a wall 95 yds. long, 9 ft. high and 3 ft. thick in 18 days, what must be the length of the wall, 11 ft. high, 3 ft. 2 in. thick, which 12 men 39 women and 48 boys each working 12 hours a day build in 15 days ? It being given that a man does thrice as much as each woman, and a woman twice as much as each boy in a day.

45. If 5 pumps each having a length of a stroke of 3 feet working 15 hours a day empty the water out of a mine in 5 days, what must be the length of a stroke of each of 15 pumps, which working 10 hours a day for 12 days, would empty the same mine. the strokes of the former set of pumps being performed four times as fast as those of the latter ?

46. An engine while driving machinery burns coal at the rate of 1 ton 12 cwt. 2 qrs. in 8 hrs. 40 min. ; when the machinery is not in motion, the consumption of coal is only $\frac{4}{11}$ths of this rate. How much coal will the engine burn in 1584 hours during $\frac{1}{9}$ of which time the machinery is at rest ?

47. If 18 pumps can raise 2140 tons of water in 10 days working 7 hrs. a day, in how many days will 16 pumps raise 1634 tons working 9 hrs. a day ?

48. A bankrupt's assets are £1472. 7s. 6d. and his debts amount to £7634. 5s. If the expenses of the bankruptcy amount to £200, what dividend will the creditors receive ?

49. If the 6d. loaf weigh 4·32 lbs. when wheat is at 5·75s. per bushel, what weight of bread, when wheat is at 18·4s. per bushel, ought to be purchased for 18·3s. ?

50. A field containing 26 acres 3 rds. 10 po. is let in equal allotments to 66 agricultural labourers at the rental of 3d. a square pole per annum, a reduction of 3s. in the pound being offered to those tenants who shall pay their rent on the day that it becomes due. When all the year's rents are paid, the landlord finds that he has received the sum of £49. 4s. 9d. How many labourers paid to the day ?

CHAIN RULE.

173. When a series of quantities of different kinds are so connected with one another, that we know how much of the first kind is equivalent to a given quantity of the second, how much of the second is equivalent to a given quantity of the third, and so on, the rule, by which we find how much of the last kind is equivalent to a given quantity of the first kind, is called *Chain Rule*. The rule is as follows :

Write the quantities equivalent to one another so that the left-hand term of each pair shall be of the same kind as the right-hand term of the pair immediately preceding. Place the term of demand, *i.e.* the required quantity of the last kind equivalent to a given quantity of the first as the left-hand term of the first pair, placing that given quantity of the first kind as the right-hand term of that pair. Then find the continued product of the right-hand numbers and divide it by the continued product of the left-hand numbers. The quotient will be the answer required.

Ex. 1. 2 cows cost as much as 7 sheep, 11 sheep as much as 3 ponies, 6 ponies as much as 22 dogs, 14 dogs as much as 3 horses. If the cost of 9 horses be Rs.432, find that of a cow.

Required cost in Rs. = 1 cow,
2 cows = 7 sheep,
11 sheep = 3 ponies,
6 ponies = 22 dogs,
14 dogs = 3 horses,
9 horses = Rs.432 ;

$$\therefore \text{ required cost} = \text{Rs.}\frac{\overset{1}{\cancel{7}}\times\overset{1}{\cancel{3}}\times\overset{\overset{1}{\cancel{2}}}{\cancel{2}}\cancel{2}\times\overset{1}{\cancel{3}}\times 432}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{11}}\times\underset{2}{\cancel{6}}\times\underset{2}{\cancel{14}}\times\underset{3}{\cancel{9}}} = \text{Rs.}\frac{432}{12} = \text{Rs.}\mathbf{36}.$$

Chain Rule is simply an abbreviated form of a series of proportions.

Ex. 2. A runs 8 yards while B runs 9, B runs 20 yards while C runs 16, C runs 18 yards while D runs 25; by how many yards can D beat A in a mile race?

Here we must first find how many yards A will run in the time in which D runs 1 mile or 1760 yards. Therefore we have

For the required distance run by A we have D's 1760 yds.
For 25 yds. run by D we have C's 18 yds.
For 16 yds. run by C we have B's 20 yds.
For 9 yds. run by B we have A's 8 yds.

$$\therefore \text{ required distance} = \frac{\overset{352}{\cancel{1760}} \times \overset{\overset{1}{\cancel{2}}}{\cancel{18}} \times \overset{4}{\cancel{20}} \times \overset{1}{\cancel{8}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{25}} \times \underset{\underset{1}{\cancel{2}}}{\cancel{16}} \times \underset{1}{\cancel{9}}} = 352 \times 4 \text{ yds.} = 1408 \text{ yds.}$$

∴ A runs 1408 yds. while D runs 1760.
∴ D will beat A by (1760 − 1408) or 352 yds.

EXERCISE LVII.

1. 25 lemons cost as much as 8 oranges, 12 oranges as much as 35 plantains, 16 plantains as much as 10 mangoes; if 2 mangoes can be bought for 3 as., what will 18 lemons be bought for?

2. If 3 horses cost as much as 5 oxen, 7 oxen as much as 16 sheep, 2 sheep as much as 5 lambs, 6 lambs as much as 7 goats, and 2 goats be worth 9 rupees, what is a horse worth?

3. A turban costs as much as 5 coats, 2 coats as much as 3 jackets, 5 jackets as much as 2 shirts; if a shirt cost R.1. 5 as., find the cost of a turban.

4. If 2 maunds of rice cost as much as 3 maunds of wheat, 4 maunds of wheat as much as 6 maunds of barley, 9 maunds of barley as much as 12 maunds of Jwari, and 3 maunds of Jwari realise 75 rupees, what would 1 maund of rice realise?

5. Seven pounds of tea cost as much as 32 lbs of sugar, 5 lbs. of sugar as much as 5 lbs. of butter, 4 lbs. of butter as much as 3 lbs. of glue; if 1 lb. of glue can be bought for 9 as., what will 1 lb. of tea cost?

6. If Rs.$3\frac{3}{4}$ = 1 honn, Rs.15 = 1 sovereign; how many honns can be exchanged for 9 sovereigns?

7. In a mile race A can beat B by 40 yds., and B can beat C by 40 yds.; by how many yards can A beat C in a mile race?

8. A counts 90 while B counts 80, B counts 60 while C counts 70, and C counts 63 while D counts 60; how many will D count in the time in which A counts 81?

9. A walks 3 miles while B walks 5 miles, B 7 miles while C 9 miles, C 10 miles while D 14 miles ; what distance will A accomplish in the time in which D goes 36 miles ?

10. A gets Rs.2 as often as B gets 3, B gets 5 as often as C gets 9, C gets 36 as often as D gets 50 ; if A gets Rs.1. 12 as. what will D get ?

11. If 15 lbs. of tea be worth 12 lbs. of coffee, 19 lbs. of coffee worth 26 lbs. of sugar, and 13 lbs. of sugar worth 38 lbs. of soap ; how many lbs. of soap may be had for 7 lbs. of tea ?

12. A man does in 5 days a piece of work which takes a woman 12 days, and a woman does in 8 days what a boy can do in 20 days ; in what time will a man do a piece of work which a boy does in 32 days ?

13. A earns in 9 days as much as B does in 12 days, B in 7 days as much as C in 15 days, C in 18 days as much as D in 28 days and D in 5 days as E in 6 days. If A's wages for a week be Rs.3; how much will E earn in 3 weeks ?

14. A runs 2 yds. while B runs 3, B runs 5 yds. while C runs 6, C runs 9 yds. while D runs 12 ; if A and D run a race of a furlong, who will win and by how many yards ?

15. A watch loses one hour in $7\frac{1}{12}$ days and 289 hours by a clock are equal to 286 hours by the watch ; find how much the clock gains per day.

PROPORTIONAL PARTS.

Ex. 1. Divide Rs.1792 into three parts in the proportion of 2, 5 and 7.

If the number to be divided had been $2+5+7=14$ the required parts would have been 2, 5 and 7. Hence to find similar parts of 1792 we have

14 : 2 : : Rs.1792 : the first part ;

$$\therefore \text{ the first part} = \text{Rs.}\frac{\overset{128}{\cancel{1792}}\times 2}{\underset{1}{\cancel{14}}} = \text{Rs.}256.$$

Again, 14 : 5 : : Rs.1792 : the second part ;

$$\therefore \text{ the second part} = \text{Rs.}\frac{\overset{128}{\cancel{1792}}\times 5}{\underset{1}{\cancel{14}}} = \text{Rs.}640.$$

Again, 14 : 7 : : Rs.1792 : the third part ;

$$\therefore \text{ the third part} = \text{Rs.}\frac{\overset{896}{\cancel{1792}}\times \cancel{7}}{\underset{2}{\cancel{14}}} = \text{Rs. } 896.$$

$\therefore$ Rs.256, Rs.640 and Rs.896 are the required parts.

Ex. 2. Divide Rs.741 into 4 parts in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{7}$.

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}=\frac{105+70+42+30}{210}=\frac{247}{210}.$$

Hence to find the required parts we have

$\frac{247}{210}:\frac{1}{2}::$ Rs.741 : the first part (=Rs.315).

$\frac{247}{210}:\frac{1}{3}::$ Rs.741 : the second part (=Rs.210).

The remaining parts should be similarly found.

Ex. 3. Divide Rs.230 between A, B and C, so that A's share may be to B's share as 2 : 3 and B's to C's as 5 : 7.

For every Rs.3 given to B Rs.2 are given to A. Let us find how much C should receive for every Rs.3 of B. To find this we have

5 : 7 :: 3 : C's corresponding share.

∴ For every Rs.3 of B's C should receive Rs.$\frac{21}{5}$.

Thus we get

A's share : B's share : C's share :: 2 : 3 : $\frac{21}{5}$
:: 10 : 15 : 21.

Hence the sum of Rs.230 is to be divided in the proportion of 10, 15 and 21.

10 + 15 + 21 = 46. Therefore we have

46 : 10 :: Rs.230 : A's share (=Rs.50).

46 : 15 :: Rs.230 : B's share (=Rs.75).

46 : 21 :: Rs.230 : C's share (=Rs.105).

∴ A's share is Rs.50, B's Rs.75 and C's Rs.105.

EXERCISE LVIII.

1. Divide the number 85 into two parts proportional to 2 and 3.

2. Divide the number 135 into three parts proportional to 3, 5 and 7.

3. Divide Rs.432 between A and B in the proportion of 7 and 5.

4. Divide Rs.123 between A and B so that A may get twice as much as B.

5. Divide Rs.392 between A and B so that as often as A gets 5 B may get 9.

6. Divide £219 between A and B so that A may get half as much as B.

7. Divide £375 between A and B so that A may get half as much again as B.

8. Divide 374 into three parts proportional to 3, 5 and 9.

9. Divide 506 in the proportion of 2, 5, 7 and 9.

10. Divide 858 into three parts proportional to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

11. Divide 489 in the proportion of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$.

12. Divide Rs.882 among A, B and C, in the proportion of 5, 7 and 9.

13. Divide £1680 among A, B, C and D, in the proportion of 7, 9, 11 and 13.

14. Divide £744 among A, B, C and D, so that their shares may be to one another as $\frac{1}{3} : \frac{1}{5} : \frac{1}{7} : \frac{1}{9}$.

15. Divide a bankrupt's assets which amount to Rs.375 net among his three creditors whose claims are Rs.123, Rs.231 and Rs.321 respectively.

16. Divide £340 among A, B and C, in the proportion of $1\frac{1}{2}$, $2\frac{1}{3}$ and $3\frac{1}{4}$.

17. A man left legacies as follows :—£426 to his wife, £537 to his son, and £341 to his daughter. At his death his estate realised only £978. How should it be divided ?

18. Divide £151. 10s. among A, B and C, so that A's share may be to B's as 3 : 5 and B's share : C's :: 7 : 9.

19. *Divide £186 among A, B and C, so that A may get twice as* much as B, and C as much as A and B together.

20. *Divide 680 mangoes among* four boys so that every time the first receives 2, the second should receive 3 ; and when the second has 9, the third should have 6 ; and when the third has 8, the fourth should have 12.

TIME AND WORK.

Ex. 1. A can do a piece of work in 8 days and B in 12 days ; in what time can they do it together ?

Since A can do the work in 8 days he can do $\frac{1}{8}$ of it in 1 day ; similarly B can do $\frac{1}{12}$ of it in 1 day.

∴ A and B can together do in 1 day $\frac{1}{8}+\frac{1}{12}=\frac{3+2}{24}=\frac{5}{24}$ of the work.

Hence we have the following proportion : If $\frac{5}{24}$ of the work can be done by the two together in 1 day, in how many days will the whole work be done ?

$$\frac{5}{24} : 1 :: 1 \text{ day} : \text{the required number of days} :$$

$$\therefore \text{ the required number of days} = \frac{1 \times 1 \times 24}{5} = 4\frac{4}{5}.$$

Ex. 2. A man is twice as good a workman as a boy. In what time will a man and a boy do a piece of work together which takes a boy 18 days to complete it ?

A man does twice as much work as a boy, *i.e.* the work of 2 boys is equal to the work of 1 man.

Therefore the work of one man and 1 boy together is equal to the work of 3 boys in 1 day. To find the required time, therefore, we have

3 boys : 1 boy :: 18 days : the required number of days ;

$\therefore$ the required number $=\frac{18}{3}=6$.

Ex. 3. A and B can together do a piece of work in 9 days, A and C in 12 days, and B and C in 15 days ; in what time can they do it conjointly and separately ?

A and B can do $\frac{1}{9}$ of the work in 1 day.
A and C ,, $\frac{1}{12}$,, ,,
B and C ,, $\frac{1}{15}$,, ,,

Therefore by adding

2A, 2B, 2C can do $\frac{47}{180}$ of the work in 1 day ;

$\therefore$ A, B and C can do $\frac{1}{2}$ of $\frac{47}{180}$ or $\frac{47}{360}$,,

To find the time when they will do the work together we have

$\frac{47}{360}:1::1$ day : the required number of days ;

$\therefore$ the required number $=\frac{1\times1\times360}{47}=7\frac{31}{47}$.

Hence A, B and C can together do the work in $7\frac{31}{47}$ days.

Again, since A, B and C can do $\frac{47}{360}$ of the work in 1 day
and A and B ,, $\frac{1}{9}$,, ,,

$\therefore$ by subtracting, we get that

C can do $\frac{47}{360}-\frac{1}{9}=\frac{7}{360}$ of the work in 1 day ;

$\therefore$ C will do the work in $\frac{360}{7}$ days $=51\frac{3}{7}$ days.

Similarly

B can do $\frac{47}{360}-\frac{1}{12}=\frac{17}{360}$ of the work in 1 day,
and A ,, $\frac{47}{360}-\frac{1}{15}=\frac{23}{360}$,, ,,

$\therefore$ B can do the work in $\frac{360}{17}=21\frac{3}{17}$ days,
and A in $\frac{360}{23}$ days $=15\frac{15}{23}$ days.

Ex. 4. The distance from Poona to Satara is 72 miles. A person starts from Satara for Poona and walks at the rate of $3\frac{1}{2}$ miles an hour. At the same time another person starts from Poona for Satara, walking at the rate of $2\frac{1}{2}$ miles an hour. When and where will they meet ?

When they meet, each will have walked for the same number of hours and between them they will have travelled the whole distance. But the distance which they can together cover in 1 hour is $3\frac{1}{2}$ m. + $2\frac{1}{2}$ m. = 6 m.

∴ we have the following proportion :

6 miles : 72 miles :: 1 hour : the required number of hours ;

∴ the required number of hours $= \frac{72}{6} = 12$;

∴ they will meet in 12 hours.

Since the person who starts from Satara walks at the rate of $3\frac{1}{2}$ miles an hour, the distance which he will travel in 12 hours is $12 \times 3\frac{1}{2}$ or 42 miles. Therefore they will meet at a distance of 42 miles from Satara, or 30 miles from Poona.

Ex. 5. A clock loses 3 minutes a day. It is set right at noon on a certain day. What time will it denote at 9 P.M. correct time ? And what will be the correct time when it is 9 P.M. by the clock ?

From noon to 9 P.M. there are 9 hours. Therefore to find how many minutes the clock will lose in 9 hours, we have

24 hrs. : 9 hrs. :: 3 mins. : the required number of mins.

∴ the clock loses in 9 hours $\frac{9 \times 3}{24} = \frac{9}{8} = 1\frac{1}{8}$ mins.

= 1 min. $7\frac{1}{2}$ secs.

Therefore the clock will show at 9 P.M. correct time, 1 min. $7\frac{1}{2}$ secs. to 9 P.M., *i.e.* 58 mins. $52\frac{1}{2}$ secs. past 8 P.M.

Again, since the clock loses 3 mins. in 24 hours of correct time $(24 \times 60 - 3)$ minutes, *i.e.* 1437 mins. of this clock are equivalent to 24 hours of correct time. Therefore to find the correct time equivalent to 9 hours of this clock we have

$\frac{1437}{60}$: 9 :: 24 : the required number of hours ;

∴ the required number $= \frac{60 \times 9 \times 24}{1437}$ hours

= 9 hrs. 1 min. $7\frac{307}{479}$ secs.

Ex. 6. At what time between 1 and 2 will the hands of a watch be together ?

At 1 o'clock the hour-hand is exactly 5 minute-spaces in advance of the minute-hand ; now the minute-hand travels 60 minute-spaces in 1 hour, while the hour-hand travels only 5 minute-spaces in that time, *i.e.* if the hour-hand be $60 - 5 = 55$ minute-spaces in advance of the minute-hand, it will take 1 hour or 60 minutes for the minute-hand to overtake it. Hence to find in how many minutes the minute-hand will make up the distance of 5 minute-spaces between itself and the hour-hand we have

55 minute-spaces : 5 minute-spaces :: 60 minutes : the required number of minutes ;

∴ the required number of minutes $= \frac{60 \times 5}{55} = 5\frac{5}{11}$;

∴ the two hands will be together at $5\frac{5}{11}$ minutes past 1.

EXERCISE LIX.

1. A man can do a piece of work in 4 days, a woman in 6 days, and a boy in 9 days. In what time can they do it together ?

2. A man can do a piece of work in 8 days. A man and a boy together do it in 6 days ; in what time can the boy alone do it ?

3. A and B complete a piece of work in 8 days ; B and C do the same in 12 days ; A, B and C finish it in 6 days. In how many days will A and C complete the work ?

4. A man can do a piece of work in 18 days, and the man and a boy can do it together in 10 days. In what time will the boy alone do it ?

5. A cistern has two pipes A and B ; A fills the cistern in 8 hours. B empties it in 12 hours ; in what time will the cistern be filled when A and B are both left open ?

6. A and B can do a piece of work in 15 days, A and C in 20 days, and B and C in 25 days. In what time can they do it together ?

7. A man and a woman can together do a piece of work in 24 days, the man and a boy together in 32 days, and the woman and the boy together in 36 days. In what time can the man, the woman and the boy do it separately ?

8. A train travels from A to B at a rate of ·625 mile per minute and performs the journey in 7·4461 hrs. ; how long does another train take, travelling at the rate of ·3919 mile per minute ?

9. A, B and C could reap a field in 18 days ; B, C and D in 20 days ; C, D and A in 24 days ; and D, A and B in 24 days ; in what time would it be reaped by them all together ?

10. Two trains start at the same time towards one another from two stations distant 480 miles. If the rates of the trains be 25 miles and 23 miles an hour respectively, when and where will they meet ?

11. A and B start at the same time from P and Q to Q and P. They meet one another in 16 hours. If the distance between P and Q be 176 miles, and if A travel at the rate of 5 miles an hour, find in how many hours more will each reach his place of destination.

12. A watch loses 3 minutes a day. How should it be set at 12 noon correct time that it may denote right time at 10 P.M. the same day ?

13. A clock gains 5 seconds in every 3 minutes. It was set right at 6 A.M. on Monday ; what time will it indicate at 6 P.M. on Monday next ?

14. A, who travels $3\frac{1}{2}$ miles an hour, starts $2\frac{1}{2}$ hours before B, who goes the same road at $4\frac{1}{2}$ miles an hour ; where will he overtake A ?

15. A local train, which travels at the rate of 24 miles an hour, leaves Lahore at 20 mins. past 8 and reaches Amritsar at 5 mins. past 10 the same morning. It stops at Minmir for 10 mins. and at each of three other stations for 5 mins. Find the distance between Lahore and Amritsar.

16. At what time between 4 and 5 will the hands of a watch be opposite ?

17. A and B start from the same place at the same time in opposite directions. If A walk at the rate of $2\frac{1}{4}$ miles an hour and B $2\frac{1}{2}$ miles, in what time will they be 78 miles apart ?

18. The hands of a watch are together between 3 and 4 ; what is the time ?

19. Two trains start at 6 A.M. from the same station in opposite directions. If the rates of the trains be 21 miles and 15 miles per hour respectively, what will be the distance between them at 3.20 P.M. on the same day ?

20. When will the hands of a watch be at right angles between 9 and 10 for the first time ?

21. A starts from Bombay to Kalyan, a distance of 34 miles, at the rate of $2\frac{1}{2}$ miles an hour ; B starts simultaneously from Kalyan to Bombay and walks at the rate of $1\frac{3}{4}$ miles an hour. Where will A pass B ? and how long will B take to reach Bombay ?

22. A policeman sets off to catch a thief, who has had 40 minutes' start ; if the thief can go 3 miles an hour and the policeman $3\frac{3}{4}$ miles an hour, how far will the thief have gone before he is caught ?

23. A can do a piece of work in 6 days, B in 8 days, and C in 12 days. B and C work together for 2 days and then C is replaced by A. Find when the work will be finished.

24. A can do $\frac{1}{3}$ of a piece of work in 2 hours, B can do $\frac{2}{5}$ of the remainder in half-an-hour ; C can then finish it in 2 hours. How long would A, B and C together take to do it ?

25. The minute-hand of a watch is 7 minutes in advance of the hour-hand between 3 and 4. Find the time.

CHAPTER XI.

AVERAGES AND PERCENTAGES.

174. The averages of any number of quantities is their sum divided by their number. Thus the average of 7 and 9 is $\frac{7+9}{2}=8$.

Ex. 1. Find the average of the quantities 3 lbs., 5 lbs., 7 lbs. and 9 lbs.

The sum of 3 lbs., 5 lbs., 7 lbs. and 9 lbs. is 24 lbs.

The number of quantities is 4.

$$\therefore \text{ the required average is } \frac{24 \text{ lbs.}}{4} = 6 \text{ lbs.}$$

Ex. 2. A village Patel kept a horse, a cow and a buffalo; the weekly expenses of feeding these animals were Rs.3. 8 as., R.1. 4 as., and Rs.2. 12 as. respectively. *Find the average expense per animal.*

The expense for the horse = Rs.3. 8 as.
,, ,, cow = R. 1. 4 as.
,, ,, buffalo = Rs.2. 12 as.
The total expense = Rs.7. 8 as.

$$\therefore \text{ the required average} = \frac{\text{Rs.7. 8 as.}}{3} = \text{Rs.2. 8 as.}$$

Ex. 3. In a class of 26 boys, 7 boys are each 14 years of age, 13 are each 12 years of age, 2 are 18 years each, and the remaining, 16 years each; find the average age per boy in the class.

The aggregate of the ages of 7 boys is 7×14 or 98
,, ,, 13 ,, 13×12 or 156
,, ,, 2 ,, 2×18 or 36
,, ,, the remaining, *i.e.* 4 ,, 4×16 or 64

$\therefore$ the aggregate of the ages of 26 boys is 354 years;

$$\therefore \text{ the required average} = \tfrac{354}{26} \text{ years} = 13{\cdot}65.$$

175. The term *per cent.* means *through hundred* (*per* = through and *centum* = hundred), *i.e.* for every hundred. Thus 4 per cent. means 4 for every 100. 8 mangoes are 4 per cent. of 200 mangoes; 14 ft. are $3\frac{1}{2}$ per cent. of 400 ft.; 60 men are 5 per cent. of 1200 men, and so on. It will be seen that a percentage is by itself an abstract number.

Ex. 4. A man allows his agent 1 anna in the rupee for collecting his rents. What per cent. of his income goes to the agent?

Out of every 16 as. of income, 1 a. goes to the agent.

$\therefore$ out of every 100 as. $\frac{100}{16}$ or $6\frac{1}{4}$ as. will go to the agent. Therefore $6\frac{1}{4}$ per cent. is the answer.

Ex. 5. A gentleman lays by 35 per cent. of his income per year: if his yearly savings amount to Rs.157 8 as., find his gross annual income.

The ratio of his savings to his annual income being 35 : 100, to find his annual income corresponding to his savings of Rs.157. 8 as. or Rs.$\frac{315}{2}$, we have

35 : 100 : : Rs.$\frac{315}{2}$: the required number of Rs. ;

$$\therefore \text{ the required number} = \frac{\overset{45}{\cancel{315}} \times \overset{\overset{10}{\cancel{20}}}{\cancel{100}}}{\underset{1}{\cancel{2}} \times \underset{\underset{1}{\cancel{7}}}{\cancel{35}}} = 45 \times 10 = 450\ ;$$

∴ his annual income is Rs.450.

Ex. 6. During a plague epidemic, out of a population of 6400 in a place, only 3072 people survived ; what per cent. of the population died of plague ?

Since 3072 people survived, 6400 − 3072 or 3328 people died of plague ; hence to find the required percentage, we have

6400 people : 3328 people : : 100 : the required percentage ;

$$\therefore \text{ the required percentage} = \frac{3328 \times 100}{6400} = 52\ ;$$

∴ 52 per cent. of the population died.

Ex. 7. A village sends $4\frac{1}{2}$ per cent. of its population to a school which admits 25 per cent. free students. If the number of free students in the school be 18, find the number of people in the village.

25 per cent. is $\frac{25}{100}$ or $\frac{1}{4}$ and $\frac{1}{4}$ of $4\frac{1}{2}$ is $\frac{1}{4}$ of $\frac{9}{2} = \frac{9}{8}$. Therefore the number of free students is $\frac{9}{8}$ per cent. of the whole population in the village. Hence to find this population we have

$\frac{9}{8}$: 100 : : 18 free students : the whole population.

$$\therefore \text{ the population of the village} = \frac{100 \times 18 \times 8}{9} = 1600.$$

176. We add here a few definitions.

1. *Insurance* is a percentage paid for securing life or property from loss arising generally from accident.

2. The *payment made for the purpose of insurance* is called the *premium*, and the legal document by which the insurer is secured from loss is called the *policy of insurance*.

Life insurance is a contract by which certain companies engage to pay a certain sum to the heirs of the person whose life is insured, in consideration of his paying a certain amount periodically.

The charge is regulated by considerations of the age and condition of health of the person, whose life is assured, at the time of the first taking out of the policy.

Fire insurance is a contract to make good any loss, accidentally sustained from fire, in consideration of receiving a certain percentage on the property insured.

EXERCISE LX.

1. Find the average of 1, 2, 3, 4 and 5.

2. Find the average of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

3. Find the average of a khandi and a maund ; a stone and a quarter.

4. A man spends Rs.912. 8 as. in a common year ; find his average daily expenditure.

5. Find the average of $3\frac{1}{2}$, 5 and 9·95.

6. A man earns Rs.75 and spends Rs.57 ; what per cent. of his income does he spend ?

7. What is the average daily income of a man who earns R.1. 8 as. 6 pies in a week ?

8. Find the commission on £485. 12s. 6d. at $8\frac{1}{3}$ p.c.

9. In a class of 24 boys 2 are each 5 ft. 2 in. in height, 7 are each 4 ft. 7 in., 12 are each 4 ft. 2 in., 1 is 3 ft. 7 in., and the rest are each 4 ft. What is the average height of the class ?

10. A man bought 5 horses at Rs.46 each, 2 horses at Rs.58 each, and 1 more for Rs.86. What was the average price per horse ?

11. A grocer bought 5 lbs. of tea at 1s. 6d. per lb., 4 lbs. at 2s. 3d. per lb., and 3 lbs. at 3s. 4d. per lb. ; find the average price of tea per lb.

12. An agent received Rs.48. 12 as. as commission on the sale of Rs.780 worth of goods ; at what rate per cent. was the commission charged ?

13. What per cent. is 9 of 72 ? What per cent. is $\frac{1}{5}$ of $\frac{1}{4}$ and ·3 of ·4 ?

14. Find the average of 5·28, 3·05, 0·975, 62·0 and 8·3125.

15. Find the average correct to two decimal places of 25·317, $37\cdot\dot{3}$, $23\cdot\dot{4}$ and $19\cdot8\dot{3}$.

16. 60 per cent. of the population of a city consists of males, and of these 40 per cent. are children ; if the number of children in the city be 1680, find the exact number of females.

17. In a dictation lesson in a class of 16 boys the following mistakes were made—4, 7, 2, 0, 18, 1, 8, 3, 0, 2, 5, 12, 0, 7, 9, 6 find the average number of errors per boy.

18. In a town of thirty-two thousand inhabitants 72 per cent. are 35 or under 35 years of age ; how many are there over that age ?

19. In a town of 1800 electors ·3 voted for one candidate and ·45 for another ; how many electors did not vote at all ?

20. A landlord after having paid income-tax at 1 per cent. on his gross income and 5 per cent. on the remainder for collecting his rents has an income of £1881 a year ; what is his gross income ?

21. Find the premium on a policy of assurance for Rs.2500 upon the life of a person aged 31, at the rate of $2\frac{1}{2}$ per cent. for that age.

22. Find the expense of insuring a vessel and cargo whose value is Rs.18450 at $3\frac{3}{4}$ per cent.

23. What is the annual payment for a house insured for Rs.7500 at $\frac{3}{4}$ per cent. ?

24. A ship is insured for $\frac{3}{4}$ of its value at $\frac{1}{2}$ per cent. If the premium be Rs.25, find the value of the ship.

25. In a school of 250 children 44 per cent. are learning Geography, 36 per cent. are learning Grammar, 12 per cent. cannot read, and 4 per cent. have advanced as far as Algebra. What are the actual numbers of each ?

CHAPTER XII.

INTEREST.

177. *Interest* is the money which the lender of a sum receives from the borrower, in consideration of the borrower using the sum.

178. The sum lent is called the *Principal.*

179. When the *rate of interest per cent.* is given, it means that the sum mentioned as the rate is the interest for 100. Thus 5 per cent. means that Rs.5 is the interest for Rs.100, £5 is the interest for £100, and so on. Generally this rate is given for 1 year, and is then called rate per cent. per annum.

180. The sum of the principal and the interest on it for any period is called the amount for that period.

SIMPLE INTEREST.

181. When the interest is calculated on the original principal only, it is called *Simple Interest.*

182. Questions in interest involve four quantities : the Principal, the rate of Interest, the Period, and the Total Interest for that period. Sometimes the Amount is given instead of the principal or of the interest on that principal.

And in these questions of Simple Interest, any three of the above quantities being given, we have to find the fourth.

Ex. 1. Find the interest on Rs.1275 for 3 years at 4 per cent. per annum Simple Interest.

Since Rs.4 is the interest for Rs.100 for one year, to find the interest on Rs.1275 for three years, we have

$$\left.\begin{array}{l}\text{Rs.}100 : \text{Rs.}1275\\ 1\text{ year} : 3\text{ years}\end{array}\right\} :: \text{Rs.}4 : \text{the required interest.}$$

$$\therefore \text{ the required interest} = \text{Rs.}\frac{4\times1275\times3}{100}$$
$$= \text{Rs.}51\times3 = \text{Rs.}153\,;$$

∴ Rs.153 is the answer.

Ex. 2. Find the amount of £1368. 15s. for 106 days at $4\frac{3}{4}$ per cent. per annum simple interest.

$$£1368.\ 15\text{s.} = £1368\tfrac{3}{4} = £\tfrac{5475}{4}\,;$$

$$4\tfrac{3}{4} = \tfrac{19}{4},\ \text{and } 106 \text{ days} = \tfrac{106}{365}\text{ year.}$$

We must find first the interest on $£\frac{5475}{4}$ for the given period; therefore we have

$$\left.\begin{array}{l}£100 \ : £\frac{5475}{4}\\ 1\text{ year} : \frac{106}{365}\text{ year}\end{array}\right\} :: £\tfrac{19}{4} : \text{the interest for the period.}$$

$$\therefore \text{ the interest for the period} = £\frac{19\times\overset{\overset{\overset{3}{\cancel{219}}}{\cancel{1095}}}{\cancel{5475}}\times\overset{53}{\cancel{106}}}{4\times4\times\underset{\underset{1}{\cancel{73}}}{\cancel{365}}\times\underset{\underset{10}{\cancel{20}}}{\cancel{100}}}$$

$$= £\frac{19\times3\times53}{4\times4\times10}\ \text{or}\ \frac{3021}{160}$$
$$= £18\tfrac{141}{160}$$
$$= £18.\ 17\text{s.}\ 7\tfrac{1}{2}\text{d.}$$

$$\therefore \text{ the required amount} = £1368.\ 15\text{s} + £18.\ 17\text{s.}\ 7\tfrac{1}{2}\text{d.}$$
$$= £1387.\ 12\text{s.}\ 7\tfrac{1}{2}\text{d.}$$

Or we may first find in this example the amount of £100 for 106 days at $4\frac{3}{4}$ per cent. and find from it the amount of £1368. 15s.

Ex. 3. What sum will amount to Rs.5311. 4 as. 9 p. in $4\frac{3}{4}$ years at $3\frac{1}{2}$ per cent. per annum ?

Since the interest of Rs.100 for 1 year is Rs.$3\frac{1}{2}$ or Rs.$\frac{7}{2}$, the interest for $4\frac{3}{4}$ years or $\frac{19}{4}$ years is Rs.$\frac{19}{4}\times\frac{7}{2}$ = Rs.$\frac{133}{8}$; therefore the amount of Rs.100 for $4\frac{3}{4}$ years is Rs.100 + Rs.$\frac{133}{8}$ = Rs.$\frac{933}{8}$; hence to find the principal which will amount in $4\frac{3}{4}$ years to Rs.5311. 4 as. 9 p., *i.e.* to Rs.$5311\frac{19}{64}$, or $\frac{339923}{64}$, we have

$$\text{Rs.}\tfrac{933}{8} : \text{Rs.}\tfrac{339923}{64} :: \text{Rs.}100 : \text{the required principal}\,;$$

$$\therefore \text{ the required principal} = \text{Rs.}\frac{339923\times100\times8}{64\times933}$$
$$= \text{Rs.}\frac{27325}{6}$$
$$= \text{Rs.}4554.\ 2 \text{ as. } 8 \text{ p.}$$

Ex. 4. At what rate per cent. Simple Interest will Rs.642. 15 as. 6 pies amount to Rs.868. 0 as. $1\frac{1}{2}$ p. in $7\frac{1}{2}$ years ?

Rs.868. 0 as. $1\frac{1}{2}$ p. is the amount ;
Rs.642. 15 as. 6 p. is the principal.

∴ by subtracting, Rs.225. 0 as. $7\frac{1}{2}$ p. is the interest on the principal.

Rs.642. 15 as. 6 p. = Rs.$642\frac{31}{32}$ = Rs.$\frac{20575}{32}$.

And Rs.225. 0 as. $7\frac{1}{2}$ p. = Rs.$225\frac{5}{128}$ = Rs.$\frac{28805}{128}$.

The principal and the total interest on it for $7\frac{1}{2}$ years or $\frac{15}{2}$ years being given, to find the rate of interest we have

$$\left.\begin{array}{l}\text{Rs.}\frac{20575}{32} : \text{Rs.}100\\ \frac{15}{2}\text{ years} : 1 \text{ year}\end{array}\right\} :: \text{Rs.}\frac{28805}{128} : \text{the interest on Rs.100.}$$

$$\therefore \text{ the interest on Rs.100} = \text{Rs.}\frac{\overset{\overset{\overset{2}{\cancel{4}}}{\cancel{20}}}{\cancel{100}}\times\overset{5761}{\cancel{28805}}\times\overset{1}{\cancel{32}}\times\overset{1}{\cancel{2}}}{\underset{\underset{\underset{1}{\cancel{2}}}{\cancel{64}}}{\cancel{128}}\times\underset{\underset{823}{\cancel{4115}}}{\cancel{20575}}\times\underset{3}{\cancel{15}}}$$

$$= \text{Rs.}\frac{2\times5761}{823\times3} = \text{Rs.}\frac{2\times7\times\cancel{823}}{\cancel{823}\times3}$$

$$= \text{Rs.}\frac{2\times7}{3} = \text{Rs.}\frac{14}{3} = \text{Rs.}4\tfrac{2}{3}.$$

∴ the required rate is $4\frac{2}{3}$ per cent.

Ex. 5. In what time will Rs.1687. 8 as. amount to Rs.2027. 1 a. 9 p. at $3\frac{1}{2}$ per cent. Simple Interest ?

Rs.2027. 1 a. 9 p. is the amount ;
Rs.1687. 8 as. is the principal ;

∴ by subtracting, Rs.339. 9 as. 9 p. or Rs.$\frac{21735}{64}$ is the interest on Rs.1687. 8 as. or Rs.$\frac{3375}{2}$ for the required time.

Since the time varies inversely as the principal and directly as the interest we have

$$\left.\begin{array}{ll}\text{Rs.}\frac{3375}{2} & : \text{Rs.}100\\ \text{Rs.}\frac{7}{2} & : \text{Rs.}\frac{21735}{64}\end{array}\right\} :: 1 \text{ year} : \text{the required time.}$$

$$\therefore \text{ the required time} = \frac{\overset{\overset{621}{\cancel{4347}}}{\cancel{21735}}\times\overset{\overset{1}{\cancel{4}}}{\cancel{100}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}}{\underset{4}{\cancel{64}}\times\underset{\underset{27}{\cancel{135}}}{\cancel{3375}}\times\underset{1}{\cancel{7}}}\text{ years}$$

$$= \frac{621}{4\times27}\text{ years} = \frac{23}{4}\text{ years}$$

$$= 5\tfrac{3}{4}\text{ years.}$$

Ex. 6. A borrowed from B £1200 at $4\frac{1}{2}$ per cent. simple interest on the 5th Jan., 1875. He repaid the loan with interest on the last day of May of the same year: what interest did he pay?

Let us first count the number of days from 5th Jan. to 31st May. In such examples of the two dates mentioned only one, *i.e.* either the first or the last is included in the period. Thus here leaving out the first and including the last we have

26 days of January
28 days of February
31 days of March
30 days of April
31 days of May
146 days altogether.

Now to find the interest on 146 days at the rate of £$4\frac{1}{2}$ or $\frac{9}{2}$ for £100 for 365 days we have

$$\left.\begin{array}{l}\pounds100 \quad : \pounds1200\\ 365 \text{ days} : 146 \text{ days}\end{array}\right\} :: \text{Rs.}\tfrac{9}{2} : \text{the interest required.}$$

$$\therefore \text{ the required interest} = \pounds\frac{\overset{\overset{12}{\cancel{240}}}{\cancel{1200}} \times 9 \times \overset{\overset{1}{\cancel{73}}}{\cancel{146}}}{\underset{5}{\cancel{100}} \times \underset{\underset{1}{\cancel{73}}}{\cancel{365}} \times \underset{1}{\cancel{2}}}$$

$$= \frac{12 \times 9}{5} = \pounds\frac{108}{5}$$

$$= \pounds21\tfrac{3}{5}$$

$$= \pounds21.\ 12\text{s.}$$

EXERCISE LXI.

Find the interest on

1. Rs.1625 for 3 years at 4 per cent. per annum simple interest.
2. Rs.325 for 9 months at 6 per cent. per annum simple interest.
3. Rs.1460 for 137 days at 5 per cent. per annum simple interest.
4. £475 for 6 years at $2\frac{1}{2}$ per cent. per annum simple interest.
5. £1285. 15s. for $3\frac{1}{4}$ years at $2\frac{2}{3}$ per cent. per annum simple interest.

Find the amount of

6. Rs.675 for 4 years at 6 per cent. per annum.
7. Rs.563. 4 as. 8 p. for 3 years at 4 per cent. per annum.
8. Rs.830. 4 as. for 4 years at 2 per cent. per annum.
9. £5027. 10s. for $4\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum.
10. £576. 18s. 6d. for 5 years at $6\frac{2}{3}$ per cent. per annum.

Find the interest and the amount of

11. Rs.1825 for 90 days at 4 per cent. per annum.

12. Rs.438 for 105 days at 3 per cent. per annum.

13. £182. 10s. for 20 days at $2\frac{1}{2}$ per cent. per annum.

14. £511. 10s. for 28 days at $6\frac{1}{4}$ per cent. per annum.

15. £310. 5s. for 60 days at 5 per cent. per annum.

16. What sum will amount to Rs.560 in 3 years at 4 per cent. simple interest ?

17. A sum of money amounts to £289. 7s. 6d. in $4\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. ; find it.

18. At what rate per cent. will Rs.450 amount to Rs.540 in 5 years ?

19. If Rs.510 amount to Rs.637. 8 as. in 5 years, what is the rate of interest ?

20. In what time will Rs.820 amount to Rs.984 at 4 per cent. ?

21. At what rate per cent. will the interest on Rs.5 amount to Rs.6. 4 as. in 20 years ?

22. In what time will Rs.350 produce Rs.360 at 8 per cent. per annum ?

23. What sum will discharge on the 5th of November a debt of £2190 borrowed on the 12th of June at 4 per cent. ?

24. A person borrowed Rs.640 on the 8th of March at 5 per cent. and repaid the loan with interest on the 20th of May of the same year ; what sum did the creditor receive ?

25. A sum of money amounts to £774 in 2 years and to £903 in 5 years ; find the sum and the rate per cent.

26. At what rate per cent. will the interest on £1690 amount to half the principal in 8 years ?

27. In what time will a sum of money double itself at $3\frac{1}{8}$ per cent. simple interest ?

28. The interest on a given sum for 3 years is £372, and the amount of the same sum at the same rate in 4 years is £3596. Find the sum and the rate per cent.

29. A person borrowed from a banker Rs.500 on the 4th of April, and Rs.300 on the 8th of June. What sum should he pay on the 8th of August in liquidation of his debt, reckoning interest at $9\frac{1}{3}$ per cent. per annum in each case ?

30. A person borrowed Rs.350, and two years after Rs.250 at twice the former rate of interest. If the amount of both the sums in 5 years from the time the first sum was borrowed be Rs.697. 8 as., find the rates of interest at which the sums were borrowed.

COMPOUND INTEREST.

183. When the interest on a given sum at the end of a fixed period is added to the sum, and the amount thus obtained is taken to be the principal for the next period, the interest thus obtained is called the *compound interest.* The compound interest on a sum for a number of periods is, therefore, the total of the various sums representing the interest for the various periods, the amount at the end of each period being the principal for the next period.

Ex. 1. Find the compound interest on £375 for 2 years at 4 per cent. per annum.

Since £4 is the interest on £100 for a year, to find the interest on the given sum for the first year we have

£100 : £375 :: £4 : first year's interest ;

$$\therefore \text{ first year's interest} = \frac{\overset{15}{\cancel{375}} \times \overset{1}{\cancel{4}}}{\underset{\underset{1}{\cancel{25}}}{\cancel{100}}} = £15.$$

$\therefore$ the principal at the beginning of the second year = £375 + £15 = £390.

Again to find the interest for the second year we have

£100 : £390 :: £4 : second year's interest ;

$$\therefore \text{ second year's interest} = £\frac{\overset{78}{\cancel{390}} \times \overset{1}{\cancel{4}}}{\underset{\underset{5}{\cancel{25}}}{\cancel{100}}} = £\frac{78}{5}$$

$$= £15\tfrac{3}{5} = £15.\ 12s.$$

$\therefore$ the total interest for the two years is £15 + £15. 12s. = £30. 12s.

184. The use of decimals greatly facilitates work in the solution of examples in compound interest.

Ex. 2. Find the amount at compound interest of Rs.750 in 3 years at 5 per cent. per annum ; also find the interest.

To find the amount of R.1 at the end of the first year we have

Rs.100 : R.1 :: Rs.105 : the required amount ;

$$\therefore \text{ the required amount} = \text{Rs.}\tfrac{105}{100} = \text{R.}1\cdot05.$$

Since the amount at the end of the *first* year becomes the principal at the beginning of the *second* year, to find the amount of R.1 at the end of the second year we have

Rs.100 : Rs.1·05 : : Rs.105 : required amount ;

$$\therefore \text{ required amount} = \text{Rs.}\frac{1\cdot05 \times 105}{100}$$

$$= \text{Rs.}1\cdot05 \times 1\cdot05 = \text{Rs.}(1\cdot05)^2.$$

Similarly to find the amount of R.1 at the end of the third year we have

$$\text{Rs.}100 : \text{Rs.}(1{\cdot}05)^2 :: \text{Rs.}105 : \text{required amount};$$

$$\therefore \text{ required amount} = \text{Rs.}\frac{(1{\cdot}05)^2 \times 105}{100}$$

$$= \text{Rs.}(1{\cdot}05)^2 \times 1{\cdot}05 = \text{Rs.}(1{\cdot}05)^3.$$

Now to find the amount of Rs.750 for the same time, *i.e.* for 3 years, we have

$$\text{R.}1 : \text{Rs.}750 :: \text{Rs.}(1{\cdot}05)^3 : \text{required amount};$$

$$\therefore \text{ required amount} = \text{Rs.}(1{\cdot}05)^3 \times 750$$

$$= \text{Rs.}1{\cdot}157625 \times 750$$

$$= \text{Rs.}868.\ 3 \text{ as. } 6 \text{ p.}$$

To obtain the interest, we subtract Rs.750 from the amount Rs.868. 3 as. 6 p.

$\therefore$ the interest = Rs.118. 3 as. 6 p.

The decimal process used in the above example may be set down thus:

Rs.1·05 = Amount of R.1 at the end of 1 year.
1·05
525
105
Rs.1·1025 = Amount of R.1 at the end of 2 years.
1·05
55125
11025
1·157625 = Amount of R.1 at the end of 3 years.
750
57881250
8103375
Rs.868·218750 = Amount of Rs.750 in 3 years.
16
3·50000
12
6·0

$\therefore$ Rs.868. 3 as. 6 p. is the amount of Rs.750 in 3 years.

185. It will be seen that for finding the amount of R.1 for three years at 5 per cent., we had to raise Rs.1·05, the amount for 1 year, to the *third power*. If the amount for 4 years had been required, Rs.1·05 would have been raised to the fourth power, and so on.

N.B.—Unless otherwise mentioned 1 year is taken as the unit of period.

186. In cases where the period given contains a fractional part we should first find the amount for the integral part of the period as explained above, and then find the amount for the fractional part of the period. Thus :

Ex. 3. Find the amount at Compound Interest of £1000 for $3\frac{1}{2}$ years at 4 per cent. per annum.

The amount of £1 in 1 year is £1·04.

∴ in three years it is $£(1{\cdot}04)^3$.

Again, £1 in $\frac{1}{2}$ year amounts to 1·02.

∴ $£(1{\cdot}04)^3$ will amount in $\frac{1}{2}$ year to $£\{(1{\cdot}04)^3 \times (1{\cdot}02)\}$;

∴ the amount of £1 in $3\frac{1}{2}$ years will be

$$£1{\cdot}124864 \times 1{\cdot}02 = £1{\cdot}14736128.$$

$$\begin{aligned} \therefore \text{ the amount of } £1000 &= £1147{\cdot}36128 \\ &= £1147.\ 7s.\ 2{\cdot}7072d. \end{aligned}$$

Ex. 4. Find the amount of £405. 12s. 6d. in 2 years at 8 per cent. compound interest, interest being payable half-yearly.

$$£405.\ 12s.\ 6d. = £405\tfrac{5}{8} = £405{\cdot}625.$$

Since interest is payable half-yearly, we have to take $\frac{1}{2}$ of 8, *i.e.* 4, as the rate in calculating interest for each period of six months. Therefore we have

$$\begin{array}{rl}
£1{\cdot}04, & \text{amount of £1 for 6 months.} \\
\underline{1{\cdot}04} & \\
416 & \\
\underline{104} & \\
£1{\cdot}0816, & \text{amount of £1 for 1 year.} \\
\underline{1{\cdot}0816} & \\
64896 & \\
10816 & \\
84928 & \\
\underline{10816} & \\
£1{\cdot}16985856, & \textit{amount of £1 for 2 years.}
\end{array}$$

Now to find the amount of £405·625 for 2 years, we have

$$£1 : £405{\cdot}625 :: £1{\cdot}16985856 : \text{the required amount}$$

$$\begin{aligned} \therefore \text{ the required amount} &= £405{\cdot}625 \times 1{\cdot}16985856 \\ &= £474{\cdot}5238784 \\ &= £474.\ 10s.\ 5{\cdot}730816d. \end{aligned}$$

Ex. 5. What sum will amount to Rs.1464. 10 as. 8 p. in 3 years at 4 per cent. Compound Interest ?

Since R.1 amounts in 1 year to Rs.$\frac{105}{100}$ = Rs.$\frac{26}{25}$, *it will amount in* 3 years to Rs.$(\frac{26}{25})^3$. Therefore, to find the sum which will amount in 3 years to Rs.$1464\frac{2}{3}$, *i.e.* to Rs.$\frac{4394}{3}$, we have

$$\text{Rs.}\left(\tfrac{23}{25}\right)^3 : \text{Rs.}\tfrac{4394}{3} :: \text{R.}1 : \text{the required sum.}$$

$$\therefore\ \text{the required sum} = \text{Rs.}\left(\tfrac{4394}{3} \div \tfrac{26\times26\times26}{25\times25\times25}\right)$$

$$= \text{Rs.}\frac{\overset{\overset{\overset{1}{\cancel{26}}}{\cancel{338}}}{\cancel{4394}}\times25\times25\times25}{\underset{2}{\cancel{26}}\times\underset{2}{\cancel{26}}\times\underset{1}{\cancel{26}}\times3}$$

$$= \text{Rs.}\frac{25\times25\times25}{3\times2\times2}$$

$$= \text{Rs.}\tfrac{15625}{12}$$

$$= \text{Rs.}1302\tfrac{1}{12} = \text{Rs.}1302.\ 1\text{ a. } 4\text{ p.}$$

Ex. 6. Find the difference between the simple and compound interest on Rs.250 for 3 years at 3 per cent. per annum.

To find the simple interest we have

$$\left.\begin{array}{l}\text{Rs.}100 : \text{Rs.}250\\ 1\text{ year} : 3\text{ years}\end{array}\right\} :: \text{Rs.}3 : \text{the required interest};$$

$$\therefore\ \text{the required interest} = \text{Rs.}\tfrac{3\times250\times3}{100}$$

$$= \text{Rs.}22\tfrac{1}{2} = \text{Rs.}22{\cdot}5.$$

To find the compound interest, it is better to proceed as in Ex. 4. Thus :

	Rs.	
Multiplying by 3 and dividing by 100	250	Original principal
	7·5	Interest for the 1st year
	257·5	Amount in 1 year
„ „	7·725	Interest for the 2nd year
	265·225	Amount in 2 years
	7·95675	Interest for the 3rd year
	273·18175	Amount in 3 years
	250	Original principal
	23·18175	Compound interest for 3 years
Subtracting	22·5	*Simple interest for 3 years*
we get	0·68175	= the required difference.
	16	
	10·90800	
	12	
	10·896	

∴ the required difference = 10 as. 10·896 pies.

Ex. 7. The difference between the simple and compound interest on a certain sum for 3 years at $6\frac{1}{4}$ per cent. is Rs.6. 2 as. ; find the sum.

$6\frac{1}{4}$ per cent. is $\frac{1}{16}$. Therefore R.1. produces R.$\frac{1}{16}$ in 1 year. Therefore it amounts in 1 year to Rs.$1\frac{1}{16}$ or Rs.$\frac{17}{16}$, and at compound interest in 3 years to Rs.$(\frac{17}{16})^3 = \text{Rs.}\frac{4913}{4096}$. Therefore the compound interest on R.1 for 3 years is

$$\text{Rs.}\tfrac{4913}{4096} - \text{R.}1 = \text{R.}\tfrac{4913-4096}{4096} = \text{Rs.}\tfrac{817}{4096}.$$

Again since R.1 produces R.$\frac{1}{16}$ at simple interest in 1 year, it produces R.$\frac{1}{16} \times 3$ or R.$\frac{3}{16}$ in 3 years. Therefore the difference between the simple and compound interest on R.1 for 3 years at $6\frac{1}{4}$ per cent. is R.$\frac{817}{4096}$ − R.$\frac{3}{16}$ = R.$\frac{49}{4096}$. Now to find the sum where such difference is Rs.6. 2 as. or Rs.$\frac{49}{8}$ we have

$$\text{R.}\tfrac{49}{4096} : \text{Rs.}\tfrac{49}{8} :: \text{R.}1 : \text{the required sum};$$

$$\therefore \text{ the required sum} = \text{Rs.}\frac{\cancel{49}\times\overset{512}{\cancel{4096}}}{\cancel{8}\times\cancel{49}}$$

$$= \text{Rs.}512.$$

Ex. 8. A person borrowed Rs.650 at 5 per cent. compound interest. He paid back at the end of the year Rs.220 to pay off the whole interest on his debt for that year and to reduce in part the principal. He did the same for the next two consecutive years. What sum will he have to pay at the end of the fourth year in liquidation of the remaining debt ?

5 per cent is $\frac{5}{100}$ or $\frac{1}{20}$. Therefore the interest for each period is $\frac{1}{20}$ of the principal. Hence we have

	Rs.	
	650	Original principal
	32·5	Interest for the 1st year
	682·5	Amount for the 1st year
	220	Sum repaid at the end of the 1st year
	462·5	Principal for the 2nd year
Subtracting	23·125	Interest for the 2nd year
we get	485·625	Amount for the 2nd year
Subtracting	220	Sum repaid at the end of the 2nd year
we get	265·625	Principal for the 3rd year
	13·28125	Interest for the 3rd year
	278·90625	Amount for the 3rd year
Subtracting	220	Sum repaid at the end of the 3rd year
we get	58·90625	Principal for the 4th year
	2·9453125	Interest for the 4th year
	61·8515625	Remaining debt
	16	
	13·6250000	
	12	
	7·500	

∴ the required sum = Rs.61. 13 as. $7\frac{1}{2}$ p.

EXERCISE LXII.

Find the compound interest on :

1. Rs.6000 for 3 years at 5 per cent. per annum.
2. Rs.1600 for 4 years at 10 per cent. per annum.
3. Rs.350 for 2 years at 4 per cent. per annum.
4. Rs.800 for 5 years at 3 per cent. per annum.
5. £1200 for 3 years at $2\frac{1}{2}$ per cent. per annum.
6. £675 for 4 years at 5 per cent. per annum.
7. £425. 10s. for 3 years at 4 per cent.
8. Rs.324. 8 as. for 4 years at 3 per cent.
9. £500 for 2 years at $2\frac{3}{4}$ per cent. payable half-yearly
10. £1000 for $1\frac{1}{2}$ years at 9 per cent. payable quarterly.
11. Rs.833. 10 as. 8 p. for 5 years at 10 per cent.
12. Rs.125 for $3\frac{1}{3}$ years at $4\frac{1}{4}$ per cent.

Find the amount on :

13. Rs.2000 for 4 years at 5 per cent.
14. £735 for 3 years at 4 per cent.
15. £1024 for 6 years at 10 per cent.
16. Rs.640 for $2\frac{1}{2}$ years at 3 per cent.
17. £960 for $3\frac{3}{4}$ years at $2\frac{1}{2}$ per cent.

18. What sum will amount to Rs.1352 in 2 years at 4 per cent. compound interest ?

19. A sum of money amounts to £5618 in 2 years at 6 per cent. compound interest ; find it.

20. What sum put out to compound interest for 3 years will produce Rs.41. 6 as. at 10 per cent. ?

21. A sum of money amounts to £1156 in 2 years at $6\frac{1}{4}$ per cent. compound interest ; find it.

22. The compound interest on a certain sum for 2 years at $8\frac{1}{3}$ per cent. is £6. 5s. ; find the sum.

23. Find the difference between the simple and compound interest on £560 for 3 years at 5 per cent.

24. The difference between the simple and compound interest on a certain sum for 2 years at 5 per cent. is R.1. 6 as. 6 p. ; find the sum.

25. A borrows a sum of Rs.840 at 4 per cent. simple interest, and lends it to B at the same rate but at compound interest ; how much will he gain by the transaction at the end of the third year ?

26. What sum will amount to Rs.9261 in 3 years at 5 per cent. compound interest ?

27. A person borrowed Rs.100 at 4 per cent. compound interest, and paid back Rs.25 at the end of each year. How much of his debt will remain at the end of the fourth year ?

28. A person lays by £500 a year, and deposits his savings in a bank at the rate of $4\frac{1}{2}$ per cent. compound interest. How much will he be worth at the end of the fourth year ?

29. A certain sum of money amounts to Rs.2756. 4 as. in 2 years, and to Rs.2894. 1 a. in 3 years, compound interest being reckoned in each case. Find the sum and the rate per cent.

30. A sum of money is put to compound interest ; the first year's interest is Rs.75, and the second year's is Rs.82. 8 as. ; find the interest for the third year.

CHAPTER XIII.

PRESENT WORTH AND DISCOUNT.

187. The Present Worth of a sum of money due at the end of a certain time is that sum which would just amount in that time at a given rate of Interest to the sum due.

188. Discount is the sum allowed for the payment of a certain sum before it is due.

Suppose A owes B Rs.210 to be paid at the end of a year. Suppose also that B requests A to make the payment at once as he cannot afford to wait for a year. It is clear that if A makes the payment at once he will not pay B the whole of Rs.210, but that he will pay him only such sum as will amount in one year to Rs.210 at the rate of interest which then prevails. If the rate of interest at the time be 5 per cent., A will give B Rs.200 because at that rate Rs.200 in 1 year will amount to Rs.210. Rs.200 here is then the present worth of Rs.210 due a year hence; and the portion cut off, *i.e.* Rs.10, is the discount allowed for making the payment at once.

189. It will be seen that the present worth and the discount together make up the debt due and that the discount is the interest on the present worth. Thus in the above example the present worth, which is Rs.200, and the discount, which is Rs.10, together make up the debt due, which is Rs.210. Also, Rs.10 is the interest on Rs.200 in 1 year at 5 per cent.

190. Questions in Discount mostly occur in connection with the payment of Bills of Exchange, generally called Bills. A bill is a document of agreement to pay a certain specified sum on a certain specified day. It is said to be *nominally* due on that day, whereas it becomes *legally* or *actually* due on the *third* day after that day, and these three days which are allowed by law as a matter of grace before payment can be enforced are called the *days of grace.*

Thus a bill drawn, *i.e.* written, on the 15th of March, undertaking to make the payment 3 months after that date, falls *nominally* due on the 15th of June, but it does not become *legally* due until the 18th of June. It may also be mentioned that if a bill appears to fall due on the 31st of a month which has only 30 days, or on the 29th, 30th, or 31st of February in a common year, or the 30th or 31st of February in a leap year, it is considered to be nominally due on the last day of the month, and therefore legally due on the 3rd of the next month.

191. When bills are discounted by bankers, they calculate and deduct interest instead of discount from the sum due. The sum so deducted by them is called *mercantile discount*, or false discount, as distinguished from *true discount*, which has been explained above, and which, as has been shown, is the interest not on the sum due but on its present worth. The difference between the false discount which a banker charges and the true discount is called the banker's gain.

Ex. 1. Find the present worth of Rs.833. 12 as. due 3 years hence at 5 per cent. per annum Simple Interest.

The interest on Rs.100 for 3 years is Rs.$5 \times 3 =$ Rs.15 ; therefore the amount of Rs.100 in 3 years will be Rs.100 + Rs.15 = Rs.115, *i.e.* Rs.100 is the present worth of Rs.115 due 3 years hence. To find therefore the present worth of Rs.833. 12 as. *i.e.*, of Rs.$\frac{3335}{4}$, we have

Rs.155 : Rs.$\frac{3335}{4}$ :: Rs.100 : the required present worth ;

$$\therefore \text{ the required present worth} = \text{Rs.}\frac{\overset{145}{\cancel{3335}} \times \overset{\overset{5}{\cancel{20}}}{\cancel{100}}}{\cancel{4} \times \underset{\underset{1}{\cancel{23}}}{\cancel{115}}}$$

$$= \text{Rs.}145. \times 5 = \text{Rs } 725$$

Ex. 2. Find the discount on Rs.734. 14 as. 9 p. due 6 months hence at $4\frac{1}{2}$ per cent. Simple Interest.

$$\text{Rs.}734.\ 14\text{ as. }9\text{ p.}=\text{Rs.}734\tfrac{59}{64}=\text{Rs.}\tfrac{47035}{64}.$$

The interest on Rs.100 for 1 year or 12 months is Rs.$4\frac{1}{2}$. Therefore the interest on Rs.100 for 6 months is $\frac{1}{2}$ of $4\frac{1}{2}=$ Rs.$2\frac{1}{4}$. Therefore Rs.100 will amount in 6 months to Rs.100 + Rs.$2\frac{1}{4}$ = Rs.$102\frac{1}{4}$, *i.e.* Rs.100 is the present worth of Rs.$102\frac{1}{4}$ due 6 months hence. Therefore Rs.$2\frac{1}{4}$ or Rs.$\frac{9}{4}$ is the discount on Rs.$102\frac{1}{4}$ or Rs.$\frac{409}{4}$ due 6 months hence. Therefore to find the discount on Rs.$\frac{47035}{64}$ we have

$$\text{Rs.}\tfrac{409}{4} : \text{Rs.}\tfrac{47035}{64} :: \text{Rs.}\tfrac{9}{4} : \text{the required discount ;}$$

$$\therefore\ \text{the required discount}=\text{Rs.}\frac{47035\times 9\times \overset{1}{\cancel{4}}}{64\times \underset{1}{\cancel{4}}\times 409}$$

$$=\text{Rs.}\frac{\overset{23}{\cancel{9407}}\times 5\times 9}{64\times \underset{1}{\cancel{409}}}$$

$$=\text{Rs.}\frac{23\times 5\times 9}{64}$$

$$=\text{Rs.}\frac{1035}{64}=\text{Rs.}16\tfrac{11}{64}$$

$$=\text{Rs.}16.\ 2\text{ as. }9\text{ p.}$$

The result may also be obtained by first finding the present worth and then subtracting it from the given sum.

Ex. 3. A bill of £45. 17s. was drawn on the 5th of June, at 4 months. It was discounted on the 24th of August at 4 per cent. simple interest. Find the discount upon the bill.

From the 5th of June to the 5th of October are 4 months. Therefore the bill falls nominally due on the 5th of October, and it becomes legally due on the 8th of October. We must therefore calculate the period between the 24th of August and the 8th of October. Leaving out the first date and including the last, we have

7 days of August,
30 „ September,
8 „ October,
45 days altogether.

We have now to find the discount for 45 days. To find the interest on £100 for 45 days, we have

$$365\text{ days} : 45\text{ days} :: £4 : \text{interest for 45 days.}$$

$$\therefore\ \text{Interest for 45 days}=£\frac{\overset{9}{\cancel{45}}\times 4}{\underset{73}{\cancel{365}}}=£\tfrac{36}{73}.$$

Therefore the amount of £100 in 45 days is £100 + £$\frac{36}{73}$ = £$\frac{7336}{73}$.

Therefore $\pounds\frac{36}{73}$ is the discount on $\pounds\frac{7336}{73}$ due 45 days hence. Hence, to find the discount on £45. 17s., *i.e.* on $\pounds\frac{917}{20}$, we have

$$\pounds\frac{7336}{73} : \pounds\frac{917}{20} :: \pounds\frac{36}{73} : \text{the required discount.}$$

$$\therefore \text{ the required discount} = \pounds\frac{\overset{1}{\cancel{917}} \times \overset{9}{\cancel{36}} \times \overset{1}{\cancel{73}}}{20 \times \underset{1}{\cancel{73}} \times \underset{\underset{2}{\cancel{1834}}}{\cancel{7336}}}$$

$$= \pounds\frac{9}{20 \times 2} = \pounds\frac{9}{40}$$

$$= 4\text{s. } 6\text{d.}$$

Ex. 4. What will a banker gain by discounting on the 15th of February at $3\frac{3}{4}$ per cent. a bill of £1853. 7s. $9\frac{1}{3}$d. drawn on the 29th of November, 1872, at 3 months ?

The bill falls nominally due on the 29th of February, 1873, which has only 28 days. Therefore the bill is nominally due on the 28th of February, and legally due on the 3rd of March. Now from the 15th of February to the 3rd of March there are 16 days. We have therefore to calculate the true and false discount for 16 days.

$$\pounds1853.\ 7\text{s. } 9\tfrac{1}{3}\text{d.} = \pounds1853\tfrac{7}{18} = \pounds\tfrac{33361}{18}.$$

Now the interest on £100 for 1 year, *i.e.* 365 days, is $\pounds\frac{15}{4}$.

$\therefore$ the interest on £100 for 16 days $= \pounds\frac{15 \times 16}{4 \times 365} = \pounds\frac{12}{73}$.

$\therefore$ the amount of £100 in 16 days $= \pounds100\frac{12}{73} = \pounds\frac{7312}{73}$.

Hence, to find the *true discount* on $\pounds\frac{33361}{18}$, we have

$$\pounds\frac{7312}{73} : \pounds\frac{33361}{18} :: \pounds\frac{12}{73} : \text{the true discount.}$$

$$\therefore \text{ the required discount} = \pounds\frac{33361 \times \overset{\overset{1}{\cancel{2}}}{\cancel{12}} \times \overset{1}{\cancel{73}}}{\underset{3}{\cancel{18}} \times \underset{1}{\cancel{73}} \times \underset{3656}{\cancel{7312}}}$$

$$= \pounds\frac{33361}{10968}$$

$$= \pounds3\frac{1}{24}$$

$$= \pounds3.\ 0\text{s. } 10\text{d.}$$

Again, to find the interest or banker's discount on $\pounds\frac{33361}{18}$, have $\pounds100 : \pounds\frac{33361}{18} :: \pounds\frac{12}{73}$: the banker's discount.

$$\therefore \text{ the banker's discount} = \pounds\frac{33361 \times \overset{\overset{1}{\cancel{2}}}{\cancel{12}}}{\underset{3}{\cancel{18}} \times 73 \times \underset{50}{\cancel{100}}}$$

$$= \pounds\frac{33361}{10950}$$

$$= \pounds3\frac{7}{150}$$

$$= \pounds3.\ 0\text{s. } 11\tfrac{1}{5}\text{d.}$$

$$\therefore \text{ the banker's gain} = \pounds3.\ 0\text{s. } 11\tfrac{1}{5}\text{d.} - \pounds3.\ 0\text{s. } 10\text{d.}$$

$$= 1\tfrac{1}{5}\text{d.}$$

EXERCISE LXIII.

Find the present worth of :

1. Rs.560 due 3 years hence at 4 per cent. simple interest.
2. Rs.640 due 2 years hence at $3\frac{1}{3}$ per cent. simple interest.
3. Rs.420 due 1 year hence at 5 per cent. simple interest.
4. £530 due $1\frac{1}{2}$ years hence at 4 per cent. simple interest.
5. £721 due 9 months hence at 4 per cent. simple interest.
6. Rs.585. 1 a. due 6 months hence at $3\frac{1}{2}$ per cent. simple interest.
7. £4221. 18s. 9d. due 3 months hence at 5 per cent. simple interest.
8. £251. 5s. due 2 months hence at 3 per cent. simple interest.
9. £92. 3s. due 90 days hence at 4 per cent. simple interest.
10. £61. 2s. 8d. due 30 days hence at 6 per cent. simple interest.

Find the discount on :

11. Rs.473 due 2 years hence at 5 per cent. simple interest.
12. £357 due 6 months hence at 4 per cent. simple interest.
13. £456 due 3 months hence at $5\frac{1}{3}$ per cent. simple interest.
14. £671 due 30 days hence at $3\frac{1}{3}$ per cent. simple interest.
15. £458. 13s. due 32 days hence at 6 per cent. simple interest.
16. £407. 13s. $1\frac{1}{3}$d. due 54 days hence at $3\frac{1}{2}$ per cent. simple interest.
17. £1218. 16s. 8d. due 20 days hence at $3\frac{1}{4}$ per cent. simple interest.
18. £915. 13s. due 28 days hence at $4\frac{1}{2}$ per cent. simple interest.
19. £731. 16s. due 15 days hence at 6 per cent. simple interest.
20. £1463 12s. due 18 days hence at 5 per cent. simple interest.

Find the discount on the following bills :

21. Rs.1830 drawn on the 4th of March at 3 months, and discounted on the 13th of May at 4 per cent. simple interest.

22. £919. 15s. drawn on the 12th of January of a common year at 4 months, and discounted on the 18th of March at 5 per cent. simple interest.

23. Rs.610. 5 as. 4 p. drawn on the 9th of June at 2 months, and discounted on the 3rd of July at 3 per cent. simple interest.

24. £1102. 18s. drawn on the 22nd of May at 5 months, and discounted on the 7th of August at $3\frac{1}{3}$ per cent. simple interest.

25. £733. 10s. 6d. drawn on the 30th of September at 3 months, and discounted on the 16th of November at $3\frac{3}{4}$ per cent. simple interest.

26. Find the difference between the true and false discount on a bill of £204. 10s. drawn on the 24th of March at 9 months, and discounted on the 22nd of May at 4 per cent.

27. If a debt of Rs.254 due on the 10th of November be paid on the 17th of June of the same year, what discount should be allowed at 4 per cent. ?

28. Find the banker's discount on a bill of £608. 6s. 8d. drawn on the 15th of December, 1895, at 90 days, and discounted on the 2nd of March of the following year at 4 per cent. simple interest.

29. A bill of £127. 10s. had 6 months to run, and the holder had it discounted at 4 per cent. for £124. 14s. 6d. How much more or less than his due did he receive ?

30. A bill for £606 was drawn on the 7th of January, 1900, at 5 months. It was discounted on the 29th of March at 5 per cent. simple interest. What did the banker gain by the transaction ?

CHAPTER XIV.

PROFIT AND LOSS.

192. *Profit* and *loss* are terms used in trade, the former when a commercial business brings in more than the outlay and the latter when it brings in less. Thus if a box bought for Rs.4 be sold for Rs.5 the transaction brings in a profit of R.1. If it be sold on the other hand for Rs.3. 8 as., it entails a loss of 8 as.

193. The terms *gain per cent.* and *loss per cent.* as used in commercial business denote the amount of gain or loss for every hundred of the outlay.

Ex. 1. A person bought a horse for Rs.84 and sold it for Rs.91; find his gain per cent.

Selling price of the horse = Rs.91
Cost price „ „ = Rs.84
By subtracting, Gain = Rs.7.

The person gains Rs.7 on an outlay of Rs.84. Therefore to find how much the man would gain for every 100 rupees of such outlay, we have

Rs.84 : Rs.100 :: Rs.7 : gain on Rs.100.

$$\therefore \text{ Gain on Rs.}100 = \text{Rs.}\frac{\overset{25}{\cancel{100}} \times \overset{1}{\cancel{7}}}{\underset{\underset{3}{\cancel{12}}}{\cancel{84}}} = \text{Rs.}\frac{25}{3}$$

$$= \text{Rs.}8\tfrac{1}{3}.$$

$\therefore$ $8\frac{1}{3}$ per cent. is the answer.

Ex. 2. A grocer bought 1 cwt. of tea for £7 and sold it at 1s. 6d. a lb.; find his whole gain and also his gain per cent.

$$1 \text{ cwt.} = 28 \times 4 = 112 \text{ lbs.}$$

$$\text{and } 1\text{s. } 6\text{d.} = 1\tfrac{1}{2}\text{s.} = \tfrac{3}{2}\text{s.}$$

$$\therefore\ 1 \text{ cwt. is sold for } 112 \times \tfrac{3}{2}\text{s.} = 168\text{s.} = £8.\ 8\text{s.}$$

$$\therefore\ \text{The grocer's whole gain} = £8.\ 8\text{s.} - £7$$
$$= £1.\ 8\text{s.}$$

And since he gains £1. 8s., *i.e.* £$\frac{7}{5}$ on an outlay of £7, to find his percentage of gain, we have

$$£7 : £100 :: £\tfrac{7}{5} : \text{gain on } £100.$$

$$\therefore\ \text{Gain on } £100 = £\frac{\overset{20}{\cancel{100}} \times \cancel{7}}{\underset{1}{\cancel{5}} \times \cancel{7}} = £20\,;$$

$$\therefore\ 20 \text{ per cent. is his gain.}$$

Ex. 3. A grocer bought a maund of sugar for Rs.8. 12 as.; how should he retail it so as to gain 25 per cent. on the whole?

To gain 25 per cent., he must realize Rs.125 for every Rs.100 of the outlay. Therefore to find how much he should sell his maund to gain 25 per cent. we have

$$\text{Rs.}100 : \text{Rs.}\tfrac{35}{4} :: \text{Rs.}125 : \text{selling price of the maund.}$$

$$\therefore\ \text{Selling price of the maund} = \text{Rs.}\frac{\overset{5}{\cancel{125}} \times 35}{4 \times \underset{4}{\cancel{100}}}$$
$$= \text{Rs.}\tfrac{175}{16}.$$

To find now the selling price per seer, we have (since 40 seers = 1 maund)

$$40 \text{ seers} : 1 \text{ seer} :: \text{Rs.}\tfrac{175}{16} : \text{selling price of 1 seer;}$$

$$\therefore\ \text{Selling price of 1 seer} = \text{Rs.}\frac{\overset{35}{\cancel{175}}}{16 \times \underset{8}{\cancel{40}}}$$
$$= \text{Rs.}\tfrac{35}{128}$$
$$= \tfrac{35}{8} \text{ as.} = 4\tfrac{3}{8} \text{ as.}$$
$$= 4 \text{ as. } 1\tfrac{1}{2} \text{ p.}$$

$$\therefore\ 4 \text{ as. } 1\tfrac{1}{2} \text{ p. is the retail price required per seer.}$$

Ex. 4. If mangoes be bought at 4 a pice and sold at 5 a pice, find the gain or loss per cent.

For 1 pice of outlay the man will buy 4 mangoes. Now to find the selling price of these 4 mangoes we have

5 mangoes : 4 mangoes : : 1 pice : the selling price of 4 mangoes.

$\therefore$ Selling price of 4 mangoes $= \frac{4}{5}$ pice.

∴ for every pice of outlay, the man suffers a loss of $1-\frac{4}{5}=\frac{1}{5}$ pice. Therefore to find the percentage of loss, we have

$$1 \text{ pice} : 100 \text{ pice} :: \tfrac{1}{5} \text{ pice} : \text{loss on } 100 \text{ pice}.$$

∴ Loss on 100 pice $=\frac{100}{5}$ pice $=20$ pice ;

∴ The loss is 20 per cent.

Ex. 5. A dairyman bought 70 seers of milk at 2 as. a seer, but 6 seers were lost by leakage ; at what price per seer must he sell the remainder so as to gain 20 per cent. on his whole outlay ?

The prime cost of 70 seers $=70\times 2$ as.

$=140$ as.

Since the dairyman has to gain 20 per cent. on the whole to find how much he must realize by sale we have

100 as. : 140 as. : : 120 as. : the required selling price ;

∴ The required selling price $=\frac{14\cancel{0}\times 12\cancel{0}}{1\cancel{0}\cancel{0}}$ as.

$=168$ as.

But as the dairyman has lost 6 seers by leakage, he has only 64 seers left, for which he must realize 168 as.

∴ each seer must be sold for $\frac{168}{64}$ as.

$=2$ as. $7\frac{1}{2}$ pies.

∴ 2 as. $7\frac{1}{2}$ pies is the selling price per seer.

EXERCISE LXIV.

1. A man bought a horse for Rs.85 and sold it for Rs.93. 8 as. ; how much per cent. did he gain ?

2. A man sold a book for Rs.1. 5 as., gaining thereby 5 per cent. ; what did it cost him ?

3. A man bought a ream of paper for Rs.1. 9 as. ; what should he sell it for to gain 4 per cent. ?

4. I bought cloth at 15 as. a yard, and lost 5 per cent. in selling it. How did I sell it ?

5. A man sold a horse for Rs.246 and lost $26\frac{1}{5}$ per cent. on what the horse cost him ; what was the original cost ?

6. If 6 per cent. be gained by selling an article for Rs.3. 5 as., what per cent. will be gained by selling it for Rs.3. 3 as. ?

7. If by selling an article for 1 guinea a shopkeeper gains 5 per cent., what per cent. will he lose by selling it for 9 florins ?

8. By selling wine at Rs.6 a gallon a merchant lost 25 per cent. ; at what price should he have sold it to gain 25 per cent. ?

9. If mangoes be bought at the rate of 3 for one anna, what must a dozen be sold for to gain 25 per cent. ?

10. A house was sold for Rs.2457, by which one-ninth of the price was lost ; what would have been the gain or loss if it had been sold for Rs.2754 ?

11. By selling a carriage for Rs.37. 6 as. 9 p. a man lost one-sixth of what it cost him ; how much would he have lost or gained by selling it for Rs.47. 6 as. 3 pies ?

12. A grazier sold 36 sheep for £74. 5s., and made thereby a profit of 10 per cent. What did he give for each sheep ?

13. A dealer buys coffee at £5. 15s. per cwt. and chicory at £2. 12s. 6d. per cwt. and mixes them in the proportion of 3 parts of chicory to 7 of coffee. Find at what price per cwt. he should sell the mixture so as to gain 20 per cent. on his outlay ?

14. A merchant bought 245 rupees worth of rice at 9 seers a rupee, and sold it at 7 seers a rupee ; find his whole gain and the gain per cent.

15. A merchant bought 1800 seers of Bajree at 12 seers a rupee ; at what rate should he sell it so as to gain 30 rupees on the whole ?

16. A person bought 42 sheep at the rate of Rs.3 each ; he lost 7 of them ; at what price should he sell the remaining stock in order to gain $11\frac{1}{9}$ per cent. on his original outlay ?

17. A milkman mixed a quantity of water with 60 seers of milk, at 2as. a seer. He then sold the mixture at 1 pice less than the cost per seer and gained R.1. 4 as. on the whole. What quantity of water did he mix ?

18. A person had goods worth Rs.150. He sold one-third of them at a loss of 5 per cent. ; what must he sell the remainder for so as to gain 5 per cent. on the whole ?

19. A person bought 40 watches at Rs.12. 8 as. each ; he sold 25 of them at a loss of 10 per cent. ; at what price must he sell the remainder so as to gain 20 per cent. on the whole ?

20. A shopkeeper purchased $250\frac{3}{4}$ yards of cloth for Rs.900 and paid expenses amounting to Rs.103 ; what must he charge per yard ? in order to make a profit of 50 per cent. ?

21. A tea dealer buys a chest of tea containing 2 maunds and 16 seers, at Rs.4. 2 as. a seer, and two chests more each containing 3 maunds and 24 seers, at Rs.4. 10 as. a seer ; at what price must he sell the whole in order to gain 576 rupees ?

22. A merchant purchases 231 gallons of spirits at Rs.10. 12 as. 4 p. per gallon ; 126 gallons at Rs.12. 11 as. 7 p. ; and 70 gallons at Rs.14. 8 as. 9 p. If he sell the mixture at Rs.13 per gallon, how much will he gain by the transaction ?

23. A merchant sold 13 cwt. 2 qrs. 9 lbs. of a commodity for £72. 17s. $7\frac{1}{2}$d., gaining thereby $3\frac{1}{2}$d. per lb. How much would he have gained per lb. if he had sold the commodity at £5. 12s. per cwt. ?

24. A merchant buys goods for £568. 4s., and sells half of them at a gain of 1d. in the shilling, one-third of them at a gain of 2d. in the shilling, and the remainder at a gain of £15. 15s. 8d. How much per cent. does he gain on the whole transaction ?

25. A dealer buys 10 horses at Rs.400 each, 8 horses at Rs.500 each, and 4 horses at Rs.600 each. He keeps the horses for 6 months, during which each costs Rs.15 a month, and then sells them all, clearing 10 per cent. on his original outlay after paying all his expenses. Find the selling price.

26. The manufacturer of an article makes a profit of 25 per cent. ; the wholesale dealer makes a profit of 20 per cent. : and the retail dealer makes a profit of 12 per cent. What is the cost to the manufacturer of an article which is retailed for Rs.56 ?

27. A merchant bought 81 marcals of rice at $6\frac{3}{4}$ padies for the rupee, and sold one-fourth at 8 as. 6 pies a padi, one-third at 4 padies for the rupee, and the remainder at 3 annas 9 pies a padi. What was the whole gain ?

28. A merchant buys 15 maunds of sugar at Rs.9. 8 as. per maund, 18 maunds at Rs.9 4 as. per maund, and 10 maunds at Rs.9 per maund. He pays Rs.4. 2 as. for carriage. He mixes the sugar. At what average price per maund must he sell the mixed sugar so as not to gain or lose ?

29. A dealer buys 60 books at the rate of R.1. 2 as. 4 p., and pays Rs.4. 12 as. 6 p. for book postage and R.1. 7 as. 6 p. for coolie hire. At what rate per book must he sell so as to make a net profit of Rs.15 ?

30. A draper buys cloth at such a price that, by selling it at 8s. 9d. per yard he will secure a profit of 5 per cent. upon his outlay. What percentage will be gained or lost upon the same cloth if it be sold at 7s. 9d. per yard ?

31. A man sells a watch for 30 p.c. more than the cost price, and allows a discount of 10 p.c. in the selling price for cash. If his profit be Rs.4. 4 as., what is the selling price of the watch ?

CHAPTER XV.

SQUARE MEASURE.

194. We propose to consider in this chapter only a few of the most common figures.

195. A rectangle is a figure of the form of a page of this book, such as the surface of a table, the walls of a house, the floor of a room, the sides of a box, etc.

196. The length and breadth of a rectangle is called its dimensions, and are measured by linear units, *i.e.* units of length, such as an inch, a foot, a metre, etc.

197. A square is a rectangle of equal dimensions, *i.e.* having the same length and breadth.

198. The perimeter of a rectangle is the sum of all its sides.

199. The area of a rectangle is the quantity of surface contained in it. It is measured by superficial or square units, such as a square inch (*i.e.* a surface of which the length is 1 inch and the breadth also 1 inch), a square foot (*i.e.* a surface of which the length is 1 ft. and the breadth also 1 ft.), etc.

200. To find the area of a rectangle.

Rule. Multiply the number of units expressing the length by the number of units expressing the breadth (the length and the breadth being expressed in the same denomination).

N.B.—Feet multiplied by feet give square feet, inches by inches give square inches, yards by yards give square yards, and so on.

On the other hand square feet divided by feet give feet, square inches divided by inches give inches, square yards divided by yards give yards, and so on.

Ex. 1. Find the area of the floor of a room 28 ft. long and 16 ft. broad.

Area $=28\times16$ or 448 sq. ft.

Ex. 2. A square plot of ground is 26 ft. 8 in. long; find its area.

26 ft. 8 in. $=26\frac{8}{12}$ ft. $=26\frac{2}{3}$ ft. $=\frac{80}{3}$ ft.

Since the piece is a square one, its length and breadth are the same.

$\therefore$ area of the plot $=\frac{80}{3}\times\frac{80}{3}$ or $\frac{6400}{9}$ sq. ft.
$=711\frac{1}{9}$ sq. ft.
$=711$ sq. ft. 16 sq. in.

Ex. 3. The perimeter of a square is 36 inches; find its area.

A square has four equal sides; therefore its perimeter is four times one of the sides.

Now the perimeter of the given square is 36 inches; therefore each of its sides is 36 in. $\div4=9$ in.

$\therefore$ the area of the square $=9\times9$ sq. in.
$=81$ sq. in.

Ex. 4. The area of the ceiling of a room is 768 sq. ft. If the length of the room be 32 ft., find its breadth.

The breadth = area ÷ length
= 768 sq. ft. ÷ 32 ft.
= 24 ft.

Ex. 5. Find the cost of glazing 5 windows, each 4 ft. long and 5 ft. wide at 9 pies per square foot.

Area of 1 window $= 5 \times 4$ or 20 sq. ft.
∴ area of 5 windows $= 20 \times 5$ or 100 sq. ft.

Therefore to find the cost of glazing 100 sq. ft. at 9 pies or $\frac{3}{4}$ anna per sq. ft. we have

1 sq. ft. : 100 sq. ft. :: $\frac{3}{4}$ a. : required cost;

∴ the required cost $= \frac{\overset{25}{\cancel{100}} \times 3}{\cancel{4}}$ as. $= 25 \times 3$ as.
= 75 as. = Rs.4. 11 as.

Ex. 6. Find the cost of covering the floor of a room 30 ft. long, 25 ft. wide, with carpet 2 ft. 6 in. wide at 2 as. 3 p. a yard.

Area of the floor $= 30 \times 25$ sq. ft. = 750 sq. ft.;
∴ area of the carpet required is 750 sq. ft.
But the width of the carpet = 2 ft. 6 in.
$= 2\frac{1}{2}$ ft.
$= \frac{5}{2}$ ft.;
∴ the length of the carpet = 750 sq. ft. ÷ $\frac{5}{2}$ ft.
= 300 ft. = 100 yds.

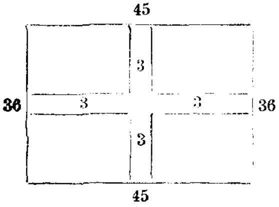
Therefore to find the cost of 100 yds. at 2 as. 8 p. or $\frac{8}{3}$ as. a yard we have

1 yd. : 100 yds. :: $\frac{8}{3}$ as. : cost of the carpet;

∴ cost of the carpet $= \frac{100 \times 8}{3}$ as. $= \frac{800}{3}$ as.
$=$ Rs.$\frac{800}{3 \times 16}$ = Rs.$\frac{50}{3}$
= Rs.16. 10 as. 8 p.

Ex. 7. A rectangular plot of ground 45 ft. long and 36 ft. broad has two paths each 3 ft. wide, running midway within it, the one parallel to the length and the other parallel to the breadth. How many flagstones each 9 in. wide, 6 in. long, will be required to pave the paths?

It will be seen that since the two paths cross one another, a small surface 3 ft. by 3 ft. is common to both. Therefore in finding the total surface of both the paths, this area, *i.e.* 9 sq. ft., should be taken into account only once, *i.e.* it should be omitted from the area of one of the paths.



Area of the lengthwise path

$= 45 \times 3$ sq. ft. $= 135$ sq. ft.

Area of the breadthwise path

$= (36 \times 3 - 9)$ sq. ft.

or $(36 - 3) \times 3$ sq. ft.

$= 33 \times 3$ sq. ft.

$= 99$ sq. ft.

$\therefore$ the total area of the paths

$= 135$ sq. ft. $+ 99$ sq. ft.

$= 234$ sq. ft.

Area of each flagstone $= 9 \times 6$ sq. in.

$= 54$ sq. in. $= \frac{54}{144}$ sq. ft.

$= \frac{3}{8}$ sq. ft.;

$\therefore$ the number of flagstones required $= 234 \div \frac{3}{8} = \frac{\overset{78}{\cancel{234}} \times 8}{\cancel{3}}$

$= 78 \times 8 = 624.$

Ex. 8. A room is 26 ft. long, 18 ft. broad and 8 ft. high. Find the area of the surface of the walls.

There are 4 walls, 2 lengthwise and 2 breadthwise.

Area of each lengthwise wall $= 26 \times 8$ sq. ft. $= 208$ sq. ft.;

$\therefore$ area of the two lengthwise walls $=$ twice 208 sq. ft. $= 416$ sq. ft.

Area of each breadthwise wall $= 18 \times 8$ sq. ft. $= 144$ sq. ft.

$\therefore$ area of the two breadthwise walls $=$ twice 144 sq. ft. $= 288$ sq. ft.;

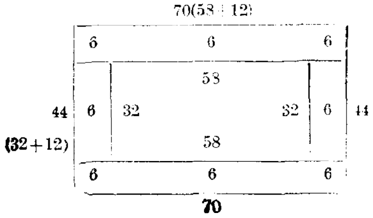
$\therefore$ total area of four walls $= (416 + 288)$ sq. ft. $= 704$ sq. ft.

Or thus: If the 4 walls be supposed to be placed in a straight line in continuation of one another, they will form one wall 8 ft. high and $26 + 26 + 18 + 18 = 88$ ft. long. And the area of the four walls being the same as the area of this one wall, made up of the four, we have

area of 4 walls $= 88 \times 8$ sq. ft.

$= 704$ sq. ft.

Ex. 9. A rectangular hall 58 ft. long and 32 ft. broad is surrounded by a verandah 6 ft. wide. Find the area of the verandah.



The given wall and the verandah round it together form another rectangle of which the length is $58 + 12 = 70$ ft. and the breadth $32 + 12 = 44$ ft. And it is obvious that the area of the verandah is equal to the difference between the areas of the larger rectangle thus formed and the given rectangular hall.

∴ area of the verandah $= 70 \times 44$ sq. ft. $- 58 \times 32$ sq. ft.
$= 3080$ sq. ft. $- 1856$ sq. ft.
$= 1224$ sq. ft.

Or we may thus proceed : If the lengthwise sides of the given hall be produced both ways, they will divide the verandah into 4 rectangles, two of which will be 70 ft. by 6 ft. each, and the other two 32 ft. by 6 ft. each.

∴ the area of the whole verandah
$= 2 \times 70 \times 6$ sq. ft. $+ 2 \times 32 \times 6$ sq. ft.
$= 840$ sq. ft. $+ 384$ sq. ft. $= 1224$ sq. ft.

Ex. 10. A room is 27 ft. 5 in. long, 22 ft. 7 in. broad and 9 ft. high. It has 3 windows each 5 ft. by 4 ft., 2 doors each 6 ft. by 3 ft., and a fire-place $5\frac{1}{2}$ ft. long by 4 ft. high ; find the cost of painting the walls at 1s. 6d. a square foot.

Area of the 4 walls $= 2 \times (27$ ft. 5 in. $+ 22$ ft. 7 in.$) \times 9$ ft.
$= 2 \times 50 \times 9$ sq. ft.
$= 900$ sq. ft.
Area of the 3 windows $= 3(5 \times 4)$ sq. ft. $= 60$ sq. ft.
Area of the 2 doors $= 2(6 \times 3)$ sq. ft. $= 36$ sq. ft.
Area of the fire-place $= 5\frac{1}{2} \times 4$ sq. ft. $= 22$ sq. ft.
Total area to be deducted $= 60$ sq. ft. $+ 36$ sq. ft. $+ 22$ sq. ft.
$= 118$ sq. ft.
∴ Total area to be painted $= 900$ sq. ft. $- 118$ sq. ft.
$= 782$ sq. ft.

Now 1s. 6d. $= \frac{3}{2}$s. ;
∴ total cost of painting $= 782 \times \frac{3}{2}$s.
$= 1173$s.
$= £58.\ 13$s.

Ex. 11. The cost of a carpet 2 ft. wide required for a square room at 1 a. 6 p. a foot amounts to Rs.9. 3 as. Find the length and breadth of the room.

Since the cost of the carpet at 1 a. 6 p. or $\frac{3}{2}$ as. a foot is Rs.9. 3 as. or 147 as., to find the length of the carpet we have

$\frac{3}{2}$ as. : 147 as. :: 1 ft. : the length of the carpet ;

∴ the length of the carpet $= \dfrac{\overset{49}{\cancel{147}} \times 2}{\underset{1}{\cancel{3}}}$ ft.
$= 49 \times 2$ ft. $= 98$ ft.

Since the carpet is 2 ft. broad,
the area of the carpet $= 98 \times 2$ sq. ft. $= 196$ sq. ft. ;
∴ the area of the floor is also 196 sq. ft.

But the floor is in the form of a square ;
∴ each side of the floor $= \sqrt{196}$ ft. ;
$= 14$ ft.

∴ the length and breadth of the room are 14 ft. each.

EXERCISE LXV.

1. Find the area of a rectangle 29 ft. long, 17 feet broad.

2. Find the perimeter of a rectangle 36 ft. long, 21 ft. broad.

3. A rectangular hall is 32 ft. long, 28 ft. broad; how many square yards of carpet will be required to cover the floor?

4. A plot of ground is 12 yards square; find its area.

5. The area of a rectangle is 364 sq. yds. Its breadth is 39 ft. Find its length.

6. What length of carpet 2 ft. 6 in. wide will be required for a room 24 ft. long and 18 ft. broad?

7. How many square yards of carpet will be required for a square room 23 ft. long?

8. Find the area of the ceiling of a room 21 ft. long and 16 ft. 8 in. broad.

9. Find the area of the walls of a room 27 ft. long, 18 ft. broad, and 7 ft. high.

10. What length of paper 2 ft. 3 in. wide will be required to paper a room 31 ft. long, 19 ft. broad, and 8 ft. high?

11. Find the cost of painting a room 22 ft. 7 in. long, 17 ft. 5 in. wide, and 7 ft. 6 in. high, at 1 a. 3 p. a square yard.

12. If it requires $163\frac{1}{3}$ yds. of carpet, 2 ft. 6 in. wide, to cover the floor of a square hall, find its length and breadth.

13. How many flagstones each 1 ft. 3 in. long and 1 ft. 1 in. broad will be required to pave a rectangular courtyard 30 feet long and 13 ft. broad?

14. A rectangular courtyard 54 ft. long and 41 ft. broad has two paths each 3 ft. wide, one parallel to the length and the other parallel to the breadth; find the area of the paths.

15. A square courtyard 47 ft. long is surrounded by a path 2 ft. 6 in. wide; find the area of the path.

16. A room is 36 ft. 8 in. long, 23 ft. 4 in. wide, and 8 ft. high; it has a doorway 6 ft. by 5 ft., and 3 windows each 4 ft. by 3 ft. Find the cost of papering the walls and the ceiling of the room at 1 a. 6 p. a square yard.

17. A piece of land is 11·916 poles broad; how long must it be to contain an acre?

18. How many paving stones, each measuring 14 in. by 12 in. are required to pave a verandah 70 ft. long and 9 ft. broad?

19. The perimeter of a square room 9 ft. high is 92 ft. Find the area of the walls, the floor, and the ceiling of the room.

20. In a rectangular area 100 yds. long and 50 yds. broad there are two paths crossing one another, each parallel to one side of a rectangle, and each 4 yds. broad. Find the cost of paving the area with stone at 12 as. per square yard, and of covering the paths with gravel at 6 as. per square yard.

21. A rectangular box is 5 ft. long, 4 ft. broad and 3 ft. high on the outside; what will it cost to paint the whole outer surface at 9 pies a square foot?

22. The area of the walls of a room is 420 sq. ft. The length of the room is 17 ft. 5 in. and the breadth 12 ft. 7 in.; find its height.

23. The cost of painting the walls of a room at 1 a. 6 p. a sq. yard is Rs.4. 11 as. The breadth of the room is 11 ft. and its height 7 ft. 6 in. Find the cost of carpeting the room at 2 as. a sq. ft.

24. A room whose height is 11 ft. and length twice its breadth takes 143 yards of paper 2 ft. wide for its four walls; how much carpet will it require?

25. A room 10 ft. 6 in. high, 22 ft. long and 14 ft. broad is painted up to one-third of the height, and the remaining two-thirds papered. The painting is charged at $7\frac{1}{2}$d. per sq. yd., the paper costs 5s. 2d. per sq. yd., and the work of papering is charged at 2d. per sq. yard. How much will the whole cost amount to?

26. It costs £643. 10s. 9d. to level and turf a square cricket ground at 9d. per sq. yd. What will it cost to enclose it with an iron railing at 7s. 6d. per yard?

27. The sides of two squares contain 77 yds. 1 ft. 9 in. and 7 yds. 2 ft. 4 in. respectively. Find the side of a square whose area is equal to the sum of the areas of the two squares.

28. A rectangular grass plot measures 320 yds. by 160 yds.; all round it is a gravel path 6 ft. broad. The price for making the grass plot is 6d. per sq. yd. What must be the price of the gravel path per sq. yd. that the path may cost £1183. 4s. less than the grass plot?

29. A person desires to paper his room with postage stamps. The room is 14 ft. 9 in. long, 9 ft. 3 in. broad and 10 ft. 6 in. high. It contains two windows, each $5\frac{1}{2}$ ft. by 4 ft., and 3 doors, each 5 ft. by 3 ft. A postage stamp is $\frac{15}{16}$ in. long and $\frac{3}{4}$ in. broad. Find the number of postage stamps required to cover the room.

30. The length of a rectangular field is twice its breadth. If the rent of the field at £3. 7s. 6d. an acre is £151. 17s. 6d., find the cost of surrounding it with a fence at $4\frac{3}{4}$d. per yard.

MISCELLANEOUS EXAMPLES.

1. Write down the greatest and least numbers of four digits which you can form with the digits 3, 6, 0 and 4; and find the product of their sum and difference.

2. The product of two numbers is 1270374 and half of one of them is 3129 ; what is the other ?

3. Multiply £63. 8s. $0\frac{3}{4}$d. by 43.

4. Divide £43009. 9s. 4d. by 64.

5. A *number diminished by one-tenth of itself* is divided by 1249 giving 144 as the quotient ; what is the number ?

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6. The sum of the two digits which form a number is 16 ; if the ten's digit exceed the unit's digit by 2 find the number.

7. Multiply by 179 the difference between the second power of 72 and the third power of 27.

8. Find what will remain after subtracting 213 as often as possible from 83216.

9. How much will a man save out of an annual income of £706, who spends a guinea and a half a day ?

10. Determine the speed in miles an hour of a train which goes a mile in 48 seconds and the number of revolutions made in a minute by a wheel of the engine 8 ft. 2 in. in diameter taking the circumference of a circle as $3\frac{1}{7}$ of the diameter.

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11. Find the number of hours from noon on December 11th, 1769, *to noon on December 11th, 1869. What would* be the amount of one farthing for each hour ?

12. How many payments of Rs.108. 10 as. 9 p. each can be made with Rs.1250, and what sum would be left ?

13. Find the least sum of money that can be exactly distributed either in half-crowns or in half-guineas.

14. Simplify $36\frac{1}{6} \div \{15\frac{2}{3} - (3\frac{7}{8} + 2\frac{3}{4})\}$.

15. What part of a sovereign is three pence three farthings and what decimal fraction of a pole is an inch ?

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16. Divide 2829 ft., 3036 sq. ft. and 7383 cub. ft. each by 23 ft.

17. Find the contents of the smallest cask that can be filled by an exact number of any one of the measures $\frac{1}{2}$ pint, $\frac{1}{2}$ gallon, 3 gallons, 5 gallons and 9 gallons.

18. Divide $\frac{7}{16}$ of $\frac{9}{11}$ by $2\frac{1}{3}$ of $1\frac{1}{4}$. How many square yards are there in the fraction of an acre which the result represents ?

19. Find the value of $\dfrac{\frac{1}{3} \text{ of } 2{\cdot}179 - \frac{5}{8} \text{ of } {\cdot}8684}{\dfrac{2\frac{1}{3}}{5} - \dfrac{2}{5\frac{5}{8}} + \dfrac{1\frac{5}{8}}{4\frac{5}{7}}}$.

20. If 22 yards = 1 chain and 4840 sq. yds. = 1 acre, how many square chains are there in 5 acres ?

21. Find the smallest number which when divided by 18, 24 and 32 gives in each case the remainder 1.

22. Find by practice the value of 3 lbs. 4 oz. 7 dwts. at £4. 2s. 6d. per lb.

23. Find the value of $\dfrac{5\frac{3}{4} - \frac{3}{7} \text{ of } 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{4}{25}}{\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}}$.

24. Find the value of ·01625 of £204. 3s. 4d. and reduce 8 lbs. 5 oz. 14 drs. to the decimal of a quarter.

25. In a manufactory there are 273 workmen, of whom 100 are paid each 12s. 6d., 100 are paid 15s. 4d. a week and the remainder 16s. 8d. Find the sum expended weekly in wages.

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26. Which of the following divisions can be effected ?

(1) 54945 as.÷495 as. (2) 54945÷495 as. (3) 54945 as.÷495. Obtain each possible quotient.

27. Find as a fraction in its lowest terms the difference between $\frac{1}{8900}$ and $\frac{1}{979}$ and then express that difference by a recurring decimal.

28. Find the prime factors of 72072.

29. How many persons can be paid 11s. 6d. each out of a sum of £170. 10s. 2d. ? And if the balance be also distributed equally among, them, how much more will each receive ?

30. Simplify $\dfrac{13}{29}$ of $\dfrac{7 \cdot \dot{1} - 3 \cdot \dot{6}}{2 \cdot 0\dot{5} + 2 \cdot 9\dot{4}}$ of $\dfrac{7 \cdot 25 \text{ of } 3 \cdot 6}{10 \cdot 725}$.

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31. What is the smallest number which when subtracted from 99099 will make it exactly divisible by 909 ?

32. Find the G.C.M. of 1071, 1092 and 2310.

33. Reduce to its simplest form $\dfrac{6 - \frac{1}{6}}{120 - \frac{1}{120}} \times \dfrac{22 - \frac{1}{22}}{51 - \frac{1}{51}} \times \frac{156}{161}$.

34. Express a guinea as a decimal of a sovereign, and a sovereign as a decimal of a guinea.

35. A man walks a certain distance and rides back in 3 hours 45 minutes. He could ride both ways in $2\frac{1}{2}$ hours. How long would it take him to walk both ways ?

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36. In a short division the successive divisors are 5, 4 and 3, and the respective remainders are 4, 1 and 0 ; if the quotient be 113, find the dividend.

37. Find the smallest number which must be added to four millions so as to make the sum exactly divisible by one hundred and twenty thousand.

38. The L.C.M. of two of the three given numbers is 48, and the L.C.M. of one of these two and the third number is 36; find the L.C.M. of the three numbers.

39. Express in seconds $\frac{7}{13}$ of a year, taking the length of a year as 365 days 6 hours.

40. Divide £29 into an equal number of half-sovereigns, crowns, half-crowns, shillings, sixpences and fourpences.

41. Find the G.C.M. and the L.C.M. of 157 days 7 hours 4 minutes 7 seconds, and 243 days 2 hours 11 minutes 49 seconds.

42. Find the value of $\frac{2\cdot25}{\cdot25}+\frac{2}{1\cdot25}+\frac{\cdot09}{\cdot00625}$.

43. A mixture is made of 297 gallons at 17s. 9d. per gallon, 132 gallons at 22s. 6d. per gallon, and 121 gallons at 19s. 4d. per gallon. Find the value of a gallon of the mixture.

44. If two persons together can perform a piece of work in $7\frac{1}{2}$ hours and one of them alone in 10 hours, how long will the other take to do it alone?

45. If the carriage of 150 feet of wood that weighs 3 stone per foot cost £3 for 40 miles, how much will the carriage of 54 feet of marble, weighing 8 stone per foot, cost for 25 miles?

46. The quotient arising from the division of 6739546 by a certain number is 1559, and the remainder is 3107; find the divisor.

47. Multiply $\frac{\text{17 yds. 2 ft. 3 in.}}{\text{11 yds. 2 ft. 10 in.}}$ by $\frac{\text{7 wks. 6 dys. 4 hrs.}}{\text{11 wks. 5 dys. 18 hrs.}}$.

48. Divide ·00729 by ·2735 correctly to 7 places of decimals.

49. Reduce to a decimal $\frac{\cdot04275}{3\cdot0\dot{5}}\times\frac{4\cdot21\dot{6}}{\cdot\dot{3}4\dot{2}}\times\frac{2\cdot\dot{7}}{1\cdot5318}$.

50. A clock which loses 1 minute in every two hours was set right at noon on Sunday; what will be the true time on the next Tuesday at night when the clock indicates half-past eleven?

51. Which is greater $\frac{1}{19}$ of a pound or $\frac{1}{20}$ of a guinea, and by how much?

52. Find the value of $\cdot0\dot{0}01122334455667778\dot{9}$ of 4 miles 4 furlongs.

53. Find the difference between ·01093125 of a mile and 3·5 of a pole.

54. Simplify $\dfrac{\frac{13}{42}\times\frac{1}{6}-\frac{11}{28}\times\frac{1}{9}}{\frac{16}{42}\times\frac{1}{6}-\frac{13}{28}\times\frac{1}{9}}$.

55. A man pays for insurance on his life 15 per cent. on his gross income; and after paying income-tax on the remainder at 2 pies in the rupee he has Rs.605 10 as. left. What is his gross income?

56. Divide the sum of $\frac{1}{15}$, $\frac{5}{24}$, $\frac{11}{36}$ and $\frac{7}{40}$ by the product of $\frac{11}{25}$ and $7\frac{11}{12}$.

57. Divide the product of 28·8 and ·0595 by 9520.

58. If 15 men can reap 20 acres of corn in 6 days working 14 hours a day, how many men must be employed to assist 10 other men to reap 6 acres in $1\frac{3}{4}$ days working 8 hours a day ?

59. A clock which was $1{\cdot}\dot{4}$ minutes fast at a quarter to 11 P.M. on December 2 was 8 minutes slow at 9 A.M. on December 7 ; when was it exactly right ?

60. If £24. 7s. $10\frac{1}{2}$d. be paid as income-tax on an income of £650. 10s., what ought to be paid at the same rate on an income of £2450. 6s. 8d. ? And at what rate in the pound is the tax levied ?

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61. Find the simple interest on £207. 12s. 6d. for $21\frac{1}{2}$ years at 3 per cent. per annum.

62. Find the amount of £1000 in 6 years at 5 per cent. compound interest.

63. The sum of £328. 3s. $6\frac{1}{2}$d. is to be divided between four men A, B, C and D in such proportion that for every £3 given to A, B is to receive £5, C £8 and D £9. What sum did each receive ?

64. If a cloth 4 yds. long and 15 in. wide cost Rs.3. 5 as., how much should you give for a cloth 19 yds. long, 12 in. wide, and every square inch of which is worth $\frac{3}{5}$ths of the value of a square foot of the former.

65. A rectangular hall is 25 ft. 4 in. long, 19 ft. $1\frac{1}{2}$ in. wide How many tiles each $9\frac{1}{2}$ in. long and $4\frac{1}{4}$ in. wide will be required to pave it ?

66. Simplify $\dfrac{8\frac{11}{63}}{\frac{7}{8}\text{ of }2{\cdot}25}$ of $\dfrac{{\cdot}1625}{\frac{1}{15}\text{ of }5\frac{4}{7}}\div\left(\frac{2}{21}+\frac{7}{81}\right)$

67. A person received on the death of his aunt $\frac{1}{10}$ of her property and spent ${\cdot}\dot{5}\dot{4}$ of it in paying off his debts ; what fraction of his aunt's property did he then possess ?

68. What sum of money put out at simple interest for $3\frac{1}{2}$ years at $4\frac{1}{4}$ per cent. will amount to £1497. 4s. 1d. ?

69. Find the difference between the simple and compound interest on £7466. 13s. 4d. for 3 years at $3\frac{3}{4}$ per cent. per annum.

70. If a tax of 12 per cent. on the income of a country brings in £520000, how much will an income-tax of 5d. in the pound bring in ?

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71. Find the least number which can be divided by 6, 9, 12, 15, 21 respectively, with remainder equal to their greatest common measure.

72. Seven bells begin to strike simultaneously and strike at intervals of 2, 3, 5, 15, 21, 65, 77 seconds respectively. After what time will they again strike simultaneously, and how often will each have struck ?

73. Divide £25. 7s. 6d. between 4 men, 3 women and 2 boys, giving each woman a fourth less and each boy a half less than the share of each man.

74. A work can be completed in 36 days by 30 men working 6 hours a day ; in what time would 54 men working 9 hours a day complete it ; supposing that in the longer days a man does only $\frac{4}{5}$ per hour of what he does per hour in the shorter days ?

75. A hare starts to run when a dog is 44 yds. off at 12 miles per hour. After half a minute the dog sees her and pursues at 16 miles per hour. How soon will he catch her ?

76. If it costs Rs.74. 8 as. to keep a horse for a year when gram is Rs.12 a palla and Rs.86. 10 as. 8 p. when gram is Rs.16 a palla, find how much gram the horse eats per day.

77. A grocer mixes 4 kinds of tea in the proportions of 1, 1·5, 2, 2·5 parts by weight of the several kinds respectively. Required the number of pounds of each kind in one hundred-weight of the mixture.

78. If it cost Rs.6. 4 as. to cover the walls of a room with paper 21 inches wide at 7 pies per yard, what would it have cost if the paper had been an inch narrower and a pie less per yard.

79. Find the discount on £808. 1s. 4d., due 3 years 9 months hence at 4 per cent. simple interest.

80. A man allows to his agent 5 per cent. on his gross income for the expense of collecting his rents. He spends $\frac{1}{7}$ of his net income in assuring his life, and this part of his income is in consequence exempt from income-tax. The income-tax being 10d. in the pound, and his income-tax amounting to £41. 6s. 10d., find his gross income.

81. Express $\frac{5}{8}$ of $\frac{11}{3}$ of $\frac{2}{7}+\frac{6}{5}\times\frac{1}{3}\div\frac{3}{4}-\frac{1}{30}$ of $31\frac{1}{7}$ as a vulgar fraction in its simplest form.

82. Add $1{\cdot}0\dot{6}$ to $\cdot4\dot{5}\dot{6}$ and divide the sum by $\cdot5\dot{0}7744\dot{1}$.

83. The sum of the ages of A and B is now 60 years, and their ages 10 years ago were as 5 : 3. Find their present ages.

84. A clock which gains 17 seconds per hour was 4 minutes 15 seconds slow at 6 A.M. ; when did it indicate right time ?

85. By selling sugar at 3 as. 9 p. a seer a grocer lost 12 per cent. What would he have gained or lost per cent. if he had raised the price to 4 as. $8\frac{1}{4}$ p. a seer ?

86. A square is made of two figures, one of which is 16 ft. square, and the other contains 144 sq. ft. ; find its dimensions.

87. Divide 5·05 into two parts, of which one shall exceed the other by 1·105.

88. Two watches are together at 12 o'clock ; one gains 75 seconds per hour and the other loses 45 seconds. When will they be together again at 12 ?

89. A grocer bought equal quantities of coffee and chicory at the respective rates of 1s. 2d. and 6d. a pound ; he mixes the two and sold the mixture at 1s. 3d. a pound ; how much per cent. did he gain ?

90. The difference between the simple interest and discount on a sum of money for 4 years at 5 p.c. simple interest is £33. 6s. 8d. ; find the sum.

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91. Express 364, 2520 and 5445 as products of powers of prime factors.

92. Find the value of

$$(1)\ \frac{\frac{2}{3}+\frac{4}{5}\text{ of }\frac{5}{9}-\frac{8}{21}}{1+\frac{2}{3}\times\frac{3}{7}-\frac{5}{9}};\qquad (2)\ \frac{5\frac{2}{5}}{7\frac{1}{2}}\text{ of }\frac{21{\cdot}25}{{\cdot}046875}.$$

93. What must be the gross produce of an estate that after paying ten per cent. income-tax and a rate of 2s. 1½d. on £1 on the residue there may remain £2574 per annum ?

94. A person with a monthly income of Rs.264 spends as much in 4 months as he earns in three. After 12 years he divides his savings amongst three children in such a manner that the eldest has twice as much as the second and thrice as much as the youngest ; how much did each receive ?

95. If the alloy in silver coin be $\frac{1}{21}$ of its weight, what would a crown be worth if it were all silver ?

96. If a man receives on a fourth of his capital 3 per cent., on two-thirds 5 per cent., on the remainder 11 per cent., what percentage does he receive on the whole ?

97. A man left £36,288 to be divided between his two sons in proportion to their ages at the time of his death, which were 19 and 23 years ; what sum did each receive ? What difference would it have made to each if the father had lived 3 years longer ?

98. Find the number of yards in the perimeter of a square field whose area is 119 acres 4 poles.

99. A certain number of articles were bought for £25. 7s. ; the total number of articles was the same as the number of farthings each cost. How many articles were bought, and what was the price of each ?

100. The rent of a house is £200 per annum, due by half-yearly payments on Jan. 1st and July 1st, and no payment has been made since the beginning of the last year. What sum will be due at the beginning of the next year, interest being added to arrears, at the beginning of each half-year, at the rate of 5 per cent. per annum ?

101. A fruiterer has 889 Spanish and 1379 Maltese oranges; he puts them into boxes, keeping the two kinds of oranges separate, and placing the same number in each box. If the number in each box is the greatest possible, find it.

102. A fraction which has the sum of its numerator and denominator equal to 100 assumes the value of $\frac{1}{2}$ if both the numerator and the denominator be increased by 1; find the fraction.

103. A can walk 11 miles in 4 hours, and his rate is $\frac{1}{2}$ as fast again as B's. How long would it take B to walk 88 miles?

104. A coal-dealer sells on 4 days 15·75 tons, 9·25 tons, 31·625 tons, $18{\cdot}\dot{3}$ tons; what is his average daily sale?

105. A and B travel together 120 miles by rail. A takes a return ticket for which he has to pay one fare and a half. Coming back they find that A has travelled cheaper than B by 4 as. 2 p. for every 100 miles. Find the fare per mile.

106. The price of oats being 30s. per quarter, it costs 17s. 6d. per week to keep a horse; if oats cost only 26s. per quarter, the expense would be 16s. $2\frac{1}{4}$d.; what quantity of oats does a horse eat per year?

107. A sum of money was lent at simple interest on the first of January, 1861, at $2\frac{1}{2}$ per cent.; in what year will the total interest first exceed the capital lent?

108. A bill for £1000 at 90 days' sight has been presented for acceptance on February 10th, 1896, and on March 13th is discharged at 5 per cent. per annum; how much will the discounter pay for the bill?

109. What fraction, when multiplied by itself, produces $\dfrac{4880\frac{1}{3}}{5043}$?

110. A debt of £18. 15s. 10d. is paid in crowns, shillings and pennies, whose numbers are proportional to 3, 2 and 1. Find the number of each coin.

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111. A man died in 1867 aged 92, his son died in 1823 at the age of 17. How old was the father when the son was born?

112. By what factor less than 1000 must 4389 be multiplied so that the last three figures of the product may be 438?

113. Complete division given below by filling in numbers omitted in first and second lines.

5) ______
7) ______ remainder 2.
1350 remainder 3.

114. Two friends during a walk take steps of $2\frac{22}{49}$ ft. and $2\frac{23}{56}$ ft.; if they start in step, how far will they have walked before they are in step again, and how many steps will each have taken?

115. A grocer mixed four kinds of tea in the proportion of 3, 4, 5, 6 parts by weight, and at the respective prices of 3s., 2s. 6d., 2s., 1s. 6d. per lb. How should he retail it in order to realize 25 per cent. on his outlay ?

116. A ship sails with a supply of biscuit for 60 days, at a daily allowance of 1 lb. a head ; after being at sea 20 days she encounters a storm, in which 5 men are washed overboard, and damage sustained that will cause a delay of 24 days, and it is found that each man's allowance must be reduced to five-sevenths of a pound. Find the original number of the crew.

117. Of the boys in a school one-third are over 15 years of age, one-third between 10 and 15. A legacy of £100 was to be exactly divided amongst them by giving 10s. to each boy over 15, 6s. 8d. to each between 10 and 15, and 3s. 4d. to each of the rest. How many boys are there in the school ?

118. The interest on a certain sum for one year is Rs.12. 8 as., and for 2 years is Rs.25. 8 as. ; find the sum and the rate per cent.

119. Find the length of the side of a square whose area is equal to that of a rectangle, the sides of which are 11·68 ft. and 6·57 ft.

120. When the rate of exchange is 1s. $1\frac{1}{8}$d. per rupee, what is the value in rupees, annas and pies of goods worth £15. 9s. 9d. ?

121. Simplify

$$(1)\ 2\tfrac{5}{6} \text{ of } 1\tfrac{4}{5} \div 6\tfrac{3}{8} \times 1\tfrac{1}{4}.$$

$$(2)\ \frac{\cdot25 \times \cdot25 - \cdot125 \times \cdot125}{\cdot25 + \cdot125}.$$

122. What part of Rs.13. 9 as. must be added to $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of Rs.4. 8 as. to make the result Rs.5 ?

123. A man left $\frac{7}{17}$ of his money to one of his sons, $\frac{4}{5}$ of the remainder to the other, and the latter had £106. 13s. 4d. more than the former. What was the value of the remainder ?

124. Find a fourth proportional to ·0429, $\cdot3\dot{8}$, and 12·87.

125. If B can do $\frac{2}{9}$ and C $\frac{1}{3}$ of the work which A, B and C can do working together, how much can A do in a day, supposing that together they can finish the work in 8 days ?

126. What sum will amount to £1061. 2s. $1\frac{11}{25}$d. in 3 years at compound interest ; the interest for the first, second and third years being 3, 2 and 1 per cent. respectively ?

127. By selling an article for Rs.7. 8 as. a person loses 10 per cent. ; at what price must he sell it to gain 15 per cent. ?

128. Find the square root to 4 decimal places of (*a*) 72471438, (*b*) $2\cdot3\dot{6}\dot{1}$.

129. The cost of a carpet $\frac{3}{4}$ yd. wide required for a rectangular room 36 ft. long is £35. 3s. at 4s. 9d. a yard; find the breadth of the room.

130. A man travelled 351 miles by sea, rail and coach. The distance by coach was $\frac{1}{3}$ that by rail, and by rail $\frac{1}{3}$ that by sea. The charges by sea were 1 anna a mile, by rail 2 pies a mile, by coach 4 annas 7 pies a mile. Find the total cost and the average per mile.

131. On measuring a distance of 32 yds. with a rod of a certain length, it was found that the rod was contained 41 times with $\frac{1}{2}$ inch over. How many inches will there be over in measuring 44 yds. with the same rod ?

132. An ounce of standard gold, $\frac{1}{12}$ of which is alloy, is worth £3. 17s. 10½d.; how many sovereigns would be coined from 36 lbs. 8 oz. of pure gold ?

133. Three men together fill a pit in an hour and a half, throwing respectively 8, 10, 12 equal shovelfuls of earth per minute. In what time could each man have filled up the pit by himself ?

134. If a coach travels at the rate of 9½ miles an hour and a railway train runs 25 miles while the coach does 8 miles, how much time would a traveller save on a journey of 152 miles by rail ?

135. A sum of money is divided between A, B and C; C gets twice as much as A; A and C together get Rs.50; B and C together get Rs.60; find the sum of money and how much each person gets.

136. If 250 rupees have the same value as 84 dollars and 36 dollars have the same value as 7 guineas, what is the value of a rupee in pounds, shillings and pence ?

137. A man travelled 72 miles in one day, half as many more on the second day, and as many miles on the third day as he had done in the two previous days taken together. What percentage of the whole journey did he travel on the second day ?

138. A piece of cloth is measured with a yard measure which is 1 inch too short, and appears to be 36 yards long. What is the true length ?

If the same piece of cloth covers the floor of a room 8 yards 1 foot by 5 yards 9 inches, find its breadth.

139. A merchant bought a quantity of grain at Rs.3. 8 as. 4 p. per maund, and by selling it at Rs.4. 1 a. per maund he gained a profit of Rs.107. 12 as. 8 p.; how many maunds did he buy ?

140. A passenger train travelling 20 miles an hour leaves Nagpur for Bombay at 10 A.M.: the mail train, travelling 35 miles an hour, leaves at 1·30 P.M. When will the mail overtake the passenger train, and at what distance from Nagpur ?

141. From 98765 subtract the smallest number consisting of the same digits, and divide the remainder by 9.

142. Find the product of $\dfrac{2}{3-\dfrac{2}{3+\dfrac{2}{3}}}$ and $\dfrac{2}{3+\dfrac{2}{3-\dfrac{2}{3}}}$.

143. Find the value of $\dfrac{\text{14 lbs. 8 oz. 18 dwt.}}{\text{1 lb. 0 oz. 10 dwt.}}$ of 5s. 2½d.

144. Find the square root of :

(1) 24·2064 ; (2) 3124·81 ; (3) 2·42064 × 312·481.

145. A ditch being dug at the rate of 81 feet per day by 54 men ; after 13 days' work 8 of them are replaced by boys, and the work goes on for 11 days more, at the end of which the whole length dug is 1889 feet. How much work per day do the boys do ?

146. A ship depreciates in value each year at the rate of 10 per cent. of its value at the beginning of the year ; and its value at the end of 3 years is £14580. What was the original value ?

147. The compound interest on a sum of money for 3 years at 5 per cent. is £331. 0s. 3d. ; what is the simple interest ?

148. An acre is 0·40467 hectare, and a pound sterling is 25·25 francs. An estate measuring 1927 hectares is sold for 10,100,000 francs. What is the selling price per acre in £ s. d. ?

149. A train starts from Bangalore to Mysore at 7 A.M., travelling at 12 miles an hour. Another train leaves Mysore for Bangalore at 9.40 A.M., and travels at 15 miles an hour. Find when and where they meet, the distance between Bangalore and Mysore being 86 miles.

150. All round the floor of a room, which is 28 feet long and 22 feet wide, there is a border 2 feet wide, which is left uncarpeted ; find the cost of staining the border at 1s. 1½d. a square yard ; find also the number of yards of carpet, 27 in. wide, required for covering the rest of the floor, and the cost of this carpet at 3s. 9d. a yard.

APPENDIX.

I. THE METRIC SYSTEM.

IN this system the unit of length is a *metre* and of weight a *gramme*. The unit of surface or square measure is called an *Are*, and the unit of volume of cubic measure is called a *stere*. The unit of capacity is called a *litre*.

A metre contains 40 inches nearly; a gramme $=15\frac{1}{2}$ grains; a litre $=1\cdot76$ pints; an Are $=120$ square yards; and a stere $=35\frac{1}{3}$ cubic feet nearly.

Units of higher denominations are formed by Greek prefixes, *Deca*, *Hecto*, *Kilo*, etc., which stand respectively for ten, hundred, thousand, etc.; while those of lower denominations are formed by Latin prefixes, *deci*, *centi*, *milli*, etc., denoting respectively one-tenth, one-hundredth, one-thousandth, etc. Thus a decimetre means one-tenth of a metre, a decalitre means a measure equal to 10 litres, a kilogramme means a weight equal to one thousand grammes, and so on. On page 122 will be found the tables of weights and linear measures. We give below a few illustrative examples in *Reduction*.

The following abbreviations may be used:

Km. for Kilometre; dm. for decimetre; Hm. for Hectometre; cm. for centimetre; Dm. for Decametre; mm. for millimetre; m. for metre.

Ex. 1. Reduce 6 metres to centimetres.

$$\begin{aligned}6 \text{ metres} &= 6\times 10 \text{ or } 60 \text{ decimetres}\\ &= 60\times 10 \text{ or } 600 \text{ centimetres.}\end{aligned}$$

Ex. 2. Reduce 7 kilometres, 2 hectometres, 5 decametres to metres.

$$\begin{aligned}7 \text{ kilometres} &= 7000 \text{ metres.}\\ 2 \text{ hectometres} &= 200 \text{ metres.}\\ 5 \text{ decametres} &= 50 \text{ metres.}\\ \therefore\ 7 \text{ Km. } 2 \text{ Hm. } 5 \text{ Dm.} &= 7250 \text{ metres.}\end{aligned}$$

The above work may be set down as follows:

$$
\begin{array}{rrl}
 & 7 & \text{Km.}\quad 2\text{ Hm.}\quad 5\text{ Dm.}\\
\times & 10 & \\ \cline{2-2}
 & 70 & \\
+ & 2 & \\ \cline{2-2}
 & 72 & \\
\times & 10 & \\ \cline{2-2}
 & 720 & \\
+ & 5 & \\ \cline{2-2}
 & 725 & \\
\times & 10 & \\ \cline{2-2}
 & 7250 & \quad \therefore\ 7250\text{ metres is the answer.}
\end{array}
$$

If it be remembered that each unit of any denomination is ten times that of the next lower one, the intermediate steps may be omitted and answer written down at once. Thus 7 Km. 2 Hm. 5 Dm. = 725 Dm. = 7250 metres.

Similarly

2 Hm. 4 Dm. 6 m. 5 dm. = 2465 decimetres.
5 Hm. 3 m. = 503 metres.
7 Km. 4 Dm. 8 m. = 7048 metres.
4 Km. 2 dm. = 40002 decimetres.

Ex. 3. Reduce 9 Kg. 5 Hg. 2 g. 6 cg. to milligrammes.

In the sum decagrammes and decigrammes do not appear, therefore we must write 0 in their places. Thus

9 Kg. 5 Hg. 0 Dg. 2 g. 0 dg. 6 cg. = 950206 cg. = 9502060 mg.

Ex. 4. Express 3145792 millimetres in each of the higher denominations.

$$
\begin{aligned}
3145792\text{ mm.} &= 31457{\cdot}92\text{ cm.}\\
&= 31457{\cdot}92\text{ dm.}\\
&= 3145{\cdot}792\text{ m.}\\
&= 314{\cdot}5792\text{ Dm.}\\
&= 31{\cdot}45792\text{ Hm.}\\
&= 3{\cdot}145792\text{ Km.}
\end{aligned}
$$

Ex. 5. Express 4753 metres and 86209 metres, each as a compound quantity consisting of kilometres and lower denominations.

In 4753 the units' figure, 3, denotes the number (less than nine) of metres; the figure in the place of tens, *i.e.* 5, denotes decametres; the figure in the hundreds' place, *i.e.* 7, denotes hectometres; while 4, which stands in the thousands' place, denotes kilometres. Therefore

4753 metres = 4 Km. 7 Hm. 5 Dm. 3 m.

Similarly

86209 metres = 86 Km. 2 Hm. 9 m.

The following are a few more examples:

Ex. 6. Add together 7 Km. 3 Hm. 2 Dm. 5 m.; 42 Km. 6 Dm. 4 m., and 9 Hm. 1 Dm. 8 m.

Here we may proceed as in compound addition. Thus:

Km.	Hm.	Dm.	m.
7	3	2	5
42	0	6	4
	9	1	8
50	3	0	7

or, since each compound quantity can be expressed as a simple quantity, the work may be set down as follows:

Metres.
7325
42064
918
50307

∴ The required sum is 50307 metres = 50 Km. 3 Hm. 7 m.

Ex. 7. Subtract 9·46 Hm. from 2·58 Km.

9·46 Hm. = 946 metres.
2·58 Km. = 2580 metres.

Metres.
2580
946
1634

∴ 1634 metres, or 1 Km. 6 Hm. 3 Dm. 4 m., is the answer.

Ex. 8. Multiply 4 Hm. 7 Dm. 6 m. by 38.

4 Hm. 7 Dm. 6 m. = 476 metres.

Metres.
476
38
3808
1428
18088

∴ The required product is 18088 metres = 18 Km. 8 Dm. 8 m.

Ex. 9. Divide 728 Km. 8 Hm. 3 Dm. 8 m. by 125.

728 Km. 8 Hm. 3 Dm. 8 m. = 728838 metres.

$$\begin{array}{rl}
 & \text{Metres.}\\
125\,) & 728838\,(\,5830 \text{ metres.}\\
 & \underline{625}\\
 & 1038\\
 & \underline{1000}\\
 & 383\\
 & \underline{375}\\
 & 88
\end{array}$$

∴ The quotient is 5830 metres, with 88 metres as remainder.

We may proceed as in Division of Decimals. Thus:

$$\begin{array}{rl}
125\,) & 728838{\cdot}000\,(\,5830{\cdot}704 \text{ metres.}\\
 & \underline{625}\\
 & 1038\\
 & \underline{1000}\\
 & 383\\
 & \underline{375}\\
 & {\cdot}880\\
 & \underline{875}\\
 & 500\\
 & \underline{500}\\
 & \ldots
\end{array}$$

∴ The required quotient is 5830·704 metres = 5 Km. 8 Hm. 3 Dm. 7 dm. 4 mm.

Ex. 10. If the carriage of one hundred Kilogrammes of goods for 35 hectometres cost 8s. 4d., what will be the cost of the carriage of the same goods for 10 kilometres 5 hectometres?

35 hectometres = 3500 metres.

10 Km. 5 Hm. = 10500 metres.

$8\text{s. } 4\text{d.} = 8\tfrac{1}{3}\text{s.} = £\tfrac{5}{12}.$

∴ 3500 m. : 10500 m. :: $£\tfrac{5}{12}$: Ans.

∴ Ans. $= £\frac{10500\times 5}{3500\times 12} = £\frac{5}{4} = £1.\ 5\text{s.}$

∴ The required cost is £1. 5s.

II. CONTRACTED MULTIPLICATION OF DECIMALS.

In multiplying two long decimals it is generally sufficient to give the product *approximately* correct, *i.e.* correct to a certain number of decimal places only. Thus if we have to multiply 3·458642 by 2·14367, we find by actual multiplication that the product is 7·41418709614. Now this is a dreadfully long fraction, so it is generally sufficient, if we give the product correct to three, four or five places of decimals only, *i.e.* if we write it as 7·414, or 7·4142 or 7·41419. When such approximately correct results are sufficient, the following rule enables us to shorten the work of multiplication :

Rule. In the decimal part of the multiplicand count from the decimal point a number of figures equal to the number of decimal places to be retained in the product, and place a mark over the last of these figures. Place under the marked figure the units' figure of the multiplier, a cipher being placed as the units' figure if the multiplier has no integral part ; to the right of this units' figure place the remaining figures, if any, of the integral part of the multiplier in the reverse order, *i.e.* the tens' figure after the units', the hundreds' figure after the tens', and so on ; and to the left of the units' figure place the figures of the decimal part of the multiplier in the reverse order, *i.e.* the tenths' figure immediately to the left of the units', the hundredths' figure to the left of the tenths', and so on. Annex, if necessary, ciphers to the multiplicand so that every figure of the multiplier shall have a figure above it. Disregard the decimal points of both the multiplicand and the multiplier, and multiply by each successive figure of the multiplier the figure of the multiplicand above it and all figures to the left, carrying to the product of each figure in the multiplier with the figure in the multiplicand above it the *nearest ten* from the product of that figure with the one to the right of the figure in the multiplicand. Place the partial products under one another so that the units' figure of each shall be in the same vertical line; and add as usual, marking off in the result the required number of decimal places.

In carrying the nearest ten, if the product be a number from zero to 4, carry nothing; from 5 to 14, carry 1; from 15 to 24, carry 2; from 25 to 34, carry 3, and so on.

Ex. 1. *Multiply* 7·4236 by 23·54, correct to two places of decimals.

$$\begin{array}{r} 7{\cdot}4\overset{|}{2}36 \\ 4\,532 \\ \hline 14\,847 \\ 2\,227 \\ 371 \\ 30 \\ \hline 174{\cdot}75 \end{array}$$

Since the product is required to be correct to two decimal places, we make a mark (a small vertical line) over 2, the second figure in the multiplicand from the decimal point. Below this marked figure we set down 3, the units' digit of the multiplier; to the right of this digit we place 2, the tens' digit; and to the left of the units' digit we set down in the reverse order the two figures of the decimal part of the multiplier. We thus have the whole order of the figures of the multiplier reversed, *i.e.*, instead of 2354, we have written 4532. We have now to multiply in succession by each one of the four figures of the multiplier, beginning with 2—the respective multiplicands being 7423, 742, 74, and 7. Take first of all 2. To its product with 3 we must carry 1, because its product with 6, the figure to the right of 3 in the multiplicand, is 12; similarly, to the product of 3 with 2, we have to carry 1, because the figure to the right of 2 is 3, and 3 multiplied by 3 is 9. In the same way to the product of 5 with 4, we have to carry 1; and to the product of 4 with 7 we have to carry 2 (since the figure to the right of 7 is 4, and 4 multiplied by 4 is

16, which is between 15 and 24). We now place the partial products above one another, so that their units' figures, viz. 7, 7, 1, and 0, are in the same vertical line, and we add. The result is 17475, in which we have to mark off two decimal places. Thus the required product is 174·75.

Note. When the number of partial products is considerable, the last figure in the decimal part of the product obtained by the contracted method cannot be always depended upon. Therefore to ensure accuracy in such cases, it is desirable to take in the multiplicand one or two decimal places more than the required number, and after performing the work to strike off from the product as many figures from the right as the number of superfluous decimal places retained in the multiplicand before marking off the decimal point.

Ex. 2. Multiply 2·46921587 by ·56427369, so that the result may be true to 5 decimal places.

$$\begin{array}{r}
2{\cdot}469215\overset{|}{8}7 \\
\underline{96\;3724650} \\
12\;346079 \\
1\;481529 \\
98768 \\
4938 \\
1728 \\
74 \\
14 \\
2 \\
\hline
1{\cdot}39331\cancel{8}\cancel{2}
\end{array}$$

Here we count in the multiplicand $(5+2=)$ 7 figures to the right from the decimal point and make a mark over 8, the figure in the 7th place; below this we place 0, because there is no units' digit in the multiplier; and to the left of 0 we place the figures of the multiplier in the reverse order. We then multiply and arrange the partial products, as stated in the rule. From the result we first strike off the last two digits, since we had taken in the multiplicand two more decimal places than required, and then counting as usual 5 places, mark the decimal point. Thus the required product is 1·39331.

We add one more example :

Ex. 3. Find the product of 25·2876 and 35·628, correct to three decimal places.

$$\begin{array}{r}
25{\cdot}287\overset{|}{6}0 \\
82653 \\
\hline
75\ 86280 \\
12\ 64380 \\
1\ 51726 \\
5057 \\
2022 \\
\hline
900{\cdot}946\cancel{5}
\end{array}$$

Here we extend work to one decimal place more, and therefore make a mark over 6, the fourth decimal figure of the multiplicand ; below this we place 5, the units' digit of the multiplier, and to the right of this units' digit, we place 3, the tens' digit. We next place to the left of 5 all the digits in the decimal part of the multiplier in the reverse order. As the last digit of the multiplier on the right has no figure above it in the multiplicand, we annex one cipher to the multiplicand. We strike off one figure from the product, because we had taken one figure more than required in the decimal part of the multiplicand. Thus the required product is 900·946.

III. CONTRACTED DIVISION OF DECIMALS.

In dividing one decimal by another, where the quotient is required to be correct only to a certain number of decimal places, the following contracted method should be used :

Rule. Determine by inspection or otherwise the number of digits in the integral part, if any, of the quotient. Add this number to the required number of decimal places in the quotient, and in the divisor, mark from the left from the first significant figure, as many places as are represented by the sum of these two, striking out the rest. In the dividend, retain as many figures as are needed to take one step in the division with this new divisor, and strike out the rest. Proceed one step in the division with the new divisor, but in multiplying its first figure by the first figure of the quotient, carry to it the nearest ten obtained from the product of the next figure on its right in the old divisor and the first figure of the quotient. To find the next figure of the quotient, instead of bringing down from the dividend a figure of the remainder, strike off the first figure from the right of the new divisor and proceed as before, until no figure is left in the divisor.

Ex. 1. Divide ·584962571 by ·6254973, correct to five places of decimals.

$$\begin{array}{lrl}
\cdot 6,2,5,4,9,\cancel{7}\cancel{3}\;) & \cdot 584962\cancel{5}\cancel{7}\cancel{1} & (\,\cdot 93519 \\
 & \underline{562947} & \\
 & 22015 & \\
 & \underline{18765} & \\
 & 3250 & \\
 & \underline{3127} & \\
 & 123 & \\
 & \underline{63} & \\
 & 60 & \\
 & \underline{56} & \\
 & 4 &
\end{array}$$

Here the quotient has no integral part, and as we require only five decimal places in the quotient, we mark off from the left 5 places in the divisor; and we retain 6 places in the dividend so as to take the first step in the division with the new divisor. The first figure in the quotient is 9, by which we multiply the new divisor 62549, carrying to the product of 9 and 9, 6 (because the next figure to the right of 9 in the old divisor is 7, and 9 multiplied by 7 is 63). At the second step we strike out 9 from the new divisor and obtain 3 as the second figure of the quotient; at the third step, we strike out 4, and so on.

Ex. 2. Divide 39·65897412 by 4·27984615, giving the result correct to 5 decimal places.

$$4\cdot27984|\cancel{6}\cancel{1}\cancel{5}\ )\ 39\cdot65897\cancel{4}\cancel{1}\cancel{2}\ (\ 9\cdot26645$$

$$\begin{array}{r}
\underline{38\ 51861} \\
114036 \\
\underline{85597} \\
28439 \\
\underline{25679} \\
2760 \\
\underline{2567} \\
193 \\
\underline{171} \\
22 \\
\underline{21} \\
1
\end{array}$$

We see here that the quotient has an integral part of one digit, and since it is to contain 5 decimal places, we mark off in all (5 + 1 =) 6 figures in the divisor from the left and 7 (for reason stated above) in the dividend.

Ex. 3. Divide ·0452143956 by ·000826147119, correct to 4 decimal places.

$$\cdot000826147|119\ )\ \cdot04521439\cancel{5}\cancel{6}\ (\ 54\cdot7292$$

$$\begin{array}{r}
\underline{4130736} \\
390703 \\
\underline{330459} \\
60244 \\
\underline{57830} \\
2414 \\
\underline{1652} \\
762 \\
\underline{743} \\
19 \\
\underline{16} \\
3
\end{array}$$

By removing the decimal point 4 places to the right in both the divisor and the dividend we get respectively 8 and 452 as integral parts, and from this we find that the quotient has 2 figures in the integral part; and since the quotient is required correct to 4 decimal places, we mark off six places in the divisor from the left, beginning with the first significant figure, *i.e.* 8.

Ex. 4. Divide ·0054608172 by 3·4820597, correct to 9 decimal places.

3·482059|7) ·005460817$\cancel{2}$ (·001568272
3482060
1978757
1741030
237727
208923
28804
27856
948
696
252
244
8
7
1

By removing the decimal point 3 places to the right in both the divisor and the dividend we determine that there will be in the quotient 2 ciphers after the decimal point; and since the quotient is to contain 9 decimal places, 9 – 2 = 7 are to be obtained by division. We, therefore, retain 7 figures in the divisor and a corresponding number of figures in the dividend, beginning in the latter case with the first significant figure, *i.e.* 5. The required quotient is ·001568272.

Ex. 5. Divide 7 by 2·534, correct to 6 decimal places.

2·534) 7·000000 (2·762430
5 068
19320
17738
15820
15204
6160
5068
1092
1014
78
76
2

We see that the quotient contains an integral part of one digit; and since it is to contain six decimal places, we must mark off from the left 7 decimal places in the divisor. But the divisor consists only of 4 figures; we, therefore, proceed as in ordinary division, until only three figures of the quotient remain to be obtained, when we resort to the contracted method.

IV. EXCHANGE.

Exchange is the term used for the payment in the money of one country, of a sum equal in value to a sum of money of another country.

The *par of exchange* is the nominal value of the standard coin of one country in the coinage of another, and is dependent on the intrinsic values of the coins concerned. This value, however, changes from a variety of causes from day to day and even from hour to hour, and is then called the *course of exchange*, which is the actual or current value of the standard coin of one country in the coinage of another.

Money is generally transmitted to foreign countries by bills called *bills of exchange.*

A bill is a paper of agreement to pay a particular sum on a particular date at a particular place for value received.

Suppose a person in Bombay wants to send £85 to a person in London. What he will do is this: he will go to one of the banks of Bombay and there purchase a bill for £85, payable in London, *at the rate of exchange at that particular time current.* Supposing that the rate of exchange at that time is 1s. 5d. per rupee, he will have to pay Rs.1200 to that Bombay bank for purchasing a bill for the required amount payable in London. He will then send the bill to the person in London who is to receive the sum, and this person will present it to the bank on which it is drawn and receive the specified amount.

FOREIGN MONEY TABLE.

GOLD CURRENCY.

Country.	Standard Coin.	Exchange Value.	Subdivisions.
England, - -	Sovereign	£1.	20s. = 240d.
United States, -	Dollar	4s. 2d.	100 cents
France - -	Franc	9½d.	100 centimes
Germany, -	Mark	11¾d.	100 pfennigs
Austria, - -	Gulden	1s. 11½d.	100 kreuzers
Italy, - -	Lira	9⅛d.	100 centesimi
Turkey, - -	Pound	18s.	100 piastres

SILVER CURRENCY.

Country.	Standard Coin.	Exchange Value.	Subdivisions.
India, - -	Rupee	1s. 4d.	64 pice = 192 pies
Ceylon, - -	Rupee	,,	100 cents
Japan, - -	Yen	2s. 1d.	100 sen
China - -	Tael	6s. 6d.	1000 cash
Russia, - -	Rouble	3s. 3d.	100 copecs

Exchange values given in the above tables are approximate.

Ex. 1. Express in Indian coinage £53. 12s. 6d. at 1s. 4¼d. per rupee.

$$1\text{s. } 4\tfrac{1}{4}\text{d.} = 16\tfrac{1}{4}\text{d.} = \frac{65}{4}\text{d.} = £\frac{65}{4} \times \frac{1}{240} = £\frac{13}{192}.$$

$$£53.\ 12\text{s. } 6\text{d.} = £53\tfrac{5}{8}\text{d.} = £\frac{429}{8}.$$

$$\therefore\ £\frac{13}{192} : £\frac{429}{8} :: \text{Re.}1 : \text{Ans.}$$

$$\therefore\ \text{Ans.} = \text{Rs.}\frac{429 \times 192}{8 \times 13} = \text{Rs.}33 \times 24 = \text{Rs.}792.$$

Ex. 2. If Re.1 = 1s. 4d. and 1 dollar = 4s. 2d., change 1000 dollars into rupees.

$$1\text{s. } 4\text{d.} = 16\text{d.};\ \ 4\text{s. } 2\text{d.} = 50\text{d.}$$

$$1000 \text{ dollars} = 1000 \times 50\text{d.} = 50000\text{d.}$$

$$\therefore\ 16\text{d.} : 50000\text{d.} :: \text{Re.}1 : \text{Ans.}$$

$$\therefore\ \text{Ans.} = \text{Rs.}\frac{50000}{16} = \text{Rs.}3125.$$

Ex. 3. What is the course of exchange between England and India, if a person paying Rs.900 in India becomes entitled to receive £56. 5s. in England ?

$$£56.\ 5s. = £56\tfrac{1}{4} = £\frac{225}{4}.$$

$$\therefore\ Rs.900 : Re.1 :: £\frac{225}{4} : Ans.$$

$$Ans. = £\frac{225}{4 \times 900} = £\frac{1}{16} = 1s.\ 3d.$$

$\therefore$ 1s. 3d. per rupee is the answer.

Ex. 4. The collector of a zilla sends to England Rs.270 per calendar month, when the course of exchange between England and India is 1s. 4d. per rupee. If the value of a rupee falls to 1s. 3d., how many rupees more per month will he have to send in order that the value in pounds of the sum sent may remain the same as before ?

$$1s.\ 4d. = £\frac{16}{240} = £\frac{1}{15}.$$

$$1s.\ 3d. = £\frac{15}{240} = £\frac{1}{16}.$$

$$Rs.270 = £\frac{270}{15} = £18.$$

He should send as many rupees as are equal in value to £18. Therefore

$$£\frac{1}{16} : £18 :: Re.1 : Ans.$$

$$\therefore\ Ans. = Rs.18 \times 16 = Rs.288.$$

Therefore he must send Rs.288 − Rs.270 = Rs.18 more every month.

Ex. 5. A man in Bombay wants to send 960 dollars to New York. How much will he save by sending the money through London instead of sending it direct, it being given that 1 dollar = Rs.3. 3 as., £1 = 4·8 dollars, and Re.1 = 1s. 4d.

Since $$1\ dollar = Rs.3.\ 3\ as. = Rs.\frac{51}{16},$$

to find the required number of rupees if the remittance were made direct, we have

$$1\ dollar : 960\ dollars :: Rs.\frac{51}{16} : \text{the required number of rupees.}$$

$$\therefore\ \text{the required number of rupees} = \frac{960 \times 51}{16} = 3060.$$

Again, since £1 = 4·8 dollars, to find the equivalent in sterling of 960 dollars, we have

4·8 dollars : 960 dollars :: £1 : the required number of £.

$\therefore$ the required amount in sterling $=\pounds\frac{960}{4\cdot 8}=\pounds 200$.

And as a rupee $=$ 1s. 4d. $=\frac{4}{3}$d. $=\pounds\frac{1}{15}$, *to find the value of £200 in rupees*, we have

$$\pounds\frac{1}{15} : \pounds 200 :: \text{Re.}1 : \text{the required number of rupees.}$$

$\therefore$ the value in rupees of £200 is Rs.200 × 15 = Rs.3000.

Thus if he sends the sum direct the bill will cost him Rs.3060. On the other hand, if he sends the sum by way of London he will have to pay Rs.3000. Therefore by sending the money by London the man will save Rs.3060 − Rs.3000 = Rs.60.

V. EXAMPLES.

1. Reduce to metres :

(1) 5 Km. ; 7 Dm. ; 75 Km. ; and 4 Hm.
(2) 4 Hm. 2 Dm. 6 m. ; and 6 Km. 2 Dm.
(3) 385 Km. 48 Dm. ; and 3 Km. 858 Hm. 95 m.

2. Reduce to millimetres :

(1) 3 dm. ; 8 Km. ; 3 m. ; 4 Hm. ; 9 cm. ; and 6 Dm.
(2) 7 Hm. 4 Dm. 2 m. ; and 2 Dm. 5 dm. 8 mm.
(3) 6 Km. 2 Dm. 4 dm. ; and 5 Hm. 7 cm. 9 mm.

3. *Express in kilometres:*

(1) 5 Dm. ; 7 Hm. ; 3 m. ; 83 Hm. ; and 725 Dm.
(2) 4 Hm. 2 Dm. 1 m. ; and 38 Hm. 95 m.
(3) 3 Dm. ; 4 dm. ; 2 Hm. ; 6 cm. ; 4375 mm.

4. Reduce to grammes :

(1) 25 Kg. ; 387 Dg. ; and 7 Hg.
(2) 2 Hg. 3 Dg. 5 g. and 821 Kg. 42 Hg. 625 g.
(3) 6 dg. ; 8 cg. ; 7 mg. ; and 34 cg. ; 281 dg. ; 7348 mg.
(4) 3 Dg. 3 dg. ; 5 Hg. 4 cg. ; and 8 Kg. 7 mg.

5. Express in metres 3·485 Km. and 24·8 Hm.

6. Add together 7 Kg. 2 Hg. 3 g. ; 4 Hg. 3 Dg. 7 g. ; 8 Kg. 2 Dg. ; and 6 Hg. 4 g.

7. Subtract 7 Hm. 2 m. from 3 Km. 2 Hm. 5 Dm.

8. Find the sum of 45 g. 3 cg. 8 mg. ; 8 Hg. 4 g. 2 dg. ; 3 Kg. 2 Dg. 1 dg. 7 mg. ; and 2 Hg. 5 cg. 8 dg. 6 mg.

9. Find the difference between 284 Km. 2 Dm. 5 cm. and 3081 Hm. 2 m. 37 mm.

10. Subtract the sum of 345 Hm. 304005 mm. from the difference between 65 Km. and 60050 dm.

11. Multiply 2 Kg. 5 Hg. 4 Dg. by 12, and express the product in grammes.

12. Multiply 5 m. 6 cm. 4 mm. by 6, 19, and 160 separately.

13. Divide 34 Km. 5 Hm. 8 Dm. by 16.

14. Divide 6 Km. 5 Dm. 2 dm. by 5, 8, and 200 separately.

15. Reduce to centimetres :

(1) 3·84 Dm. × 8 ; 38·04 Hm. × 6 ; and ·0846 Km. × 48.
(2) 33·76 Km. ÷ 4 ; 3·135 dm. ÷ 15 ; and 4·096 Hm. ÷ 32.

16. How many decigrammes are there in 500 Kg. ?

17. Express 23 Kg. 8 Dg. 3 g. 5 cg. in terms of (1) hectogrammes, (2) decigrammes, and (3) milligrammes.

18. Reduce to the decimal of a Kilogramme 38465 cg. ; 708004 mg. ; 81030675 Dg.

19. Express in grammes :

(1) 432·5 Hg. + 3·542 Dg. (2) 3·852 Kg. − 29·04 Hg.

20. If a Kilometre be taken as containing 1093·4 English yards, what fraction will 7 decametres be of 49 miles ?

21. The distance between Berlin and Paris is 1120 Km. Express it in miles if 1 metre = 39·6 inches.

22. Reduce 5 miles to Kilometres if 1 metre = 40 inches.

23. Reduce 160 hectometres to miles if 1 inch = 2·5 cm.

24. Reduce 2 cwt. 3 qr. 21 lbs. to grammes if 1 cwt. = 50·8 Kg.

25. Express 3425·48 Kg. in tons, cwts., etc., if 1 Kg. = 2·2 lbs.

26. Find the cost of 8·2 Kg. if sugar is at 4·5d. a Kg.

27. A bar of iron 1·1 metre long weighs 1 Kg. ; what length (expressed in feet) will weigh a pound ? if 1 metre = 39·36 in. and 1 lb. = ·454 Kg.

28. A man travels 7·5 Km. an hour ; how far will he go in 8 hrs. 45 minutes ?

29. Find the cost of 82 Kg. 125 g. of butter (in £. s. d.) at 3 francs 44 centimes per Kg. if 1 franc = 100 centimes = 25s.

30. Find to the nearest pound the cost of 5122·7 Kg. of tea at 3·43 francs per Kilogramme, reckoning 25·42 francs as equivalent to one pound sterling.

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31. Multiply 8·7612 by 5·63, correct to 2 places of decimals.

32. Multiply 6·3481 by 4·865, correct to 3 places of decimals.

33. Multiply 23·17503625 by 6·80162, correct to 3 places of decimals.

34. Multiply ·5276 by ·238, correct to 4 places of decimals.

35. Multiply ·854 by 736·2, correct to 3 places of decimals.

36. Multiply ·906 by ·00847, correct to 4 places of decimals.

37. Multiply 97·26414 by ·04529, correct to 5 places of decimals.

38. Multiply 123·456 by 654·321, correct to the nearest integer.

39. Multiply 42·36068 by ·0236068, correct to 6 places of decimals.

40. Multiply 1·236068 by 3·236068, correct to an integer.

41. Divide 39·71903 by 6·412804, correct to 5 places of decimals.

42. Divide ·43429448 by ·54930615, correct to 5 places of decimals.

43. Divide ·1289585 by 1·01729, correct to 7 places of decimals.

44. Divide 92·68806 by ·04382016, correct to 4 places of decimals.

45. Divide ·006894835 by ·000741285, correct to 4 places of decimals.

46. Divide ·0262894759 by 6·137, correct to 8 places of decimals.

47. Divide 1 by 2·82842714, correct to 7 places of decimals.

48. Divide 2 by ·7320508, correct to 7 places of decimals.

49. Divide 3 by 3·65028157, correct to 8 places of decimals.

50. Divide 6·10878 by ·0234, correct to an integer.

Find the value of

51. 5·5 of £50·056, correct to a penny.

52. 1·157625 of Rs.750, correct to a pie.

53. £1790 × 143 ÷ 2400, correct to a penny.

54. £1·16985856 × 405·625, correct to a penny.

55. £512·225 × 1·098 ÷ 73, correct to a penny.

56. 3·16227 × 4·472136 ÷ 6·3245553, correct to 5 decimal places.

57. 7·0710678 × 3·4641016 ÷ 8·660254, correct to 4 decimal places.

58. 12·247448 × 15·81138 ÷ 17·3205081, correct to 6 decimal places.

59. 4·2426 × 18·7082869 ÷ 5·477, correct to 3 decimal places.

60. 3·70271 × ·64732 ÷ ·043679, correct to 5 decimal places.

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61. Exchange Rs.2437 for English money at 1s. 4d. per rupee.

62. If 25·5 francs be equivalent to £1 sterling, find the value, in English money, of 673·2 francs.

63. If Rs.16 be worth 21s. and 20s. be equivalent to 25 francs, how many rupees must be exchanged for 5775 francs ?

64. Find the value, in Indian coinage, of goods sold for 800 dollars, exchange being 1 dollar = 4s. 2d. and Re.1 = 1s. 4d.

65. Find the course of exchange between London and Paris, if 1 American dollar is worth 4s. 2d. in London and 5·4 francs in Paris.

66. Find the equivalent, in British currency, of 839·85 marks when £1 = 20·36 marks.

67. The value of Rs.531. 4 as. being remitted from Bombay to Tokio realizes there 170 yens. Find the rate of exchange in rupees per yen.

68. What is the face value in Pekin of a bill for Rs.2379 sent from Bombay through Hong-Kong, the course of exchange being £1 = Rs.16 and 1 tael = 6s. 6d. ?

69. What was the value in Bombay of a sight draft to London for £471. 1s. $6\frac{3}{4}$d. when the course of exchange was at 1s. $3\frac{31}{32}$d. per rupee ?

70. If 6 roubles be worth 21s., 20s. = 22 marks, 14 marks = 3 dollars, and 11 dollars = 12 yens, how many yens are worth one thousand roubles ?

71. Convert £65. 17s. 6d. into dollars at 4s. 3d. each.

72. Express 105·4 marks into Indian coinage when £1 = Rs.15 and 20s. = 21·08 marks.

73. If 5 francs are worth 4s., how many francs are equal in value to £18. 12s. ?

74. The value of a dollar being 4s. 2d., how many dollars should be given in exchange for Rs.1234. 2 as. 8 pies if Re.1 = 1s. 3d. ?

75. If the *hundee* rate between Bombay and Madras be Rs.100. 2 as. and that between Madras and Colombo be Rs.100 4 as., what sum must be paid in Bombay to remit Rs.8000 to Colombo ?

76. A person in India sends per month £42 to his son in London. If the course of exchange be 1s. $3\frac{3}{4}$d. per rupee, how many rupees does he pay to buy a bill required for the purpose ?

77. An Indian officer whose pay was estimated in rupees lost £45. 10s. in a certain year by a fall in the value of a rupee from 1s. $9\frac{1}{8}$d. to 1s. $8\frac{1}{4}$d. What was his salary per month ?

78. A debt of £68. 5s. is paid in francs valued at $9\frac{3}{4}$d. each at a time when 25 francs are worth £1. What does the creditor gain or lose ?

79. If exchange be at the rate of 18s. for 1 Turkish pound and Rs.1 = 1s. $2\frac{15}{16}$d., how many rupees will discharge a debt of 127500 piastres ?

80. A person enters France with a one-thousand-rupee note. There he spends 562 francs 5 centimes ; how many sovereigns will he receive in exchange for the remainder if £1 = 25 francs ?

ANSWERS.

Ex. 1. Page 5.

1. Five. 2. Twenty-five. 3. One hundred and twenty-five.

4. One thousand two hundred and fifty.

5. Ten thousand two hundred and five. 6. Fifty.

7. One hundred. 8. Two thousand 9. Thirty thousand.

10 Four hundred thousand.

11. (1) Five units ; (2) five tens ; (3) five units ; (4) five hundreds ; (5) five thousands ; (6) five thousands.

12. 234. **13.** 324. **14.** 4032. **15.** 3420. **16.** 10986.

17. 200407. **18.** 468935. **19.** 1002578. **20.** 12013014.

21. Four hundred and thirty-five.

22. Nine thousand six hundred and seventy-six.

23. Twelve thousand three hundred and forty-nine.

24. Sixty thousand seven hundred and five.

25. Forty-three thousand two hundred and sixteen.

26. Two hundred and forty-eight thousand five hundred and thirty-two.

27. Four millions three hundred and seventy-two thousand one hundred and eighty.

28. Six millions twenty thousand and seventy-five.

29. One thousand and one.

30. Thirty thousand four hundred and two.

31. Seven millions six hundred thousand and sixty.

32. Three thousand and twenty millions one hundred and six thousand eight hundred and nine.

33. 999 ; 1000. **34.** 330. **35.** 4000. **36.** 800000.

37. 10000000. **38.** 80900. **39.** 99999999 ; 10000000.

40. Three hundred and fifty-seven millions nine hundred and fourteen thousand seven hundred and nineteen.

41. One hundred and twenty-three millions four hundred and fifty-six thousand seven hundred and eighty-nine.

42. Two billions three thousand millions four hundred thousand and five.

43. Six billions four hundred and two thousand and one hundred millions seven hundred and six thousand and seventy.

44. One trillion twenty thousand and three hundred billions four hundred and fifty thousand and sixty millions eight thousand and seven.

45. 1002300. **46.** 4005850075. **47.** 351609040008.

48. 50000040000003001001.

49. Four hundred and six thousand and eighty.

50. 3080102 ; *Three millions eighty thousand one hundred and two.*

Ex. 2. Page 8.

1. 108. 2. 878. 3. 1079. 4. 1082.
5. 98789. 6. 86597. 7. 127598. 8. 130362.
9. 179653. **10.** 179795. 11. 2669691. 12. 2354761.
13. 170734. **14.** 1859264. 15. 2180626. 16. 300017709.
17. 6415607595. **18.** 9951. 19. 2806. 20. 6642. 21. 6050.
22. 206171532255. **23.** 3259403141. 24. Ten millions.
25. Nine hundred and ninety-nine thousand nine hundred and ninety-nine.

Ex. 3. Page 11.

1. 22. 2. 333. 3. 282. 4. 1425.
5. 6093. 6. 20917. 7. 1076. 8. 53634.
9. 191427. **10.** 31405. **11.** 25794. **12.** 88677.
13. 109494. **14.** 104866. **15.** 2632809. **16.** 7946458.
17. 769485. **18.** 1172577. **19.** 16560003. **20.** 900891.
21. 990. **22.** 5400 more. **23.** 39960. **24.** 30.
25. One million and two hundred is the greater ; by 799900.

Ex. 4. Page 12.

1. 156 boys. 2. 261 sheep, 44 cows, 83 horses; 388 animals.
3. 18004 rupees. 4. 84 years. 5. 1276 rupees.
6. 91 rupees. 7. 687 trees. 8. 139 mangoes.
9. The first 229, the second 166, and the third 172 mangoes.
10. 1986 rupees. 11. 24 years. 12. 69 years old.
13. 247 animals. 14. In 1954. 15. 54 years.
16. 262 passengers. 17. 45 rupees.
18. Rama has 199 and Govinda 203 marbles. 19. 34 years old.
20. 66 years. 21. 1825 ; 111385 inhabitants.
22. 79 mangoes, 92 oranges and 66 plantains. 23. 52966 inhabitants.
24. 3725 rupees. 25. 1273 boys. 26. 113 rupees.
27. 3212 pice. 28. 1998 men. 29. 2169 men. **30. 3009.**

Ex. 5. Page 18.

1. 684.
2. 3969.
3. 164884.
4. 1638.
5. 190120.
6. 453615.
7. 47520.
8. 423514.
9. 3282360.
10. 7795008.
11. 9382864.
12. 26688830.
13. 12386860.
14. 202013040.
15. 687461397.
16. 285714 ; 428571 ; 571428 ; 714285 ; 857142 ; 999999.
17. 230769 ; 307692 ; 692307 ; 923076 ; 999999.
18. 11111111 ; 22222222 ; 33333333 ; 44444444 ; 55555555 ; 66666666 ; 77777777 ; 88888888 ; 99999999.
19. 96552 ; 128736 ; 225288 ; 253449.
20. 340464 ; 907904 ; 1569456 ; 1151488 ; 1480880 ; 1793664.
21. 308610.
22. 675360.
23. 1825920.
24. 1280187936.
25. 13164006500.
26. 2401453530.
27. 81965453598.
28. 29354213674308672.
29. 16777216.
30. 6289223.
31. 7925 multiplied by itself is the greater ; by 1.
32. 360000.
33. 21025 ; 116964 ; 164025 ; 81162081.
34. 46656 ; 614125 ; 24389000 ; 1030301.
35. 1600 ; 1990656.

Ex. 6. Page 26.

1. 321.
2. 6231.
3. 241324.
4. 3125.
5. 4112.
6. 51210.
7. 3121.
8. 3309.
9. $41900\frac{4}{17}$.
10. $5016\frac{14}{19}$.
11. 9051.
12. $5138\frac{1}{29}$.
13. 760687.
14. 305345.
15. 10647022.
16. $1629\frac{2}{23}$.
17. 5431.
18. 127587.
19. $273\frac{186}{293}$.
20. 704.
21. $1163\frac{2369}{4173}$.
22. 45067.
23. 7777
24. 32643.
25. 130130.
26. 40960.
27. 2005.
28. $761\frac{16}{21}$.
29. $7402\frac{67}{77}$.
30. $1004\frac{45}{84}$.
31. $22484\frac{7}{105}$.
32. $322591\frac{244}{2431}$.
33. $1606282\frac{6}{7}$; $1404876\frac{2}{3}$; $549738\frac{6}{23}$.
34. 403.
35. 728.

Ex. 7. Page 27.

1. 13572 oranges.
2. 157 trees.
3. 910 rupees.
4. 14790 lines.
5. 37 mangoes.
6. 43 oranges.
7. 700 miles.
8. 27324 steps.
9. 11 rupees.
10. 37 loaves.
11. 36800 pice.
12. 227.
13. 14007 miles.
14. 119.
15. 11 regiments.
16. 2580 stones.
17. 768 persons.
18. 30150 passengers.
19. 66528 men.
20. 34875 bricks.
21. 15 pice.
22. 13 rupees.
23. 5148 persons.
24. 17.
25. 37 rupees.
26. 16 mangoes.
27. 1081 miles ; 15 days.
28. 25 rupees.
29. 52150 yards.
30. 300 sheep.

Ex. 8. Page 32.

1. 622. **2.** 1126. **3.** 5554. **4.** 209. **5.** 2104.
6. 399 **7.** 468. **8.** 800. **9.** 270. **10.** 66.
11. 67. **12.** 233. **13.** 1931. **14.** 23. **15.** 116.
16. 14120. **17.** 100. **18.** 8. **19.** 54. **20.** 3.

Ex. 9. Page 34.

1. 10011010.

2. Four million three hundred thousand seven hundred and two; six million seventy thousand five hundred and six; three hundred and forty-one thousand three hundred and eighteen; ten million seven hundred and twelve thousand five hundred and twenty-six.

3. 1235. **4.** 3660. 5. 179.

6. Fifteen million six thousand and twenty-one; the number is increased 99999 times itself.

7. 764. 8. The square of twice 16 is the greater.
9. 1528234000000000000. 10. 4083.
11. Forty-nine thousand nine hundred and fifty; 49950. 12. 1886.
13. 18 years. 14. 3. 15. 52. **16.** 600000; 5000; 200; 9.
17. Thirty-three thousand two hundred and nineteen.
18. Sixteen hundred and seventy-nine. **19.** Nine thousand and sixty.
20. 101 times. 21. 99999. **22.** One million.
23. Two thousand nine hundred and ten. **24.** 0.
25. 60, 060, 060, 060. 26. 1554. **27.** 317450.
28. 60768396; 129847 and 40068. **29.** 236561164.
30. 208117, 29731, $2973\frac{1}{10}$. **31.** 1235; 5321.
32. 121; 63. **33.** Two, 8, 1; 9. **34.** $1+4+9=14$.
35. 958; 12. **36.** 4071348. **37.** 6617862.
38. $70993\frac{5}{84}$; $124237\frac{41}{48}$; $45177\frac{53}{132}$; $23294\frac{153}{256}$.
39. 99. **40.** 100101; 6.
41. Twenty-two thousand two hundred and sixty-five; 61.
42. 998000. **43.** 4322; 4321; 4320. **44.** 8789. **45.** 6024847158.
46. 26 rupees. 47. 15. **48.** 230. **49.** 5.
50. 659305. **51.** 1591. **52.** (1) 394974; (2) 1292; (3) 12; (4) 1627.
53. (1) 0; (2) 6; (3) 4; (4) 343; (5) 33; (6) 1173; (7) 236304.
54. 11 o'clock. 55. 0. **56.** 3330. 57. 36288.
58. A scores 36 runs; B, 49 runs; C, 52 runs. **59.** 378 rupees. **60.** **35.**

Ex. 10. Page 39.

1. $2 \times 2 \times 3 \times 7$. 2. $2 \times 2 \times 3 \times 11$. 3. $2 \times 3 \times 13$.
4. $2 \times 2 \times 2 \times 3 \times 7$. 5. $2 \times 3 \times 19$. 6. $2 \times 3 \times 5 \times 7$.
7. $2 \times 2 \times 2 \times 7 \times 7$. 8. $3 \times 3 \times 3 \times 5 \times 5$. 9. 2^{10}.
10. $2^3 \times 13^2$. 11. $2^4 \times 3^3 \times 5$. 12. $2^3 \times 3^2 \times 7$.
13. 5×113. 14. $2^2 \times 7 \times 17$. 15. $3 \times 3 \times 11 \times 17$.
16. $3 \times 3 \times 3 \times 5 \times 11$. 17. $5 \times 5 \times 7 \times 13$. 18. $11 \times 17 \times 17$.
19. 2^{12}. 20. $2^3 \times 3^6$. 21. 2×3^8. 22. $2 \times 3^2 \times 5 \times 7^2 \times 11$.
23. $3 \times 5 \times 29^2$. 24. $2 \times 3 \times 7^2 \times 13^2$. 25. $2^{10} \times 3^2 \times 7$.
26. $2^4 \times 3^2 \times 7^2 \times 11$. 27. $5^4 \times 2^2 \times 11 \times 3$. 28. $2^4 \times 3^2 \times 13 \times 5$.
29. $5^2 \times 37^2$. 30. 61, 67, 71, 73 ; 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223 ; 839, 853, 857, 859, 863, 877, 881, 883, 887, 907.

Ex. 11. Page 41.

1. 6 ; 7 ; 8. 2. 12. 3. 17. 4. 14. 5. 13.
6. 29. 7. 34. 8. 36. 9. 82. 10. 722.
11. 23. 12. 68. 13. 3. 14. 8371. 15. 2134.
16. 46. 17. 476. 18. 8910. 19. 18. 20. 2003.

Ex. 12. Page 44.

1. 24. 2. 36. 3. 60. 4. 42. 5. 72.
6. 48. 7. 24. 8. 75. 9. 24. 10. 24.
11. 8125. 12. 1200. 13. 1716. 14. 336. 15. 1575.
16. 1008. 17. 3360. 18. 1560. 19. 756. 20. 11424.
21. 480. 22. 42504. 23. 10101. 24. 2464770. 25. 3051540.

Ex. 13. Page 46.

1. 6. 2. 212 and 265. 3. 632 and 474. 4. 19.
5. 432. 6. 8. 7. 117, 23400. 8. 19, 88179.
9. 799. 10. 27720. 11. 7200 rupees. 12. 504 rupees.
13. 9, 7 and 11. 14. After 180 minutes ; 4 times and 3 times.
15. 289. 16. After 60 minutes. 17. 7875.
18. 336, 384. 19. 580. 20. 78 miles an hour. 21. 47.
22. 11951. 23. 90. 24. 104. 25. 990. 26. 210
27. 192. 28. 1260. 29. 720. 30. 5040 rupees.
31. 37 beggars. 32. After 80 minutes ; 5 shots and 4 shots.

Ex. 14. Page 49.

1. A proper fraction. **2.** A mixed number.
3. A proper fraction. **4.** An improper fraction.
5. A complex fraction. **6.** A mixed number.
7. A compound fraction. **8.** A complex fraction.
9. $\frac{3}{7}$. 10. $8\frac{2}{3}$. 11. $\frac{9}{5}$. 12. $\frac{12}{37}$.
13. $\frac{9}{10}$. 14. $\frac{75}{36}$. 15. $10\frac{1}{2}$. 16. $\frac{1}{125}$.
17. Three-eighths. 18. Seven and four-fifths.
19. Fifty-five ninths. 20. Forty-one fiftieths.
21. Five and three-fourths. 22. Ninety-three upon one-half.
23. Sixty-three hundred and eighths.
24. Five hundred and sixty-one thousandths. 25. $\frac{57}{83}$; $\frac{150}{83}$.

Ex. 15. Page 49.

1. $\frac{17}{7}$. 2. $\frac{52}{9}$. 3. $\frac{67}{5}$. 4. $\frac{263}{132}$. 5. $\frac{527}{125}$.
6. $\frac{75}{4}$. 7. $\frac{415}{16}$. 8. $\frac{3653}{60}$. 9. $\frac{35}{6}$. 10. $\frac{10001}{100}$.

Ex. 16. Page 50.

1. $3\frac{1}{8}$. 2. $8\frac{3}{5}$. 3. $9\frac{2}{3}$. 4. $105\frac{3}{7}$. 5. $2\frac{1}{11}$. 6. $3\frac{2}{3}$.
7. $1\frac{7}{9}$. 8. $6\frac{1}{2}$. 9. $76\frac{12}{13}$. 10. $15\frac{5}{6}$. 11. $100\frac{4}{5}$.

Ex. 17. Page 51.

1. $\frac{18}{9}, \frac{45}{9}, \frac{63}{9}$. 2. $\frac{1450}{50}, \frac{1600}{50}, \frac{2150}{50}$.
3. $\frac{207}{9}, \frac{54}{9}, \frac{288}{9}$. 4. $\frac{1000}{8}$. 5. $\frac{792}{8}, \frac{1188}{12}, \frac{6237}{63}, \frac{9999}{101}$.

Ex. 18. Page 52.

1. $\frac{3}{4}$. 2. $\frac{3}{4}$. 3. $\frac{3}{5}$. 4. $\frac{13}{19}$. 5. $\frac{27}{31}$. 6. $\frac{5}{7}$.
7. $\frac{4}{5}$. 8. $\frac{114}{145}$. 9. $\frac{2}{3}$. 10. $\frac{15}{16}$. 11. $\frac{3}{4}$.
12. $\frac{9}{20}$. 13. $\frac{21}{25}$. 14. $\frac{285}{296}$. 15. $\frac{9}{11}$. 16. $\frac{3}{5}$.
17. $\frac{2314}{3721}$. 18. $\frac{7}{9}$. 19. $\frac{21}{22}$. 20. $\frac{11}{13}$. 21. $\frac{2592}{3457}$.
22. $\frac{557}{839}$. 23. $\frac{68}{125}$. 24. $\frac{1211}{1389}$. 25. $\frac{11}{12}$. 26. $\frac{61}{73}$.
27. $\frac{10}{77}$. 28. $\frac{7}{13}$. 29. $\frac{17}{143}$. 30. $\frac{3}{7}$. 31. $\frac{10}{13}$.

Ex. 19. Page 53.

1. $\frac{5}{9}$. 2. $\frac{7}{18}$. 3. $2\frac{1}{4}$. 4. 32. 5. $1\frac{3}{4}$.
6. $\frac{1}{5}$. 7. $1\frac{1}{4}$. 8. 8. 9. 5. 10. 7.
11. 45. 12. $48\frac{3}{7}$. 13. $\frac{2}{7}$. 14. $\frac{1}{2}$. 15. $3\frac{3}{8}$.
16. $48\frac{2}{5}$. 17. 4. 18. $\frac{23}{45}$. 19. 6. 20. $4\frac{13}{18}$.

Ex. 20. Page 55.

1. $\frac{8}{12}, \frac{9}{12}$. 2. $\frac{28}{35}, \frac{15}{35}$. 3. $\frac{15}{36}, \frac{8}{36}$.
4. $\frac{84}{175}, \frac{85}{175}$ 5. $\frac{45}{240}, \frac{44}{240}$. 6. $\frac{7}{504}, \frac{6}{504}$.
7. $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$. 8. $\frac{126}{315}, \frac{135}{315}, \frac{175}{315}$. 9. $\frac{12}{144}, \frac{9}{144}, \frac{8}{144}$.
10. $\frac{9}{21}, \frac{84}{21}, \frac{5}{21}$. 11. $\frac{2}{18}, \frac{63}{18}, \frac{36}{18}$. 12. $\frac{63}{1575}, \frac{70}{1575}, \frac{135}{1575}$.
13. $\frac{420}{720}, \frac{405}{720}, \frac{36}{720}, \frac{200}{720}$. 14. $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{48}{60}$.
15. $\frac{15}{21}, \frac{84}{21}, \frac{133}{21}, \frac{2}{21}$. 16. $\frac{198}{660}, \frac{30}{660}, \frac{108}{660}, \frac{77}{660}$. 17. $\frac{7}{16}$,
18. $\frac{3}{4}$ of $\frac{8}{15}$. 19. $\frac{12}{25}$ is the greatest, $\frac{3}{8}$ the least.
20. $\frac{8}{15}$ of $1\frac{1}{4}, \frac{9}{20}, \frac{5}{12}, \frac{7}{18}$. 21. $\frac{4}{5}, \frac{4+5}{5+7}, \frac{5}{7}$.

Ex. 21. Page 58.

1. $\frac{5}{6}$. 2. $1\frac{11}{18}$. 3. $1\frac{19}{30}$. 4. $1\frac{7}{48}$. 5. 1.
6. $\frac{5}{96}$. 7. $2\frac{13}{60}$. 8. $1\frac{8}{45}$. 9. $1\frac{23}{30}$✓ 10. $3\frac{163}{240}$.
11. $\frac{533}{10080}$. 12. $8\frac{73}{200}$. 13. $5\frac{3}{4}$. 14. $1\frac{13}{30}$. 15. $2\frac{1613}{3400}$.
16. $22\frac{2}{3}$. 17. $\frac{14}{15}$. 18. $16\frac{9}{20}$. 19. $2\frac{1}{6}$. 20. 1.
21. $11\frac{31}{96}$. 22. $14\frac{37}{60}$. 23. $5\frac{3}{32}$. 24. 670.

Ex. 22. Page 61.

1. $\frac{1}{5}$. 2. $\frac{1}{12}$. 3. $\frac{1}{48}$. 4. $\frac{25}{288}$. 5. $\frac{1}{60}$.
6. $\frac{1}{2907}$. 7. $1\frac{5}{12}$. 8. $\frac{9}{20}$. 9. $7\frac{1}{24}$. 10. $1\frac{61}{63}$.
11. $\frac{9}{16}$. 12. $\frac{134}{675}$. 13. $\frac{3}{224}$. 14. $\frac{19}{2618}$. 15. $\frac{1}{56}$.
16. $\frac{5}{9}$. 17. $\frac{47}{48}$. 18. $\frac{1}{24}$.
19. $\frac{6331}{15584}$; by $\frac{1}{96}$. 20. $2\frac{211}{240}$. 21. $\frac{4}{21}$.
22. $10\frac{6}{7}$. 23. $16\frac{59}{60}$. 24. $\frac{7}{13}$.
25. $2\frac{1}{3}$ of $5\frac{1}{6}$; by $\frac{49}{72}$. 26. 28. 27. $9\frac{85}{126}$.
28. $1\frac{1}{16}$. 29. $\frac{512}{1275}$. 30. $\frac{17}{60}$.

Ex. **23**. Page **63**.

1. $1\frac{1}{2}$. 2. $4\frac{2}{3}$. 3. $3\frac{1}{3}$. 4. $\frac{2}{5}$. 5. $\frac{3}{8}$.
6. $\frac{14}{15}$. 7. $1\frac{1}{3}$. 8. $6\frac{1}{4}$. 9. $1\frac{1}{8}$. 10. 22.
11. $\frac{4}{9}$. 12. $\frac{8}{105}$. 13. $3\frac{3}{4}$. 14. $1\frac{1}{18}$. 15. $\frac{4}{15}$.
16. $5\frac{9}{11}$. 17. $\frac{17}{144}$. 18. $\frac{2}{125}$. 19. $\frac{13}{120}$. 20. 1.
21. $234\frac{2}{3}$. 22. 3. 23. 1. 24. 64. 25. $5\frac{3}{40}$.

Ex. **24**. Page **66**.

1. $\frac{1}{14}$. 2. $\frac{2}{33}$. 3. $\frac{3}{52}$. 4. $1\frac{1}{2}$. 5. $1\frac{1}{4}$.
6. $1\frac{7}{20}$. 7. $2\frac{2}{3}$. 8. $\frac{2}{3}$. 9. $\frac{4}{5}$. 10. 16.
11. $\frac{7}{40}$. 12. $8\frac{4}{7}$. 13. $1\frac{32}{45}$. 14. $16\frac{1}{2}$. 15. $\frac{75}{112}$.
16. $1\frac{3}{7}$. 17. $\frac{19}{24}$. 18. $\frac{2}{3}$. 19. $1\frac{1}{9}$. 20. $\frac{2}{93}$.
21. $7\frac{1}{2}$; $\frac{2}{9}$; $\frac{7}{25}$. 22. $\frac{45}{841}$; $\frac{255}{364}$. 23. $9\frac{10}{27}$.
24. $\frac{1191}{1862}$. 25. 999. 26. $1\frac{1}{5}$; $\frac{5}{7}$; $\frac{24}{77}$; $\frac{49}{52}$.
27. $1\frac{2}{3}$. 28. $1\frac{17}{100}$. 29. 3. 30. 6.

Ex. **25**. Page **69**.

1. $\frac{1}{36}$. 2. $\frac{1}{161}$. 3. $\frac{5}{66}$. 4. $\frac{2}{9}$.
5. $\frac{1}{8}$. 6. $\frac{1}{108}$. 7. $\frac{2}{1827}$. 8. $\frac{5}{1984}$.
9. $3\frac{3}{5}$. 10. $33\frac{1}{3}$. 11. $7\frac{7}{33}$. 12. $53\frac{1}{3}$.
13. $\frac{2}{3}$. 14. $\frac{1}{8}$. 15. $404811109\frac{3}{4}$. 16. $\frac{5}{72}$; $405166\frac{2}{3}$.
17. $\frac{3}{16}$. 18. $41\frac{1}{7}$. 19. $58\frac{1}{3}$. 20. $\frac{16}{35}$.

Ex. **26**. Page **72**.

1. $12\frac{7}{8}$ rupees. 2. $\frac{13}{60}$. 3. 720. 4. $\frac{1}{12}$. 5. 450.
6. $\frac{135}{315}$, $\frac{175}{315}$, $\frac{126}{315}$, $\frac{105}{315}$. 7. $4\frac{22}{51}$. 8. $16\frac{29}{40}$. 9. $\frac{3}{8}$.
10. $4\frac{5}{24}$, $2\frac{23}{24}$, $2\frac{23}{96}$. 11. 2187 rupees. 12. $\frac{37}{78}$, $\frac{41}{78}$.
13. 18. 14. $\frac{5}{18}$. 15. $\frac{51}{65}$. 16. 4 times. 17. $\frac{20}{63}$. 18. $5\frac{31}{40}$.
19. $\frac{1424}{3725}$. 20. $6\frac{35}{299}$. 21. 1. 22. $\frac{10}{9}$. 23. 1. 24. 1.
25. $\frac{39}{161}$, $\frac{47}{194}$, $\frac{8}{33}$; $\frac{33}{161}$. 26. $2\frac{71}{91}$. 27. 2. 28. 18.
29. $7\frac{1}{9}$. 30. 360 gallons. 31. $\frac{1}{6}$. 32. $1\frac{95}{147}$. 33. $2\frac{7}{9}$.
34. $1\frac{7}{9}$. 35. $1\frac{2}{3}$. 36. 8. 37. $1\frac{1}{3}$. 38. $3\frac{1}{9}$. 39. $5\frac{1}{4}$.
40. $2\frac{25}{64}$. 41. $\frac{4}{9}$. 42. 1. 43. $\frac{56}{143}$ is the greater; 221.
44. $\frac{5}{2574}$; 1. 45. $\frac{7}{44}$. 46. $1\frac{313}{500}$. 47. 8901.

48. $\frac{68}{123}$; $\frac{1760}{2261}$. 49. $\frac{43}{88}$, $\frac{45}{88}$; 1. 50. 15. 51. 1.
52. $\frac{3}{8}$. 53. 66. 54. $\frac{1}{7}$. 55. 1. 56. $2\frac{1}{6}$. 57. $\frac{65}{168}$.
58. 4. 59. $\frac{1}{1512}$. 60. 2. 61. 9. 62. 2.
63. 1. 64. 1. 65. $\frac{1}{9}$. 66. $\frac{1}{32}$. 67. $\frac{1}{16}$.
68. 855 rupees. 69. $\frac{3}{13}$, $\frac{5}{13}$, $\frac{7}{13}$ and $\frac{11}{13}$. 70. 36. 71. $\frac{2}{41}$.
72. Between $5\frac{2}{5}$ and $7\frac{1}{2}$. 73. 20. 74. 19. 75. 30.

Ex. 27. Page 78.

1. 45·3. 2. ·09. 3. 1·9. 4. ·753.
5. ·0053. 6. ·0001. 7. ·85779. 8. ·026.
9. 2060. 10. 5·017. 11. 189·223. 12. ·0111.
13. ·423. 14. ·0999. 15. 16·05. 16. ·000049.
17. ·0123456. 18. ·479. 19. 25·305007.
20. Twenty; five-hundredths; 1st six, six-units; 2nd six, six ten-thousandths.

Ex. 28. Page 79.

1. $\frac{1}{5}$. 2. $2\frac{1}{2}$. 3. $\frac{2}{25}$. 4. $4\frac{1}{20}$.
5. $1\frac{3}{4}$. 6. $\frac{1}{250}$. 7. $\frac{5}{16}$. 8. $6\frac{3}{16}$.
9. $\frac{7}{80}$. 10. $40\frac{1}{80}$. 11. $\frac{5}{64}$. 12. $\frac{32}{78125}$.
13. $2\frac{64}{625}$. 14. $25\frac{36}{625}$. 15. $28\frac{32}{125}$. 16. $\frac{5}{32}$.
17. $\frac{2041}{200000}$. 18. $\frac{9}{160}$. 19. $3\frac{81}{6250}$. 20. $\frac{31}{3200}$.

Ex. 29. Page 81.

1. 65·218; 6521·8. 2. 450·32, 4503·2; ·85, 8·5.
3. 2·43812; ·243812; ·0243812.
4. 3·7921, ·37921, ·037921; ·0008453, ·00008453, ·000008453.
5. 16000. 6. ·000001. 7. 5·60000; ·07500; 12·45600.
8. 24·000; 2·400. 9. ·13000. 10. Two.

Ex. 30. Page 82.

1. 183·7275. 2. 156·22385. 3. 100·077.
4. 999·9999. 5. 13·88889. 6. 101·01.
7. 781·46875. 8. 171·641984. 9. 28·2003.
10. 1001. 11. 388·1115. 12. 172·454.
13. 452·089. 14. 100·00001. 15. 355·087.
16. 1·371602. 17. 109728·3594. 18. 74249·089299.
19. 247370·28. 20. 1030·179561.

Ex. 31. Page 83.

1. 11·11. 2. ·9249. 3. 100·9899. 4. 36·891.
5. 99·999. 6. ·04995. 7. 111·111. 8. 2222·214.
9. 10·8889. 10. 76·77777. 11. 7·00605. 12. 1·00243.
13. ·032967. 14. 3·6. 15. ·9996. 16. 8546·68464.
17. ·015. 18. 16·5509. 19. 168·2095. 20. 11·945388.

Ex. 32. Page 85.

1. 23·2766. 2. 128·36025. 3. 14530·625.
4. ·0323703. 5. 9·090564. 6. 48·0384.
7. ·0207414. 8. 2·712464. 9. 161·68972.
10. ·14186. 11. ·1. 12. 4·4106306765.
13. 3·34931292023. 14. ·049265256. 15. 10·84588703.
16. 1·154212711. 17. ·1481882853. 18. ·412340073159.
19. ·5201607228. 20. 58·101136574. 21. 8·556.
22. 41107·5. 23. 4·75. 24. 16.
25. 1·0098. 26. ·01 ; ·008. 27. 1562·5.
28. ·01. 29. 86·4475. 30. 1.

Ex. 33. Page 88.

1. 9·236. 2. ·0718. 3. 4. 4. 30·1. 5. ·25.
6. ·00025. 7. 400. 8. ·00716. 9. ·64. 10. ·0390625.
11. 1·90451660l5625. 12. ·0057373046875. 13. ·00008.
14. 800. 15. ·008. 16. ·002. 17. 3122. 18. 119·94.
19. 13316·875. 20. ·15279375. 21. ·4. 22. 47·64375.
23. ·15625. 24. 1963·4375. 25. 1804·8. 26. 170790.
27. 1200000. 28. ·0039. 29. ·00357. 30. 376. 31. 108·8781609.
32. 6·7246422. 33. 5·9106713. 34. 19·7946859. 35. ·3183101.

Ex. 34. Page 90.

1. ·08. 2. ·28. 3. ·08. 4. ·26.
5. ·9. 6. ·022. 7. ·24. 8. ·003.
9. 138·6. 10. 6·8. 11. 12·5. 12. ·36.
13. 21·6. 14. 42. 15. 147. 16. 546.
17. ·008 ; 22·4. 18. ·014 ; 196. 19. ·02 ; 2. 20. ·05 ; 3.

Ex. 35. Page 95.

1. ·6 ; ·15625.   2. ·34 ; ·4.   3. $\cdot 58\dot{3}$; ·69921875.
4. 1·432.   5. ·5796875.   6. 1·392578125.
7. $\cdot\dot{3}$; $\cdot\dot{1}4285\dot{7}$.   8. ·125 ; $\cdot\dot{0}7692\dot{3}$.   9. 6·9136.
10. $\cdot 641\dot{6}$.   11. $\cdot 00130208\dot{3}$.   12. $3\cdot 15\dot{9}\dot{0}$.
13. Recurring ; terminating ; recurring ; terminating ; terminating ; recurring.
14. $\cdot 2\dot{7}$; ·28 ; $\cdot 541\dot{6}$; ·00032 ; ·996 ; $\cdot 9\dot{1}4285\dot{7}$.
15. $\frac{7}{20}$; $\frac{3}{4}$.   16. $\frac{1}{8}$; $1\frac{1}{4}$.   17. $\frac{1}{3}$; $\frac{7}{111}$.   18. $\frac{13}{80}$.
19. $\frac{7}{80}$.   20. $6\frac{31}{6250}$.   21. $\frac{11}{30}$.   22. $\frac{3}{11}$; $\frac{5}{18}$.
23. $\frac{8}{45}$; $\frac{1}{75}$.   24. $\frac{4}{9}$; $\frac{2}{495}$.   25. $\frac{1}{11}$; $\frac{20}{33}$.   26. $\frac{13}{33}$; $\frac{2}{5}$.
27. $\frac{49}{222}$.   28. $\frac{51}{550}$.   29. $\frac{24}{27775}$.   30. $\frac{7}{55}$.
31. $\frac{17}{72}$.   32. $\frac{161}{300}$.   33. $\frac{13}{396}$.   34. $\frac{115}{132}$.
35. $\frac{119}{120}$.   36. $\frac{17463}{55550}$.   37. $\frac{491}{630}$.   38. $\frac{1}{73}$.
39. $\frac{2849}{799200}$.   40. $\frac{3}{28}$.   41. ·04513.   42. 13·14159.
43. 1·76430.   44. ·428.   45. ·0075.   46. ·2963.
47. ·714 ; ·230 ; ·941.   48. 3·1415929.   49. ·2631579.
50. 2·6470588.

Ex. 36. Page 97.

1. $10\cdot\dot{2}\dot{4}$.   2. $2\cdot 3\dot{6}$.   3. $4\cdot\dot{9}02\dot{4}$; $1\cdot\dot{4}57\dot{9}$.
4. $7\cdot 6\dot{4}2857\dot{1}$; $2\cdot 9\dot{2}8571\dot{4}$.   5. $1\cdot 08\dot{8}4\dot{5}$; $0\cdot 43\dot{8}2\dot{0}$.
6. $0\cdot 0\dot{1}\dot{6}$.   7. $0\cdot\dot{2}$.   8. 0·25.
9. $34\cdot\dot{3}98\dot{6}$.   10. $0\cdot 0\dot{3}\dot{5}$.   11. $0\cdot 198\dot{3}$.
12. 1·5.   13. ·25.   14. $0\cdot 091\dot{6}\dot{3}$.
15. 1·1.   16. 117·9056.   17. 13·5111.
18. 6·8291.   19. 5·5814.   20. 2·0906.
21. $\frac{37}{54}$.   22. $\frac{3}{8}$.   23. $\frac{1}{9}$.   24. $7\frac{3}{55}$.   25. $6\frac{19}{100}$.   26. 2.
27. $20\frac{1}{4}$.   28. $\frac{19}{60}$.   29. $5\frac{37}{55}$.   30. 1.

Ex. 37. Page 100.

1. ·92852.   2. ·229975.   3. $73\cdot 52\dot{5}2\dot{4}$.
4. $\cdot 02\dot{0}2\dot{7}$.   5. $\frac{2}{55}$, $\frac{3}{1600}$, $\frac{217}{555}$, $\frac{25}{27}$.   6. $6\cdot\dot{0}\dot{9}$.
7. $1\cdot\dot{2}6373\dot{6}$.   8. 10 ; ·1 ; $1\cdot\dot{0}4761\dot{9}$.   9. ·007.
10. ·3.   11. 2.   12. 44·153157 ; 11·569328 ; $\cdot\dot{4}950\dot{0}$; 50000.

13. $6{\cdot}5\dot{5}0\dot{7}$. 14. 10·532646. 15. ·06321782 ; ·000006321782.
16. 60320000 ; 20500. 17. 292·5. 18. ·16. 19. ·999.
20. ·407597. 21. $\cdot079\dot{2}$. 22. 4·05536 : $5{\cdot}07\dot{9}6\dot{2}$; $\frac{15}{16}$, $\frac{133}{270}$.
23. ·00018. 24. ·3125. 25. $\cdot0348\dot{3}$. 26. 1·499314.
27. ·0884. 28. $\cdot30\dot{5}$. 29. ·04 ; 18. 30. $7{\cdot}\dot{3}\dot{0}$.
31. ·199999. 32. 270·33006. 33. 2·05467. 34. $\cdot18\dot{3}$.
35. 1·04907.... 36. $\cdot000\dot{5}3\dot{5}$; $\frac{107}{199800}$. 37. ·4047. 38. $\cdot\dot{4}$.
39. 3 ; $267{\cdot}\dot{4}0259\dot{7}$. 40. $\cdot\dot{4}5\dot{9}$; ·046875. 41. $925{\cdot}53\dot{1}$.
42. 3333. 43. 5·90625. 44. 6. 45. $10313{\cdot}\dot{3}$. 46. $5{\cdot}\dot{0}731\dot{7}$.
47. ·0481. 48. ·0209. 49. ·08. 50. $36{\cdot}21\dot{4}\dot{6}$. 51. $3{\cdot}7\dot{6}$.
52. 3·2. 53. 562·1. 54. 1. 55. ·12. 56. 90·625.
57. $11344{\cdot}\dot{6}$. 58. $41{\cdot}\dot{6}$. 59. $\cdot8\dot{3}$. 60. 998·001.

Ex. 38. Page 108.

1. 165. 2. 371. 3. 199. 4. 1234. 5. 345.
6. 592. 7. 908. 8. 499. 9. 9876. 10. 740.
11. 723. 12. 789. 13. 6201. 14. 9045. 15. 75416.
16. 50406. 17. 2809. 18. 2178476. 19. 8·04. 20. 45·6.
21. 6·78. 22. 1·018. 23. 924·5. 24. 32·7. 25. ·107.
26. ·1879. 27. 2·826. 28. $\frac{13}{16}$, $1\frac{1}{4}$. 29. ·774 ; $3\frac{1}{3}$. 30. ·721, 1·712.
31. ·3162; $\cdot\dot{6}$. 32. 5·982. 33. $50\frac{3}{19}$. 34. $1{\cdot}33\dot{6}$; ·8. 35. 705.
36. 34. 37. 208. 38. 432. 39. 729. 40. ·50636.
41. ·83847. 42. 1·118034. 43. ·65674. 44. 1·87417. 45. ·96177.
46. ·88192. 47. 96. 48. 482. 49. 507. 50. 3001.

Ex. 39. Page 112.

1. 24. 2. 56. 3. 68. 4. 32. 5. 73.
6. 84. 7. 144. 8. 125. 9. 187. 10. 121.
11. 345. 12. 567. 13. 503. 14. 678. 15. 408.
16. 3579. 17. 6543. 18. 1111. 19. 8729. 20. 1043.
21. 7777. 22. 40506. 23. 87435. 24. ·43. 25. ·91.
26. 2·49. 27. 85·7. 28. 4·05. 29. ·013. 30. 7·061.
31. 300·2. 32. $\frac{6}{7}$; $1\frac{1}{2}$. 33. $\frac{4}{5}$, $\frac{11}{15}$. 34. ·961 ; $\frac{1}{3}$.
35. 1·259 ; 1·442. 36. ·908, ·895, ·986.
37. 23. 38. 57. 39. 1·01. 40. 4·68.

Ex. 40. Page 125.

1. 3456 p. **2.** 168 p. **3.** 71424 p. **4.** 924 p.
5. 1764 p. **6.** 12624 p. **7.** 8640 d. **8.** 204 d.
9. 2076 d. **10.** 2940 d. **11.** 10812 d. **12.** 5136 d.
13. 2100 d. **14.** 1356 d. **15.** 1788 d. **16.** 2080 farthings.
17. 17652 farthings. **18.** 876 farthings. **19.** 24988 farthings.
20. 1017 p. **21.** 7363 farthings. **22.** 868 lbs.
23. 11424 lbs. **24.** 20216 lbs. **25.** 44576 lbs.
26. 564 lbs. **27.** 177 lbs. **28.** 7904681 drs. **29.** 55219 grs.
30. 180 in. **31.** 132 in. **32.** 156 in. **33.** 55512 in.
34. 609840 in. **35.** 158400 in. **36.** 232419 in. **37.** 7 pts.
38. 12 pts. **39.** 42 pts. **40.** 40 pts. **41.** 2096 pts.
42. 456 pts. **43.** 79 pts. **44.** 2556 pts. **45.** 150 srs.
46. 30720 secs. **47.** 1689 secs. **48.** 201600 secs. **49.** 1728000 secs.
50. 346740 secs. **51.** 64815 secs. **52.** 337778 secs. **53.** 1919 dys.
54. 42350 sq. yds. **55.** $246730\frac{1}{2}$ sq. ft. **56.** 43296 cub. in.
57. 152880 secs. **58.** 512 gunjas. **59.** 260 srs. **60.** 48960.

Ex. 41. Page 128.

1. 8 as. ; 14 as. ; 21 as. ; 63 as. **2.** 11 s. ; 28 s. ; 36 s. ; 43 s. ; 79 s.
3. Rs.11 ; Rs.14 ; Rs.19 ; Rs.48 ; Rs.256.
4. 21 guin. ; 35 guin. ; 47 guin. ; 59 guin. ; 84 guin.
5. 11 qrs. ; 28 qrs. ; 112 qrs. ; 158 qrs.
6. 16 dwts. ; 24 dwts. ; 96 dwts. **7.** 21 yds. ; 36 yds. ; 144 yds.
8. 64 gals. ; 128 gals. ; 968 gals. ; $1200\frac{1}{2}$ gals.
9. Rs.2. 3 as. 1 p. ; Rs.2. 12 as. 2 p. ; Rs.3. 12 as. 11 p. ; Rs.4. 6 as. 7 p. ; Rs.5. 3 as. 4 p.
10. Rs.4. 13 as. 4 p. ; Rs.6. 14 as. 4 p. ; Rs.9. 7 as. 9 p. ; Rs.10. 6 as. 8 p. ; Rs.282. 14 as. 9 p.
11. Rs.31. 4 as. ; Rs.46. 10 as. ; Rs.515. 10 as. ; Rs.771. 9 as. ; Rs.637. 11 as.
12. £2. 18s. 4d. ; £3. 10s. 5d. ; £3. 17s. 2d. ; £4. 5s. ; £167. 18s. 5d.
13. £29. 13s. ; £32. 8s. ; £356. 4s. ; £9234. 12s.
14. 3 cwts. 3 qrs. 3 lbs. ; 3 tons 6 cwts. 3 qrs. 19 lbs. ; 1 ton 4 cwts. 1 qr. 14 lbs. 9 oz. ; 8 cwts. 1 qr. 5 lbs. 9 oz. 13 drams.
15. 2 tons 9 cwts. 2 qrs. ; 48 tons 7 cwts. ; 4 tons 14 cwts. 0 qr. 4 lbs.; 322 tons 11 cwts.
16. 17 po. 4 yds. 0 ft. 7 in. ; 2 fur. 4 po. 2 yds. ; 2 fur. 13 po. 4 yds. 0 ft. 3 in. ; 1 m. 4 fur. 17 po. 2 yds. 1 ft. 6 in.

17. 1 fur. 26 po. 2 yds.; 45 m. 1 fur.; 19 m. 3 fur. 16 po. 4 yds.; 1 m. 4 fur. 18 po.

18. 3 po. 4 yds. 2 ft. 3 in.; 7 fur. 36 po.; 2 fur. 14 po. 3 yds.; 29 m. 6 fur.; 2 m. 7 fur. 39 po.

19. 3 yrs. 20 wks. 6 dys.; 14 yrs. 13 wks.; 2 yrs. 38 wks. 6 dys.; 11 yrs. 23 wks. 1 dy. 16 hrs.

20. 2 dys. 0 hr. 43 mins. 49 secs.; 5 dys. 20 hrs. 15 mins.; 19 yrs. 49 wks. 2 dys.

21. 27 yrs. 3 wks.; 1685 yrs. 34 wks.; 12 wks. 4 dys. 14 hrs. 12 mins. 1 sec.

22. 42 lbs. 9 oz.; 28 lbs. 10 oz. 1 dwt.; 14 lbs. 1 oz. 17 dwts. 19 grs.

23. 10 lbs. 3 oz.; 1 lb. 10 oz. 16 dwts.; 13 lbs. 8 oz. 7 dwts. 22 grs.

24. 4 ac. 0 ro. 8 po.; 1 ac. 1 ro. 18 po. 30 sq. yds. 72 sq. in.; 1 ac. 0 ro. 10 po. 26 sq. yds. 4 sq. ft. 72 sq. in.

25. 68 ac. 3 ro.; 3 ac. 1 ro. 7 po.; 1 ac. 2 ro. 8 po. 18 sq. yds.

26. 6 sq. po. 8 sq. yds. 8 sq. ft. 66 sq. in.; 1 ac.; 46 ac. 2 ro. 17 po. 26 yds. 2 sq. ft. 108 sq. in.

27. 16 cub. yds. 6 cub. ft.; 39 cub. yds. 1 cub. ft.; 1 cub. yd. 18 cub. ft. 1652 cub. in.

28. 64 cub. yds. 26 cub. ft.; 8 cub. yds. 5 cub. ft. 507 cub. in.

29. 25 maunds.

30. 4 lbs. 2 oz.

31. 432 lbs.

32. 6820 guin.

33. 16 English ells 1 qr.

34. 10 tolas 5 mâsâs.

35. 9 khandis 3 maunds 5 seers.

36. £40. 19s.

37. 48 guin. 12s.

38. £22.

39. Rs.4005.

40. Rs.105.

Ex. 42. Page 131.

1. Rs.110. 10 as. 10 p.

2. £166. 8s. 11d.

3. £156. Rs.0. 0. as. 1 p.

4. 200 guin. 0s. 6d.

5. £16. 2s. $2\frac{1}{12}$d.

6. £20000.

7. 161 khandis 0 maunds 25 srs.

8. 29 lbs. 7 oz. 16 dwts. 15 grs.

9. 48 tolâs 5 masas.

10. 25 oz. 1 dr. 2 scrs. 7 grs.

11. 151 yds. 0 ft. 5 in.

12. 201 miles 0 fur. 0 po. 4½ yds.

13. 100 sq. yds.

14. 171 ac. 0 ro. 27 sq. po.

15. 10 ro. 0 sq. po. 29½ sq. yds.

16. 186 cub. yds. 11 cub. ft. 172 cub. in.

17. 58 hrs. 6 mins. 50 secs.

18. 14500 tons 1 cwt. 0 qr. 0 lb. 15 oz. 13 drs.

19. 80 miles 6 fur. 3 po. 2 yds. 1 ft. 5 in.

20. 19 wks. 5 dys. 15 hrs. 20 mins. 21 secs.

21. 31 qrs. 0 bus. 0 pk. 0 gal. 2 qts. 0 pt. 3 gills.

Ex. 43. Page 132.

1. Rs.12. 6 as. 3 p.
2. £88. 8s. 8d. 3 farthings.
3. Rs.5. 13 as. $10\frac{5}{8}$ p.
4. £24. 6s. $10\frac{13}{20}$d.
5. £9. 9 rs. 9 as. 9 p.
6. 4 guin. 3s. 2d. 1 farthing.
7. 14 cwt. 0 qr. 15 lbs.
8. 13 cwt. 0 lb. 13 oz.
9. 19 khandis 19 maunds 39 seers.
10. 10 lbs. 0 oz. 17 dwts. 22 grs.
11. 1 tola 9 mâsâs 6 gunjas.
12. 7 drs. 0 ser. 3 grs.
13. 9 yds. 0 ft. 9 in.
14. 8 miles 0 fur. 4 po. 1 yd.
15. 18 sq. yds. 8 sq. ft. 63 sq. in.
16. 3 sq. miles 318 ac. 0 r. 34 po.
17. 100 cub. yds. 10 cub. ft. 1000 cub. in.
18. 36 tons 17 cwt. 1 qr. 25 lbs. 7 oz. 4 drs.
19. 263 yrs. 11 wks. 4 dys. 22 hrs. 49 mins. 16 secs.
20. 7 fur. 20 po. 4 yds. 2 ft. 3 in.
21. 5 ac. 1 ro. 36 sq. po. 29 yds. 1 ft. 141 in.

Ex. 44. Page 135.

1. Rs.177. 10 as. 11 p.; Rs.248. 12 as. 1 p.; Rs.319. 13 as. 3 p.
2. Rs.938; Rs.1172. 8 as.; Rs.1407.
3. Rs.1151. 11 as. $11\frac{1}{4}$ p.; Rs.1316. 4 as. 6 p.; Rs.1535. 10 as. 7 p.
4. £211. 10s.; £317. 5s.; £793. 2s. 6d.
5. £417. 16s. $4\frac{1}{2}$d.; £1740. 18s. $2\frac{3}{4}$d.; £3342. 11s.
6. £1451. 6s. 1d.; £2475. 15s. 1d.; £4524. 13s. 1d.
7. 187 lbs. 14 oz. 8 drs.; 328 lbs. 13 oz. 6 drs.; 610 lbs. 11 oz. 2 drs.
8. 928 tons 9 cwt. 2 qrs. 11 lbs.; 2971 tons 2 cwt. 2 qrs. 24 lbs.; 4456 tons 14 cwt. 0 qr. 8 lbs.
9. 426 lbs. 1 oz. 17 dwts. $16\frac{2}{3}$ grs.: 891 lbs. 0 oz. 13 dwts. $8\frac{2}{3}$ grs.
10. 59 yds. 0 ft. $3\frac{1}{2}$ in.; 206 yds. 2 ft. $6\frac{1}{4}$ in.; 496 yds. 1 ft. 3 in.
11. 239 fur. 31 po. 1 yd. 1 ft. 6 in.; 299 fur. 29 po. 0 yd. 1 ft. 6 in.; 382 fur. 6 po. 1 yd. 1 ft. 6 in.
12. 1986 miles 2 fur. 4 po. 3 yds. 2 ft.; 2553 miles 6 fur. 6 po.; 18443 miles 7 fur. 3 po. 1 yd. 2 ft. 6 in.
13. 99 sq. yds. 8 sq. ft. 30 sq. in.; 299 sq. yds. 6 sq. ft. 90 sq. in.; 1198 sq. yds. 8 sq. ft. 72 sq. in.
14. 743 ac. 2 ro. 8 po. 12 sq. yds.; 1115 ac. 1 ro. 12 po. 18 yds.; 1394 ac. 0 ro. 25 po. 22 sq. yds. 4 sq. ft. 72 sq. in.
15. 2569 ac. 25 sq. po. $3\frac{1}{4}$ sq. yds.; 5260 ac. 2 ro. 24 sq. po. $22\frac{1}{2}$ sq. yds.; 5811 ac. 30 sq. po. 3 sq. yds. 108 sq. in.

16. 2670 cub. yds. 8 cub. ft. 1048 cub. in. ; 3624 cub. yds. 0 cub. ft. 188 cub. in.

17. 46655 cub. ft. **18.** 932 dys. 8 hrs. 36 mins.

19. 71 gals. 1 qt. ; 106 gals. 3 qts. 1 pt. ; 130 gals. 2 qts. 1 pt.

20. 592 bus. ; 906 bus. 2 pks.

21. 249 qrs. 1 bus. 1 pk. 1 gal. ; 276 qrs. 0 bus. 3 pks. 1 gal. ; 397 qrs. 2 bus. 2 pks. 1 gal.

22. 49 dys. 1 hr. 6 mins. 37 secs. ; 71 dys. 16 hrs. 23 mins. 31 secs. ; 116 dys. 22 hrs. 57 mins. 19 secs.

23. 40712 yrs. 339 dys. 12 hrs. ; 221229 yrs. 269 dys. 21 hrs.

24. 42148 khandis 19 maunds 15 seers ; 82611 khandis 19 maunds 23 seers.

25. 23729 tolas 2 masas ; 65255 tolas 2 masas 4 gunjâs.

Ex. 45. Page 139.

1. Rs.4. 6 as. 7 p. **2.** Rs.43. 13 as. $1\frac{1}{12}$p.

3. £17. 8s. 3d. **4.** £43. 1s. $0\frac{1}{2}$d.

5. £1364. 17s. $6\frac{1}{2}$d. **6.** £1365. 17s. 11d.

7. 1 ton 13 cwts. 0 qr. 10 lbs. 6 oz. 4 drs.

8. 11 cwts. 2 qrs. 18 lbs. 10 oz. 8 drs.

9. 2 cwts. 1 qr. 22 lbs. 9 oz. $13\frac{1}{9}$ drs.

10. 3 lbs. 7 oz. 17 dwts. 19 grs. **11.** 4 miles 7 fur. 8 po.

12. 15 yds. 2 ft. 11 in. **13.** 1 mile 3 fur. 10 po. 5 yds. 0 ft. 6 in.

14. 5 sq. yds. 4 sq. ft. 83 sq. in. **15.** 209 ac. 3 ro. 25 po. 3 yds.

16. 3 cub. yds. 16 cub. ft. 52 cub. in.

17. 4 yrs. 3 wks. 2 dys. 0 hr. 53 min. 20 secs.

18. 1 gal. 1 qt. 1 pt. **19.** 1 bus. 3 pks. 1 gal. 3 qts. 1 pt.

20. 1 khandi 4 maunds $35\frac{285}{392}$ seers.

21. 1 tola 8 masas $2\frac{39}{40}$ gunjas. **22.** 95. **23.** 38.

24. 168. **25.** 231. **26.** £57. 3s. $9\frac{21}{50}$d.

27. Rs.68. 6 as. $10\frac{29}{100}$ p. **28.** £48. 15s. $4\frac{2}{5}$d.

29. £43. 12s. $7\frac{227}{500}$d. **30.** 13.

Ex. 46. Page 142.

1. 13 as. 4 p. **2.** 12s. **3.** 2 as. $7\frac{1}{2}$ p.

4. 9s. **5.** R.1. 15 as. 6 p. **6.** 3s. 9d.

7. $2\frac{7}{24}$ pies. **8.** $6\frac{11}{48}$d. **9.** 1 a. 6 p. **10.** 8 cwts.

11. 3 qrs. 14 lbs. **12.** 8 lbs. **13.** 9 grs.

14. 1 maund $26\frac{2}{3}$ seers. **15.** 2 masas.
16. 2 fur. 35 po. 3 yds. 0 ft. 2 in. **17.** 30 po. **18.** 4 yds. 9 in.
19. 18 hrs. 40 mins. **20.** 52 secs. **21.** 6 dys.
22. Rs.2. 0 a. 7 p. **23.** £2. 12s. $11\frac{3}{4}$d.
24. 17 cwts. 3 qrs. $18\frac{6}{7}$ lbs. **25.** 1 khandi 2 maunds $7\frac{2}{3}$ seers.
26. 16 dys. 23 hrs. 2 mins. 21 secs.
27. 2 ac. 0 r. 35 po. 18 sq. yds. 3 sq. ft. 108 sq. in.
28. 6 fur. 20 po. 2 yds. 0 ft. $6\frac{54}{175}$ in. **29.** $\frac{1}{20}$. **30.** $\frac{5}{36}$.
31. $\frac{1}{20}$. **32.** $\frac{3}{22}$. **33.** $\frac{1}{8}$. **34.** $\frac{1}{6}$. **35.** $\frac{9}{56}$.
36. $\frac{1}{160}$. **37.** $\frac{1}{40}$. **38.** $\frac{11}{28}$. **39.** $\frac{1}{72}$. **40.** $\frac{13}{75}$.
41. $\frac{1}{8}$. **42.** $\frac{1}{8}$. **43.** $\frac{2}{3}$. **44.** $\frac{8}{125}$. **45.** $\frac{3}{2}$. **46.** £5.
47. £5. **48.** £10. 10s. $4\frac{7}{12}$d. **49.** £1. 5s. $8\frac{1}{2}$d. **50.** £10. 8s.

Ex. 47. Page 146.

1. 10 as. **2.** 2s. 6d. **3.** 9s. 5·4d. **4.** 6 pies.
5. 4·2d. **6.** 3 farthings. **7.** 12 cwts.
8. 3 qrs. 11 lbs. 3 oz. 3·2 drs. **9.** 25 lbs. 3 oz. 3·2 drs.
10. 9·888 grs. **11.** 9 masas 4·8 gunjas. **12.** 7 maunds 20 seers.
13. 14·4 srs. **14.** 2·16 gunjas. **15.** 12·8 drs.
16. 3 fur. 22 pos. 1 yd. 0 ft. 8 in. **17.** 6 pos. 3 yds. 2 ft.
18. 2 ft. 6 in. **19.** $2\frac{2}{9}$ pts. **20.** 1 pk. 0 gal. 1 qt. $\frac{22}{45}$ pt.
21. 1·6 pts. **22.** 21 pos. 24 yds. 6 sq. ft. 108 sq. in.
23. 120·96 cub. in. **24.** 16 mins. 30 secs. **25.** 10 as. 6·7875 pies.
26. £1. 0s. 2·5d. **27.** 3 qrs. 0 lb. 9·024 oz. **28.** 7 dwt. 16·96 grs.
29. 2 fur. 12 po. 1 yd. **30.** 1 sq. yd. 2 sq. ft. 81 sq. in.
31. ·21875. **32.** $\cdot 588541\dot{6}$. **33.** ·034375.
34. ·735. **35.** ·00234375. **36.** $\cdot 0069\dot{4}$.
37. $\cdot 0011\dot{3}\dot{6}$. **38.** $\cdot 221\dot{6}$. **39.** 6·4765625. **40.** 1·2.
41. Rs.68. 3 as. 1·2 p. **42.** 4·25d. **43.** Rs.4. 3 as. 1 p. **44.** 5·5 p.
45. 3 cwts. 2 qrs. 12 lbs. 2·4 oz. **46.** 6 hrs. 41 mins. 52·8 secs.
47. 2 pks. 0 gal. 2 qts. 1·11906 pts. **48.** 1 ro. 31·44 po.
49. ·375. **50.** 10.

Ex. 48. Page 150.

1. £26. 2s. 2d. 3 farthings. 2. 61600.
3. 189 tons 19 cwts. 1 qr. 19 lbs. ; 6 lbs. $6\frac{317}{324}$ oz.

4. 11 hrs. 1 min. 5. 128. 6. 30. 7. 2 pies.
8. 39600. 9. £21. 10. Rs.42. 8 as. 11. 1 anna.
12. 2 as. 8 p. 13. Rs.8566. 1 a. 6 p. 14. £4. 14s. $1\frac{1}{2}$d. gain.
15. R.1. 1 a. 6 p. 16. 119 ; 17. 17. £3215. 14s. 3d.
18. R.1. 5 as. 3 p. 19. 42 secs. 20. Rs.172. 11 as. 7 p.
21. Rs.68. 22. Rs.96. 10 as. $7\frac{3}{4}$ p. 23. 3s. 6d. 24. 5 lbs.
25. 6 as. 6 p. 26. Horses, £68. 6s. 5d., £94. 13s. 7d. ; carriage £87.
27. Rs.16. 4 as. 28. Rs.17.
29. Men, Rs.687. 8 as. ; women, Rs.802. 1 a. 4 p.
30. 2nd of April ; 1st of April. 31. 8s. $4\frac{5}{7}$d. 32. £4. 7s. $2\frac{1}{4}$d.
33. 7s. $10\frac{7}{8}$d. 34. £22. 16s. 35. $\cdot 000134150\dot{4}\dot{0}$.
36. 8s. $7\frac{3}{4}$d. ; $\frac{5}{24}$. 37. 12 days 2 hours. 38. $\frac{2}{5}$.
39. $\cdot 048958\dot{3}$. 40. 10s. $3\frac{3}{4}$d. ; ·515625.
41. ·000022957 or ·000022958. 42. $\frac{73}{234320}$. 43. 8 lbs.
44. £3. 6s. $4\frac{1}{4}$d. ; ·298828125. 45. ·00015625 ; 1 cwt. 1 qr. 20 lbs.
46. ·864. 47. 1s. $5\frac{1}{2}$d. ; 4s. $11\frac{1}{4}$d. 48. 34s. ; 4·08.
49. 17·8609375 ; 14·697. 50. £6. 5s. $2\frac{1}{4}$d. 51. 5.
52. $33\frac{1}{3}$. 53. £603. 5s. 54. 103·1515. 55. £200.
56. $\frac{4}{423}$. 57. £84. 16s. 6d. 58. 960.
59. A's share is Rs.5. 9 as. 3 p.; B's, R.3. 7 as. 9 p. ; C's, Rs.4. 13 as. 6 p.
60. 120. 61. 8, 5.

Ex. 49. Page 155.

1. Rs.73. 10 as. 2. £23. 4s. 3. Rs.77. 5 as. 6 p.
4. £98. 8s. 5. £57. 10s. 4d. 6. Rs.29. 11 as.
7. £14. 9s. 10d. 8. £254. 0s. 2d. 9. Rs.48. 0 a. 6 p.
10. Rs.257. 13 as. 4 p. 11. Rs.425. 4 as. 12. £625. 7s. 8d.
13. Rs.11. 3 as. 8 p. 14. £120. 5s. 15. £28. 0s. 6d.
16. £142. 10s. 17. £520. 18. £83. 6s. 6d.
19. Rs.964. 3 as. 6 p. 20. Rs.581. 4 as. 21. £6534.
22. £6238. 13s. $6\frac{1}{2}$d. 23. Rs.20110. 7 as. 4 p. 24. £9132.
25. £24466. 6s. 26. £33133. 2s. 6d. 27. £8715.
28. £356792. 16s. 29. £92239. 8s. 8d. 30. £24381. 18s. 9d.

Ex. 50. Page 157.

1. £53. 15s. $4\frac{7}{8}$d. 2. Rs.320. 13 as. $5\frac{503}{800}$ p. 3. Rs.1831. 4 as. $3\frac{23}{30}$ p.
4. Rs.190. 5 as. $4\frac{23}{24}$ p. 5. £7. 6s. 3d. 6. £5417. 13s. 8d.
7. £2764. qrs. 5 bus. $0\frac{1}{4}$ pk. 8. £8. 3s. $6\frac{1}{4}$d.

9. £6. 9s. $8\frac{29}{56}$d.
10. £54. 13s. 9d.
11. Rs.44. 13 as. $7\frac{17}{32}$d.
12. Rs.494. 3 as. $4\frac{1}{4}$ p.
13. 11s. 1d.
14. £603. 2s. 6d.
15. £52. 15s. 6d.
16. £90. 11s. $4\frac{11}{16}$d.
17. £622. 16s.
18. £20. 4s. $10\frac{1}{2}$d.
19. £723. 5s. $4\frac{1}{2}$d.
20. £1810. 13s. 6d.

Ex. 51. Page 159.

1.	£	s.	d.
	1	12	8
	0	2	$0\frac{3}{4}$
	0	2	$0\frac{1}{16}$
	1	8	$10\frac{1}{2}$
	0	15	9
	4	1	$4\frac{5}{16}$

2.	£	s.	d.
	0	10	$11\frac{3}{4}$
	1	3	$7\frac{1}{2}$
	0	10	6
	1	15	$7\frac{1}{2}$
	4	0	$8\frac{3}{4}$

3.	£	s.	d.
	48	2	6
	1	7	$6\frac{3}{4}$
	13	19	$9\frac{3}{4}$
	4	1	8
	67	11	$6\frac{1}{2}$

4.	£	s.	d.
	0	19	3
	20	10	0
	3	14	9
	25	4	0

5.	£	s.	d.
	0	6	$0\frac{1}{2}$
	8	6	$6\frac{3}{4}$
	0	1	$2\frac{1}{4}$
	1	1	$3\frac{3}{4}$
	9	15	$1\frac{1}{4}$

6.	£	s.	d.
	0	4	2
	1	2	0
	0	10	8
	0	5	9
	0	3	9
	0	8	3
	2	14	7

7.	£	s.	d.
	46	17	6
	17	6	6
	16	11	6
	6	17	6
	87	13	0

8.	£	s.	d.
	4	10	0
	0	14	8
	1	9	2
	1	1	0
	0	5	10
	8	0	8

9.	£	s.	d.
	0	6	$8\frac{3}{4}$
	0	17	$5\frac{1}{4}$
	0	8	$5\frac{1}{4}$
	0	7	$10\frac{1}{2}$
	0	1	11
	0	7	7
	0	11	$5\frac{1}{2}$
	3	1	$5\frac{1}{4}$

10.	£	s.	d.
	81	4	0
	130	13	4
	12	8	3
	23	12	0
	247	17	7
Deducting	24	15	$9\frac{1}{10}$
	223	1	$9\frac{9}{10}$

Ex. 52. Page 162.

1. Rs.3. 2 as.
2. Rs.5. 13 as.
3. Rs.185. 8 as.
4. 40 boys.
5. 28 st. $13\frac{1}{3}$ lbs.
6. Rs.5. 9 as. $7\frac{1}{5}$ p.
7. 75 miles.
8. 10 as.
9. 4 pies.

10. Rs.67. 8 as. **11.** Rs.963. 3 as. **12.** $11\frac{1}{4}$d.
13. Rs.241. 0 as. 3 p. **14.** Rs.63. **15.** Rs.8.
16. 30 days. **17.** 12 oxen. **18.** 21 ac.
19. Rs.672. 11 as. 3 p. **20.** Rs.364. **21.** 105 men.
22. 8 days. **23.** £441. **24.** Rs.5. 12 as. **25.** 5 months.
26. $20\frac{4}{7}$ miles. **27.** £40. 5s. **28.** £13. 0s. $7\frac{1}{2}$d. **29.** 5s. 9d.
30. £3. 4s. $7\frac{1}{2}$d. **31.** $35\frac{1}{4}$ miles. **32.** £2. 5s. 9d **33.** 24 men.
34. Rs.3827. 8 as. **35.** $203\frac{1}{16}$ miles. **36.** $11\frac{1}{5}$ hours. **37.** £10. 1s. $8\frac{3}{8}$d.
38. 6 hrs. **39.** £50. 8s. 9d. **40.** £50. 10s.

Ex. 53. Page 166.

1. 3 : 2. **2.** 5 : 3. **3.** 2 : 1. **4.** 7 : 11. **5.** 28 : 39.
6. 9 : 20. **7.** 4 : 5. **8.** 2 : 3. **9.** 21 : 1. **10.** 3 : 8.
11. 3 : 1. **12.** 5 : 8. **13.** $\frac{10}{3}$. **14.** $\frac{19}{23}$. **15.** $\frac{3}{2}$.
16. 6. **17.** 15. **18.** 4. **19.** 3. **20.** 5.
21. 98 : 103. **22.** Rs.4. 8 as. **23.** 232. **24.** 3. **25.** Rs.4. 10 as.

Ex. 54. Page 172.

1. 3645. **2.** Rs.15. **3.** 56 lbs.
4. Rs.587. 8 as. **5.** 2s. $8\frac{1}{2}$d. **6.** 4 as. 3 p.
7. £32. 5s. $0\frac{1}{2}$d. **8.** 55080. **9.** 2508. 5 as. 4 p.
10. £26. 6s. **11.** Rs.90. **12.** Rs.1128. 12 as.
13. £35. 9s. $4\frac{7}{8}$d. **14.** $2\frac{1}{2}$d. **15.** £591. 8s.
16. 72 days. **17.** 84 men. **18.** $44\frac{1}{3}$ days.
19. 20 more. **20.** 3 months. **21.** 144 yds.
22. 2 qrs. 19 lbs. **23.** 1 lb. 2 oz. **24.** Rs.433. 7 as.
25. £13. 10s. **26.** 70 miles. **27.** 16 days.
28. 24 days. **29.** 1 hr. $57\frac{1}{7}$ m. **30.** 160 ft.
31. 7 yds. 2 ft. $7\frac{1}{2}$ in. **32.** Rs.500. **33.** $1503\frac{1}{3}$ tons.
34. £500. **35.** £1134. 18s. $3\frac{1}{4}$d. **36.** Rs.160. 13 as. 4 p.
37. $6\frac{3}{4}$ srs. a rupee. **38.** 15 measures a rupee. 39. $6\frac{5}{6}$ months.
40. Rs.15. **41.** $9\frac{1}{3}$ hrs. **42.** 32 days.
43. 407 yds. **44.** 11 mths. $13\frac{2}{9}$ dys. **45.** £2142. 5s. $4\frac{1}{5}$d.
46. Rs.6. **47.** 5 as. $7\frac{4}{5}$ p. ; Rs.5498. 7 as. **48.** 6 miles.
49. £81. **50.** 63 hrs. **51.** 18 men.
52. Rs.13. **53.** Rs.2164. 6 as. $2\frac{1}{2}$ p. **54.** 1 ft. $9\frac{1}{7}$ in.
55. 2666. **56.** Rs.640. **57.** 200 days.
58. 24 seers. **59.** Rs. 15. **60.** 7.

Ex. 55. Page 179.

1. Rs.72. **2.** 12 days. **3.** $22\frac{1}{2}$ acres. **4.** Rs.22. 8 as.
5. Rs.640. **6.** $17\frac{1}{2}$ days. **7.** 4 cwt. 2 qrs. 1 lb. **8.** $82\frac{1}{2}$ miles.
9. 90 days. **10.** $14\frac{27}{35}$ days. **11.** $10\frac{2}{7}$ days. **12.** 20.
13. $10\frac{1}{8}$ days. **14.** 9 days. **15.** Rs.25600. **16.** 180 men.
17. £21. 7s. 6d. **18.** £50. **19.** $20\frac{1980}{3101}$ hrs.
20. 3 tons 18 cwts. 3 qrs. **21.** $12\frac{1}{2}$ days. **22.** 100800 gals.
23. 20 days. **24.** 2500 quires. **25.** 4s.
26. $9\frac{1}{3}$ oz. **27.** $7\frac{5}{7}$d. **28.** 20 days.
29. Rs.6. **30.** 6 men. **31.** $2\frac{1}{2}$ days. **32.** $9\frac{2}{3}$ hrs.

Ex. 56. Page 185.

1. 36 miles. **2.** 35 acres. **3.** $643\frac{1}{2}$ miles. **4.** $14\frac{1}{2}$ mins.
5. 32 days. **6.** Rs.4. 6 as. **7.** £230. **8.** 54.
9. $3\frac{9}{13}$ months. **10.** 160 men. **11.** 2 gals.
12. 100 men, 200 women and 400 boys.
13. 12. **14.** 36 men. **15.** £20. **16.** 17 dys.
17. 425 men. **18.** £4. 4s., £2. 2s., £1. 1s.
19. 9 miles. **20.** £272. 5s. **21.** 5 hrs. **22.** 2s. 4d.
23. Rs.645. **24.** Rs.801. **25.** 10 : 3. **26.** Rs.250.
27. 25 beggars. **28.** 35 seers. **29.** Rs.91. 10 as. 8 p. **30.** $\frac{15}{16}$ seer.
31. 9 dys. **32.** Rs.160. **33.** Rs.45.
34. 55 mins. past 10 o'clock. **35.** Rs.1466. 10 as. 8 p.
36. Rs.45. **37.** $\frac{3}{4}$ tola. **38.** £154. 3s. 4d. **39.** 6 hrs.
40. $7\frac{1}{5}$ oz. **41.** 150 men. **42.** 221 days. **43.** 150 men.
44. $92\frac{4}{7}$ yds. **45.** $2\frac{1}{2}$ ft. **46.** 276 tons. **47.** $6\frac{583}{856}$ days.
48. 3s. 4d. in the pound. **49.** 49·41 lbs. **50.** 36.

Ex. 57. Page 190.

1. $15\frac{3}{4}$ as. **2.** Rs.50. **3.** Rs.3. 15 as. **4.** Rs.75.
5. R.1. $1\frac{1}{7}$ as. **6.** 36 honns. **7.** $79\frac{1}{11}$ yards. **8.** 80.
9. 12 miles. **10.** Rs.6. 9 as. **11.** $22\frac{2}{5}$ lbs. **12.** $5\frac{1}{3}$ days.
13. Rs.1. 11 as. **14.** D will win by $128\frac{1}{3}$ yds. **15.** 6 mins. $32\frac{8}{11}$ secs.

Ex. 58. Page 192.

1. 34, 51. **2.** 27, 45, 63. **3.** Rs.252 ; Rs.180.
4. Rs.82 : Rs.41. **5.** Rs.140 ; Rs.252. **6.** £73 ; £146.

7. £225 ; £150. **8.** 66, 110, 198.
9. 44, 110, 154 and 198. **10.** 396, 264, 198.
11. 90, 120, 135, 144. **12.** Rs.210 ; Rs.294 ; Rs.378.
13. *£294 ; £378 ; £462 ; £546.* **14.** *£315 ; £189 ; £135 ; £105.*
15. Rs.68. 5 as. 4 p. ; Rs.128. 5 as. 4 p. ; Rs.178. 5 as. 4 p.
16. £72 ; £112 ; £156.
17. £319. 10s. ; £402. 15s. ; £255. 15s.
18. £31. 10s ; £52. 10s ; £67. 10s.
19. £62 ; £31 ; £93. **20.** 136, 204, 136, 204.

Ex. 59. Page 196.

1. $1\frac{17}{19}$ days. **2.** 24 days. **3.** 8 days. **4.** $22\frac{1}{2}$ days.
5. 24 hours. **6.** $12\frac{36}{47}$ days. **7.** $44\frac{4}{13}$ days ; $52\frac{4}{11}$ days ; $115\frac{1}{5}$ days.
8. 11 hrs. 52 mins. 30 secs. **9.** $15\frac{15}{17}$ days.
10. After 10 hrs. ; 250 miles from the former.
11. $19\frac{1}{5}$ hrs. ; $13\frac{1}{3}$ hrs. **12.** $1\frac{1}{4}$ min. past 12. **13.** 11 p.m.
14. At the distance of $39\frac{3}{8}$ miles from the starting-point.
15. 32 miles. **16.** $54\frac{6}{11}$ mins. past 4.
17. $16\frac{8}{19}$ hours. **18.** $16\frac{4}{11}$ mins. past 3.
19. 336 miles. **20.** $32\frac{8}{11}$ mins. past 9.
21. 14 miles from Kalyan ; $11\frac{3}{7}$ hours. **22.** 8 miles.
23. 4 days. **24.** $1\frac{1}{9}$ hours. **25.** 24 mins. past 3.

Ex. 60. Page 200.

1. 3. **2.** $\frac{13}{36}$. **3.** 10·5 m., 21 lbs. **4.** Rs.2. 8 as.
5. 6·15. **6.** 76 p.c. **7.** 3 as. 6 p. **8.** £40. 9s. $4\frac{1}{2}$d.
9. 4 ft. 4 in. **10.** Rs.54. **11.** 2s. $2\frac{1}{2}$d. **12.** $6\frac{1}{4}$ p.c.
13. $12\frac{1}{2}$ p.c. ; 80 p.c. ; 75 p.c. **14.** 15·9235. **15.** 26·48.
16. 2800. **17.** $5\frac{1}{4}$. **18.** 8960. **19.** 450.
20. £2000. **21.** Rs.62. 8 as. **22.** Rs.691. 14 as. **23.** Rs.56. 4 as.
24. Rs.6666. 10 as. 8 p. **25.** 110, 90, 30, 10.

Ex. 61. Page 204.

1. Rs.195. **2.** Rs.14. 10 as. **3.** Rs.27. 6 as. $4\frac{4}{5}$ p.
4. £71. 5s. **5.** £111. 8s. $7\frac{3}{5}$d. **6.** Rs.837.
7. Rs.630. 14as. $2\frac{6}{25}$p. **8.** Rs.396. 10 as. $8\frac{16}{25}$p. **9.** £5593. 1s. $10\frac{1}{2}$d.

10. £769. 4s. 8d. **11.** Rs.18 ; Rs.1843.
12. Rs.3. 12 as. $5\frac{19}{25}$ p. ; Rs.441.12 as. $5\frac{19}{25}$ p. **13.** 5s. ; £182. 15s.
14. £2. 9s. $0\frac{42}{73}$d. ; £513. 19s. $0\frac{42}{73}$d. **15.** £2. 11s. ; £312. 16s.
16. Rs.500. **17.** £250. **18.** 4 p.c. **19.** 5 p.c.
20. 5 years. **21.** $6\frac{1}{4}$ p.c. **22.** $12\frac{6}{7}$ years.
23. £2225. 0s. $9\frac{3}{5}$d. **24.** Rs.6466 as.$4\frac{4}{5}$ p. **25.** £688 ; $6\frac{1}{4}$ p.c.
26. $6\frac{1}{4}$ p.c. **27.** 32 years. **28.** £3100 ; 4 p.c.
29. Rs.820. 4 as. **30.** 3 p.c. ; 6 p.c.

Ex. 62. Page 211.

1. Rs.945. 12 as. **2.** Rs.742. 8 as. 11·52 p.
3. Rs.28. 8 as. 11·52 p. **4.** Rs.127. 6 as. 8·498 p.
5. £92. 5s. 4·5d. **6.** £145. 9s. 4·0125d.
7. £53. 2s. 7·11168d. **8.** Rs.40. 11 as. 7·704... p.
9. £28. 1s. 5·377d. **10.** £142. 16s. 6·106d.
11. Rs.508. 15 as. 4·67264 p. **12.** Rs.18. 10 as. $1\frac{354721}{32000000}$ p.
13. Rs.2431. 0 as. 2·4 p. **14.** £826. 15s. 6·0096d.
15. £1814. 1s. 6·83136d. **16.** Rs.689. 2 as. 6·84288 p.
17. £1053. 3s. 11·7675d. **18.** Rs.1250. **19.** £5000.
20. Rs.125. **21.** £1024. **22.** £36. **23.** £4. 5s. 4·8d.
24. Rs.562. 8 as. **25.** Rs.4. 1 a. 4·46592 p. **26.** Rs.8000.
27. Rs.10. 13 as. 2·257152 p. **28.** £2139. 1s. 10·935d.
29. Rs.2500 ; 5 p.c. **30.** Rs.90. 12 as.

Ex. 63. Page 216.

1. Rs.500. **2.** Rs.600. **3.** Rs.400. **4.** £500.
5. £700. **6.** Rs.575. **7.** £4169. 16s. $3\frac{5}{9}$d.
8. £250. **9.** £91. 5s. **10.** £60. 16s. 8d.
11. Rs.43. **12.** £7. **13.** £6. **14.** £1. 16s. 8d.
15. £2. 8s. **16.** £2. 2s. **17.** £2. 3s. 4d. **18.** £3. 3s.
19. £1. 16s. **20.** £3. 12s. **21.** Rs.5. **22.** £7. 5s.
23. Rs.2. **24.** £7. 18s. **25.** £3. 10s. 6d. **26.** 2s. $3\frac{243}{400}$d.
27. Rs.4. **28.** £1. **29.** 5s. 6d. less. **30.** 1s. $2\frac{2}{5}$d.

Ex. 64. Page 219.

1. 10 p.c. **2.** R.1. 4 as. **3.** R.1. 10 as.
4. 14 as. 3 p. **5.** Rs.333. 5 as. 4 p. **6.** 2 p.c.

7. 10 p.c. 8. Rs.10. 9. 5 as.
10. Rs.24 gain. 11. Rs.2. 7 as. 9 p. gain. 12. £1. 17s. 6d
13. £5. 15s. 6d. 14. Rs.70 ; $28\frac{4}{7}$ p.c. 15. 10 seers a rupee.
16. Rs.4. 17. 20 seers. 18. Rs.110.
19. Rs.21. 4 as. 20. Rs.6. 21. Rs.6
22. Rs.441. 7 as. 23. 4d. 24. $12\frac{1}{2}$ p.c.
25. Rs.619 each horse. 26. Rs.33. 5 as. 4 p. 27. Rs.107. 5 as. 6 p.
28. Rs.9. 6 as. 29. R.1. 8 as. 30. 7 p.c. loss. 31. Rs.29. 4 as.

Ex. 65. Page 226.

1. 493 sq. ft. 2. 114 ft. 3. $99\frac{5}{9}$ sq. yds.
4. 144 sq. yds. 5. 28 yds. 6. $172\frac{4}{5}$ ft.
7. $58\frac{7}{9}$ sq. yds. 8. 350 sq. ft. 9. 630 sq. ft.
10. $355\frac{5}{9}$ ft. 11. Rs.5. 3 as. 4 p.
12. 35 ft. length and 35 ft. breadth. 13. 288. 14. 276 sq. ft.
15. 495 sq. ft. 16. Rs.18. 3 as. $7\frac{1}{9}$ p. 17. 13·426... po.
18. 540. 19. 828 sq. ft. ; 529 sq. ft. ; 529 sq. ft.
20. Rs.219 ; Rs.2312. 21. Rs.4. 6 as. 6 p. 22. 7 ft.
23. Rs.26. 2 as. 24. 338 sq. ft. 25. £15. 16s. 2d.
26. £196. 10s. 27. 77 yds. 2 ft. 11 in
28. 1s. per yard. 29. 84992. 30. £39. 3s. 9d.

Miscellaneous Examples. Page 227.

1. 6430, 3046 ; 32066784. 2. 203. 3. £2726. 6s. $8\frac{1}{4}$d.
4. £672. 0s. $5\frac{1}{2}$d. 5. 199840. 6. 97.
7. 2595321. 8. 146. 9. £131. 2s. 6d.
10. 75 m. ; $257\frac{1}{7}$ revolutions. 11. 876576 ; £913. 2s.
12. 11, Rs.54. 9 as. 9 p. 13. £2. 12s. 6d. 14. 4. 15. $\frac{1}{64}$; ·015
16. 123, 132 ft.; 321 sq. ft. 17. 45 gals. 18. $\frac{27}{220}$; 594 sq. yds
19. $\cdot 005\dot{3}$. 20. 50 sq. chains. 21. 289.
22. £13. 7s. $4\frac{7}{8}$d. 23. $\frac{157}{783}$. 24. £3. 6s. $4\frac{1}{4}$d. ; ·298828125
25. £200. 26. The 1st and the 3rd. ; 111, Rs.6. 15 as
27. $\frac{1}{1100}$; $\cdot 00\dot{0}\dot{9}$. 28. 2^2, 3^2, 7, 11, 13.
29. 296 ; $\frac{1}{4}$d. 30. $\frac{124}{165}$. 31. 18. 32. 21.
33. $\frac{27}{1331}$. 34. 1·05 ; $\cdot\dot{9}5238\dot{0}$. 35. 5 hours
36. 6789. 37. 80000. 38. 144
39. $16992553\frac{11}{13}$ secs. 40. 30

41. 14 dys. 7 hrs. 11 mins. 17 secs. ; 2674 dys. 0 hrs. 9 mins. 59 secs.

42. 25. **43.** 19s. $2\frac{43}{50}$d. **44.** 30 hrs. **45.** £1. 16s.

46. 4321. **47.** $\frac{213}{215}$. **48.** ·0266545. **49.** ·3125.

50. 12 P.M. midnight. **51.** £$\frac{1}{19}$; $\frac{1}{380}$s. **52.** 2 ft. 8 in.

53. ·002 po. **54.** $\frac{2}{3}$. **55.** Rs.720. **56.** $\frac{136}{627}$.

57. ·00018. **58.** 17. **59.** At 3 P.M. December 3rd.

60. £91. 17s. 9d. ; 9d. **61.** £133. 18s. $4\frac{7}{20}$d. **62.** £1340. 1s. $10\frac{763}{800}$d.

63. A, £39. 7s. $7\frac{1}{2}$d. ; B, £65. 12s. $8\frac{1}{2}$d. ; C, £105. 0s. 4d. ; D, £118. 2s. $10\frac{1}{2}$d.

64. Rs.1087. 8 as. $11\frac{13}{25}$ p. **65.** 1728. **66.** 10. **67.** $\frac{1}{22}$.

68. £1303. 6s. 8d. **69.** £31. 17s. $10\frac{1}{2}$d.

70. £90277. 15s. $6\frac{2}{3}$d. **71.** 1263.

72. 30030 seconds after ; 15015, 10010, 6006, 2002, 1430, 462, 390 times respectively.

73. Each man, £3. 10s. ; each woman, £2. 12s. 6d. ; each boy, £1. 15s.

74. $16\frac{2}{3}$ days. **75.** 1 min. $52\frac{1}{2}$ secs. **76.** 1 seer.

77. 16 lbs. ; 24 lbs. ; 32 lbs. ; 40 lbs.

78. Rs.5. 10 as. **79.** £105. 8s. **80.** £1148.

81. $\frac{3}{20}$. **82.** 3. **83.** 35 ; 25. **84.** 9 P.M.

85. 10 p.c. gain. **86.** Each side, 20 ft. **87.** 1·9725 ; 3·0775.

88. 360 hrs. **89.** 50 p.c. **90.** £1000.

91. $2^2 \times 7 \times 13$; $2^3 \times 3^2 \times 5 \times 7$; $3^2 \times 5 \times 11^2$.

92. 1 ; $326\frac{2}{5}$. **93.** £3200.

94. The first, Rs.5184 ; the second, Rs.2592 ; and the third, Rs.1728.

95. 5s. 3d. **96.** 5 p.c.

97. The younger £16416, and the elder £19872. The younger would have got £216 more, and the elder £216 less.

98. 3036 yards. **99.** 156 articles ; 3s. 3d. **100.** £425. 12s. 3·08d.

101. 7. **102.** $\frac{33}{67}$. **103.** 48 hrs. **104.** $18{\cdot}73958\dot{3}$ tons.

105. 1 p. **106.** 17 qrs. $3\frac{1}{2}$ pks. **107.** 1902.

108. £991. 13s. 4d. **109.** $\frac{121}{123}$. **110.** 66, 44, 22

111. 31 years. **112.** 942. **113.** 5) 47267

114. $154\frac{2}{7}$ ft. ; 63, 64. **115.** 2s. $7\frac{1}{3}$d. 7) 9453 rem.2.

116. 40. **117.** 300. **118.** Rs.312. 8 as. ; 4 p.c. 1350 rem. 3.

119. 8·76. **120.** Rs.283. 3 as. $2\frac{2}{5}$ p. **121.** (1) 1, (2) ·125.

122. $\frac{2}{7}$. **123.** £213. 6s. 8d. **124.** $116{\cdot}\dot{6}$.

125. $\frac{1}{18}$. **126.** £1000. **127.** Rs.9. 9 as. 4 p.

128. (*a*) 8513·0157 ; (*b*) 1·5367 **129.** 27 ft. 9 in.

130. Rs.23. 12 as. 3 p. ; 1 a. 1 p. **131.** $11\frac{9}{41}$ in. **132.** 1869

133. 5 hrs. $37\frac{1}{2}$ mins. ; 4 hrs. 30 mins. ; 3 hrs. 45 mins.

134. 10 hrs. 52 mins. 48 secs.

135. £76. 13s. 4d. ; £16. 13s. 4d. ; £26. 13s. 4d. ; £33. 6s. 8d.

136. 1s. $4\frac{58}{125}$d. **137.** 30 p.c. **138.** 35 yds. ; $1\frac{1}{4}$ yds.

139. 199. **140.** 6.10 P.M. ; $163\frac{1}{3}$ miles. **141.** 4664.

142. $\frac{308}{729}$. **143.** £3. 13s. $8\frac{1}{2}$d. **144.** 4·92, 55·9, 27·5028.

145. 7 ft. **146.** £20000. **147.** £315. **148.** £84.

149. 11·40 A.M., at 30 miles from Mysore. **150.** £1. 3s., 64 yds. ; £12.

Appendix. Page 252.

1. (1) 5000; 70; 75000; 400. (2) 426; 6020. (3) 385480; 88895.

2. (1) 300 ; 8000000 ; 3000 ; 400000 ; 90 ; 60000.
(2) 742000 ; 20508. (3) 6020400 ; 500079.

3. (1) ·05 ; ·7 ; ·003 ; 8·3 ; 7·25. (2) ·421 ; 3·895.
(3) ·03 ; ·0004 ; ·2 ; ·00006 ; ·004375.

4. (1) 25000 ; 3870 ; 700. (2) 235 ; 825825.
(3) ·6 ; ·008 ; ·0007 ; ·034 ; 28·1 ; 7·348.
(4) 30·3 ; 500·04 ; 8000·007.

5. 3485 ; 2480. **6.** 16 Kg. 2 Hg. 6 Dg. 4 g.

7. 2 Km. 5 Hm. 4 Dm. 8 m. **8.** 4 Kg. 7 Dg. 2 dg. 1 mg.

9. 24 Km. 8 Dm. 1 m. 9 dm. 8 cm. 7 mm.

10. 29 Km. 5 Hm. 9 Dm. 5 m. 4 dm. 9 cm. 5 mm. **11.** 30480.

12. 3 Dm. 3 dm. 8 cm. 4 mm. ; 9 Dm. 6 m. 2 dm. 1 cm. 6 mm. ; and 8 Hm. 1 Dm. 2 dm. 4 cm.

13. 2 Km. 1 Hm. 6 Dm. 1 m. 2 dm. 5 cm.

14. 1 Km. 2 Hm. 1 Dm. 4 cm.; 7 Hm. 5 Dm. 6 m. 2 dm. 7 cm. 3 mm.; and 3 Dm. 2 dm. 5 cm.

15. (1) 30720 ; 2282400 ; 406080. (2) 844000 ; 2·09 ; 1280.

16. 5000000. **17.** 230·83050 Hg. ; 230830·5 dg. ; 23083050 mg.

18. ·38465 ; ·708004 ; 810306·75. **19.** 43285·42 ; 948. 20. $\frac{71}{800000}$.

21. 700 miles. 22. 7·92. **23.** 10 m. of 32 po. 1 yd. 2 ft. 4 in

24. 149225. **25.** 3 tons 7 cwt. 1 qr. 4·032 lbs. 26. 3s. 9d.

27. 1·64 ft. nearly. 28. 65·625 Km. 29. £11. 8s. 4·896d

30. £691. **31.** 49·33. **32.** 30·882. **33.** 157·628.

34. ·1255. **35.** 628·715. **36.** ·0076. **37.** 4·40509.

38. ·80780. **39.** ·999999. **40.** 4. **41.** 6·19.

42. ·95424. **43.** ·1267667. **44.** 2115·1921. **45.** 9·3012

46. ·00428376. 47. ·3535534. 48. 2·7320508. 49. ·82225443.
50. 261. 51. £25. 0s. 7d. 52. Rs.868. 3 as. 6 p.
53. £106. 13s. 10d. 54. £474. 10s. 5d. 55. £7. 14s. 1d.
56. 2·23606. 57. 2·8284. 58. 11·180339. 59. 14·491.
60. 54·873. 61. £162. 9s. 4d. 62. £26. 8s.
63. Rs.3520. 64. Rs.2500. 65. £1 = 25·92 francs.
66. £41. 5s. 67. Rs.3. 2 as. 68. 457 taels 500 cash.
69. Rs.7080. 70. 900 yen. 71. 330. 72. Rs.75.
73. 465 francs. 74. 370·25. 75. Rs.8030. 5 pies nearly
76. Rs.640. 77. Rs.1040. 78. £1. 1s. loss.
79. Rs.17280. 80. £40.